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THEORY

OF

TRANSVERSE STRAINS

AND ITS APPLICATION

IN THE

CONSTRUCTION OF BUILDINGS.

INCLUDING A FULL DISCUSSION OF THE THEORY AND CONSTRUCTION OF FLOOR BEAMS, GIRDERS, HEADERS, CARRIAGE BEAMS, BRIDGING, ROLLED-IRON BEAMS, TUBULAR IRON GIRDERS, CAST-IRON GIRDERS, FRAMED GIRDERS, AND ROOF TRUSSES; WITH

TABLES,

Calculated and prepared expressly for this Work,

OF THE DIMENSIONS OF FLOOR BEAMS, HEADERS AND ROLLED-IRON BEAMS; AND TABLES SHOWING RESULTS OF ORIGINAL EXPERIMENTS ON THE TENSILE, TRANSVERSE, AND COMPRESSIVE STRENGTHS OF AMERICAN WOODS.

BY

R. G. HATFIELD, ARCHITECT.

FELLOW AM. INST. OF ARCHITECTS; MEM. AM. SOC. OF CIVIL ENGINEERS;
AUTHOR OF "AMERICAN HOUSE CARPENTER."

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PREFACE.

This work is intended for architects and students of architecture.

Within the last ten years, many books have been written upon the mathematics of construction. Among them are several of particular excellence. Few, however, are of a character adapted to the specific wants of the architect. The subject is treated, by some, in the abstract, and in a manner so diffuse and general as to be useful only to instructors. In other works, where a practical application is made, the wants of the civil engineer rather than of the architect are consulted. Writers of scientific books, as well as the public at large, have failed to appreciate the wants of the architect. Indeed, many architects are content to forego a knowledge of construction; following precedent as far as precedent will lead, and, for the rest, trusting to the chances of mere guess-work. For such, all scientific works are alike useless; but there is a class of architects who, through a faulty system of education, have failed to obtain, while students, the knowledge they need; and who now have little time and less inclination to apply themselves to abstract or inappropriate works, although feeling keenly the need of some knowledge which will help them in their daily duties.

For this class, and for students in architecture, this book is written. In fitting it for its purpose, the course adopted has been to present an idea at first in concrete form, and then to lead the mind gradually to the abstract truth or first principles upon which the idea is based. This method, or the manner in which it is executed, may not meet the approval of all. Nevertheless, it is hoped that those for whom the work is written may, by its help, acquire the knowledge they need, and be enabled to solve readily the problems arising in their professional practice.

To adapt the work to the attainments of younger students, the attempt has been made to present the ideas, especially in the first chapters, in a simple manner, elaborating them to a greater extent than is usual.

The graphical method of illustration has been employed largely, and by its help some of the more abstruse parts of the science of construction, it is thought, have been made plain. Results obtained by this method have been analyzed and shown to accord with the analytical formulas heretofore employed. In a discussion of the relation between strength and stiffness, a method has been developed for determining the factor of safety in the rules for strength. Rules for carriage beams with two and three headers are given. The subject of bridging has been discussed, and the value of this system of stiffening floors defined.

Especial attention has been given to the chapters on tubular iron girders, rolled-iron beams, framed girders and roofs; and these chapters, it is hoped, will be particularly acceptable to architects.

The rules for the various timbers of floors, trussed girders, and roof trusses, are all accompanied by practical examples worked out in detail. Tables are given containing the dimensions of floor beams and headers for all floors. These tables are in two classes; one for dwellings and assembly rooms, the other for first-class stores; and give dimensions for beams of Georgia pine, spruce, white pine and hemlock, and for rolled-iron beams.

Immediately following the tables will be found a directory, or digest, by which the more important formulas are so classified that the proper one for any particular use may be discerned at a glance.

The occurrence recently of conflagrations, resulting in serious loss of life, has shown the necessity of using every expedient calculated to render at least our public buildings less liable to destruction by fire. To this end it is proposed to construct timber floors solid, laying the beams in contact, so as to close the usual spaces between the beams, and thus prevent the passage of air, and thereby retard the flames. The strength of these solid floors has been discussed in Article 702, and a rule been obtained for the depth of beam or thickness of floor. By this rule the depths for floors of various spans have been computed, and the results recorded in table XXI.

Tables XXIII. to XLVI. contain a record of experiments made, expressly for this work, upon six of our American woods. In these experiments and in computations, the author has been assisted by his son, Mr. R. F. Hatfield.

In the preparation of the work, he has had recourse to the works of numerous writers on the strength of materials, to whom he is under obligation, and here makes his acknowledgments. The following are the works which were more particularly consulted:—

Baker on Beams, Columns, and Arches.
Barlow on Materials and on Construction.
Bow on Bracing.
Bow's Economics of Construction.
Campin on Iron Roofs.
Cargill's Strains upon Bridge Girders and Roof Trusses.
Clark on the Britannia and Conway Tubular Bridges.
Emerson's Principles of Mechanics.
Fairbairn on Cast and Wrought Iron.
Fenwick on the Mechanics of Construction.
Francis on the Strength of Cast-Iron Pillars.
Haswell's Engineers' and Mechanics' Pocket-Book.
Haupt on Bridge Construction.
Hodgkinson's Tredgold on the Strength of Cast-Iron.
Humber on Strains in Girders.
Hurst's Tredgold on Carpentry.
Kirkaldy's Experiments on Wrought-Iron and Steel.
Mahan's Civil Engineering.
Mahan's Moseley's Engineering and Architecture.
Moseley's Engineering and Architecture.
Poisson's Traité de Mécanique.
Ranken on Strains in Trusses.
Rankine's Applied Mechanics.
Robison's Mechanical Philosophy.
Rondelet sur le Dôme du Panthéon Français.
Sheilds' Strains on Structures of Ironwork.
Styffe on Iron and Steel.
Tarn on the Science of Building.
Tate on the Strength of Materials.
Tredgold's Carpentry.
Unwin on Iron Bridges and Roofs.
Weisbach's Mechanics and Engineering.
Wood on the Resistance of Materials.

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INTRODUCTION.

ART. 1.—The science of Construction, as the term is used in architecture, comprehends a knowledge of the forces tending to destroy the materials constituting a building, and of the capacities of resistance of the materials to these forces.

2.—One of the requisites of good architecture is Stability. Without this the beautiful designs of the architect can have no lasting existence beyond the paper upon which they are delineated.

3.—The force of Gravity is inherent not only in the contents of a building, but also in the materials of which the building itself is constructed; and unless these materials have an adequate power of resistance to this force, the safety of the building is endangered. Hence the necessity of a knowledge of the laws governing the force of gravity in its action upon the several parts of a building, and of the expedients to be resorted to in order to resist its action effectually.

4.—It may be objected by some that this knowledge pertains rather to building than to architecture, and that the architect is required merely to indicate the outlines of his plans, leaving to the builder the work of determining the arrangement and dimensions of the materials. This objection is not well founded. Between the duties of

the architect and those of the builder there is a well-defined line. This may be shown by a consideration of the operation of building as it is usually conducted. The builder is selected generally from among those who compete for the work. Each builder competing fixes the amount for which he is willing to erect the building, after an examination of the plans and specifications and an estimate of the cost of the work. To arrive at this cost the arrangement and dimensions of the materials must be fixed; and if not fixed by the plans and specifications, in what way shall they be determined? Shall it be by the builder? The builder has not yet been selected. Shall each builder estimating be permitted to assign such dimensions as his caprice or cupidity shall dictate? The evil effect of such a course is apparent. The only proper method is to have the arrangement and dimensions of the materials all definitely settled by the architect in his plans and specifications.

Moreover, the necessity for a knowledge of this subject by the architect is manifest in this, that he is constantly liable, without this knowledge, to include in his plans such features as the action of gravity would render impossible of production in solid material, or which, if executed, would not possess the requisite degree of stability.

5.—In considering the requisites for stability in a building, the various parts need to be taken in detail: such as Walls, Piers, Columns, Buttresses, Foundations, Arches, Lintels, Floors, Partitions, Posts, Girders and Roofs.

6.—It is the purpose of the present work to treat principally of those parts which are subjected to transverse strains.

7.—In the construction of a floor, the safety of those who are to trust themselves upon it is the first consideration.

8.—Floors are not always made sufficiently strong. Scarcely a year passes without its record of deaths consequent upon the failure of floors upon which people should have assembled with safety. Many floors now existing, and not a few of those annually constructed, are deficient in material, or have an improper arrangement of it.

9.—The strength of a floor consists in the strength of its timbers.

The dimensions of the timbers for any given floor may be ascertained, practically, by an examination of other similar floors which have been tried and found sufficiently strong. But if no *similar* floor is found, how is the problem to be solved?

10.—The amount of material required may be found by constructing one or more experimental floors, and testing them with proper weights; but this would be attended with great expense, and probably with the loss of more time than could be spared for the purpose.

11.—There is a simple method, which is quite as certain and less expensive. The chemist, from a small specimen, makes an analysis sufficient to determine the character of whole mines of ore or quarries of rock. So we, by proper tests of a small piece of any building material, may determine the characteristics of all material of that kind.

12.—To obtain, then, the requisite knowledge of the strength of floor timbers, let us adopt a piece of convenient size as the unit of material. Let it be a piece one inch square and one foot long in the clear between the bearings. This we will submit to a transverse force, applied at the middle of its length, sufficient to break it crosswise, and learn from the result the power of resistance it possesses.

Numerous experiments of this nature have been made

upon all the ordinary kinds of timber, stone and iron, and the average results collected in tabular form. (See Table XX.) A few results are here given.

13.—The unit of material, when of

Hemlock,	breaks	with	450	pounds:
White Pine,	"	"	500	"
Spruce,	"	"	550	"
White Oak,	"	"	650	"
Georgia Pine,	"	"	850	"
Locust,	"	"	1200	"
Cast-Iron	"	"	2100	"

These figures give the average unit of strength for these several kinds of material, when exposed to a transverse strain at the middle of their length.

CHAPTER I.

THE LAW OF RESISTANCE.

ART. **14.—Relation between Size and Strength.**—Having ascertained, by careful experiment, the power of resistance in a unit of any given material, the next question is: What is the existing relation between size and strength? Is the increase of one proportionate to that of the other?

In two square beams of equal length, but of different sectional area, the larger one will bear more than the smaller. From this it appears that the resistance *is*, to a certain degree at least, in proportion to the quantity of material, or to the area of cross-section. There is an element of strength, however, other than this, and one which modifies the proportion very materially.

15.—Strength not always in Proportion to Area of Cross-section.—That the strength of any two pieces of equal length is not always in proportion to the area of cross-section, is shown by attempts to break two given pieces. For example, take two beams of equal length, but of differing area of cross-section; the one being 3 × 8, and the other 5 × 6 inches. The former has 24 and the latter 30 inches of sectional area. If the strength be in proportion to the sectional area, the weights required to break these two pieces will be in the proportion of 24 to 30—their relative areas of cross-section; but they will be found (the pieces being placed upon edge) to be in the proportion of 24 to 22½; the smaller piece being actually stronger than the larger!

16.—Resistance in Proportion to Area of Cross-section. —Preliminary to seeking the cause of this apparent want of proportion, it will be well to show first that, under certain conditions, the resistance of beams *is* directly proportional to their area of cross-section.

Let there be twenty pieces of smooth white pine, each one inch square, and one foot long in the clear between the bearings. The resistance of any one of these pieces is limited to 500 pounds. This has been ascertained by experiment as before stated in *Art.* **13.**

Let four of these pieces be placed side by side upon the bearings. The resistance of the four is evidently just four times the resistance of one piece; or $4 \times 500 = 2000$ pounds.

Let four more pieces be placed upon the first four: the strength of the eight amounts to $2 \times 2000 = 4000$ pounds.

Add four more, and the combined resistance of the twelve pieces will be $3 \times 2000 = 6000$ pounds.

The resistance of four tiers of four each, or of sixteen pieces, will be $16 \times 500 = 8000$ pounds.

The total strength of the twenty pieces, piled up five tiers high, will be $20 \times 500 = 10{,}000$ pounds.

Thus we see that the resistance is exactly in proportion to the amount of material used.*

17.—Units may be Taken of any Given Dimensions.— In this trial we have taken as the unit of material a bar one inch square. We might have taken this unit of any other dimension, as a half, a quarter, or even a tenth of an inch square, and, after finding by trial the strength of one of

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* The truth of this proposition depends upon obtaining, in the experiment, pieces of wood so smooth that, in being deflected by the weight, they will move upon each other without friction; a condition not quite possible in practice to obtain. This friction restrains free action, and, as a consequence, the weight required to effect the rupture will be somewhat greater than is stated.

these units, could have as readily known the strength of the whole pile by merely multiplying the number of units by the strength of one of them.

We will now consider the relation between breadth and depth.

18.—Experience Shows a Beam Stronger when Set on Edge.—One of the first lessons of experience with timber of greater breadth than thickness, is the fact of its possessing greater strength when placed on edge than when laid on the flat. As an example: a beam of white pine, 3 × 8 inches, and 10 feet long between bearings, will require 9600 pounds to break it when set on edge; while three eighths of this amount, or 3600 pounds, will break it if it be laid upon the flat. Here again, as in *Art.* **15**, we have a fact seemingly at variance with the one but just previously established—namely, that of the resistance being in proportion to the area of cross-section. We will now investigate the apparent anomaly.

19.—Strength Directly in Proportion to Breadth.—First, as to the breadth of a beam. If two beams of like size are placed side by side, the two will resist just twice that which one of them alone would. Three beams will resist three times as much as one beam would. So of any number of beams, the resistance will be in proportion directly as the breadth.

This is found by trial to be true, whether the beams are separate or together, solid; for a 6 × 8 inch beam will bear as much, and only as much, as three beams 2 × 8 inches set side by side, and, in both cases, on the edge. In other words, when the depths and lengths are equal, a beam of six

inches breadth will bear just three times as much as a beam of two inches breadth, or twice as much as one of three inches breadth.

So this fact appears established, that the resistance of beams is directly in proportion to their breadth.

20.—By Experiment Strength Increases more Rapidly than the Depth.—In regarding the depth of beams, another law of proportion is found. Having two beams of the same breadth and length, but differing in depth, we find the strength greater than in proportion to the depth. If it were in this proportion, a beam nine inches high would bear just three times as much as one three inches high, whereas experiment shows it to bear much more than this.

21.—Comparison of a Solid Beam with a Laminated one.—To test this, let there be two beams of equal length, breadth and depth, one of them being in one solid piece

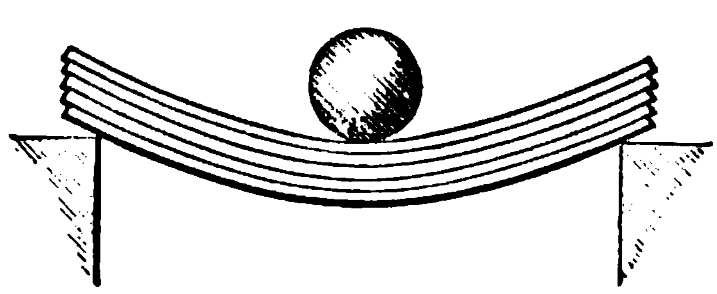

FIG. 1.

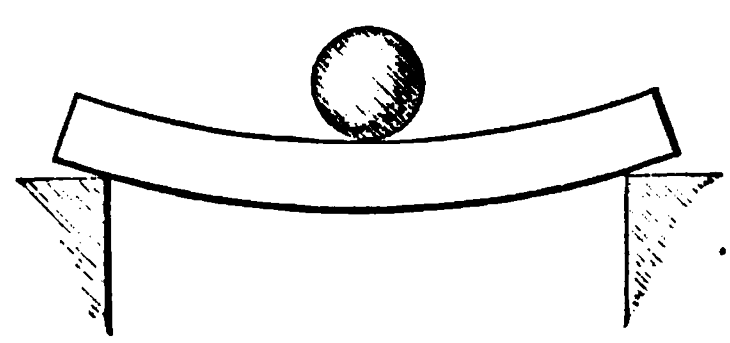

FIG. 2.

(*Fig.* 2), while the other is made up of horizontal layers or veneers, laid together loosely (*Fig.* 1). Placing weights upon these two beams, it is seen that, although they contain a like quantity of material in cross-section, and are of equal height, the solid beam will sustain much more weight than the laminated one. Let the several parts of the latter beam be connected together by glue, or other cementing material,

and again applying weights, it will be found that it has become nearly, if not quite, as strong as the beam naturally solid.

From these results we infer that the increased strength is due to the union of the fibres at each juncture of the horizontal layers. But why does this result follow? If the simple knitting together of the fibres is the cause, then why, in considering the breadth, is a solid beam no stronger than two beams, each of half the breadth, as has been shown?

22.—Strength due to Resistance of Fibres to Extension and Compression.—An examination of the action of the beams under pressure in *Figs.* 1 and 2 may explain this. The weights bending the beams make them concave on top. In *Fig.* 1 the ends of the veneers or layers remain in vertical planes, while, in the other case, the end of the solid beam is inclined, and normal to the curve. It is also seen that the upper surface in *Fig.* 2 is shorter than the lower one, although the two surfaces were of the same length before bending. This change in length has occurred during the process of bending, and could only happen through a change in length of the fibres constituting the beam.

In the operation of bending, one of two things must of necessity take place: either the fibres must slide upon each other, as in *Fig.* 1, or else the length of the fibres must be changed, as in *Fig.* 2; and since in practice it is found that the fibres are so firmly knit as effectually to prevent sliding, we have only to consider the effects of a change in the length of the fibres. The resistance to this change is an element of strength other than that due to quantity of material, and its nature will now be examined.

23.—Power Extending Fibres in Proportion to Depth of Beam.—If a beam be made of four equal pieces, as in *Fig.* 3, and be held together by an elastic strap firmly attached to the under side of the beam, and by two cross pieces let into the horizontal joint and closely fitted; and if upon this beam a weight be laid at the middle sufficient to elongate the strap and open the vertical joint at the bottom a given distance—say an eighth of an inch; then, if the weight and the two upper quarters of the beam be removed, and a weight laid on at the middle sufficient to open the joint to the like distance as before, it will be found that this weight is just one half of that before used. In this experiment, the strap may be taken to represent the fibres at the lower edge of the beam.

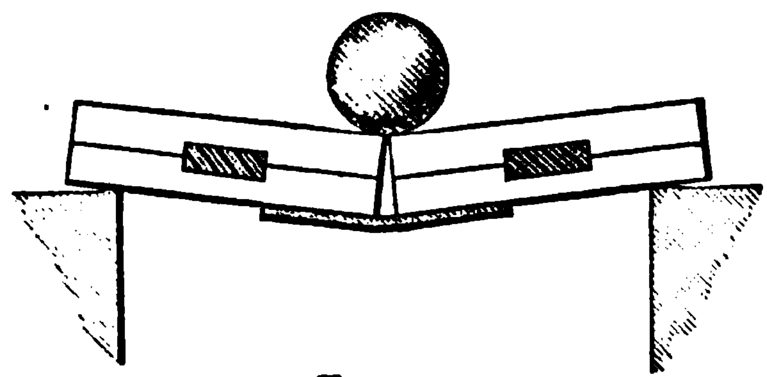

FIG. 3.

We here find a relation between the weight and the height of the beam. The greater the height, the greater must be the weight to produce a like effect upon the fibres of the lower edge. Double the height requires double the weight. Three times the height requires three times the weight. Therefore we decide that, in elongating the fibres at the bottom, the weight and the height are directly in proportion.

It must be observed that *Fig.* 3 and its explanation are not to be taken as a representation of the full effect of a transverse strain upon a beam. The scope of the experiment is limited to the action of the fibres at the lower edge. The other fibres, all contributing more or less to the resistance, are, for the moment, neglected, in order to show this one feature of the strain—namely, the manner in which fibres at any point contribute to the general resistance.

Galileo, of Italy, who, two hundred and fifty years since,

was the first to show the connection of the theory of transverse strains with mathematics, not recognizing in his theory the compressibility of the fibres at the concave side of the beam, supposed that in a rupture by cross strain *all* the fibres were separated by pulling apart; as might be shown in *Fig.* 3, in case the rubber were extended up each side to the top, instead of being confined to the lower edge. We are greatly indebted to Galileo for his studies in this direction; but Hooke, Mariotte, and Leibnitz, about 1680, found the theory of Galileo to be defective, and showed that the fibres were elastic; that only those fibres at the convex side of the beam suffer extension; that those at the concave side suffer compression and are shortened; and that, at the line separating the fibres which are extended from those which are compressed, they are neither lengthened nor shortened, but remain at their natural length. This line is denominated the *neutral* line or surface.

It will here be observed that the *amount* of extension or compression in any fibre is proportional to its distance from the neutral line.

CHAPTER II.

APPLICATION OF THE LEVER PRINCIPLE.

ART. **24.—The Law of the Lever.**—The deduction drawn from the experiment named in *Art.* **23** depends for its truth upon what is known as the law of the lever. This law, in so far as it applies to transverse strains, will now be considered.

25.—Equilibrium—Direction of Pressures.—When equal weights, suspended from the ends of a beam supported upon a fulcrum, as at *W* in *Fig.* 4, are in equilibrium, it is found that the point of support is just midway between the two weights, provided that the beam be of equal size and weight throughout its length.

It will be observed that the directions of the strains upon the beam are vertical, those at the ends being downward, while that at the middle is upward; also that the strains are evidently equal, the upward pressure at the middle being just equal to the sum of the two weights at the ends; for if unequal, there would be no equilibrium, but a movement in the direction of the greater power.

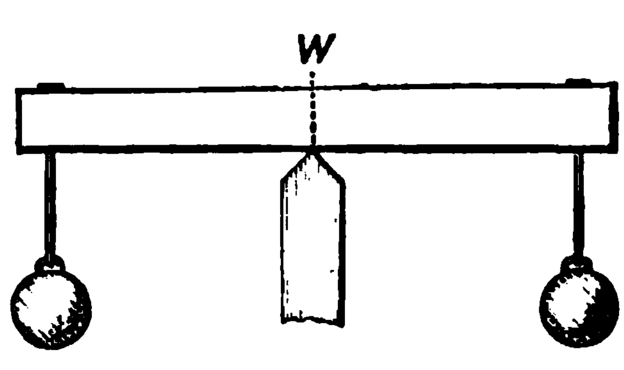

FIG. 4.

We decide, then, that the pressure upon the fulcrum is equal to the sum of the two weights.*

* In ascertaining the pressure at the fulcrum, the weight of the beam itself should be added to the sum of the two weights, but to simplify the question, the beam, or lever, is supposed to be without weight.

26.—Conditions of Pressure in a Loaded Beam.—In *Fig.* 5 we have a beam supported at each end, and a weight W laid upon the middle of its length.

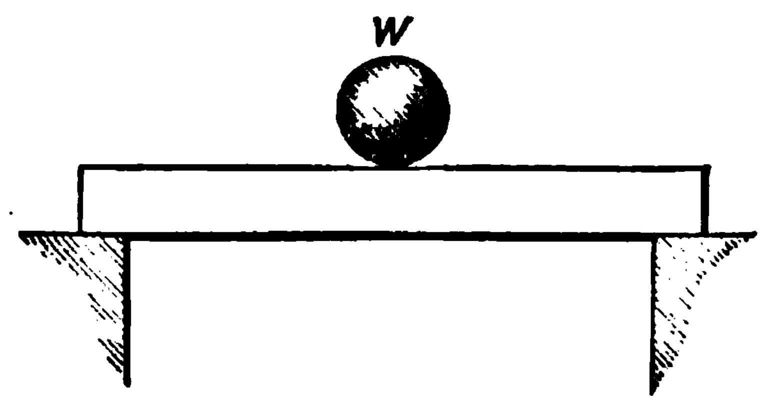

FIG. 5.

Comparing this with *Fig.* 4 we see that the strains here are also vertical but in reversed order, the one at the middle being downwards, while those at the ends are upwards. In other respects we have here the same conditions as in *Fig.* 4.

The downward pressure at the middle is equal to the upward pressures or reactions at the ends; and, since the weight is placed midway between the points of support, the reactions at these points are equal, and each is equal to one half the weight at the middle.

27.—The Principle of the Lever.—In *Fig.* 6 is shown a lever resting upon a fulcrum W, and carrying at its ends the weights R and P.

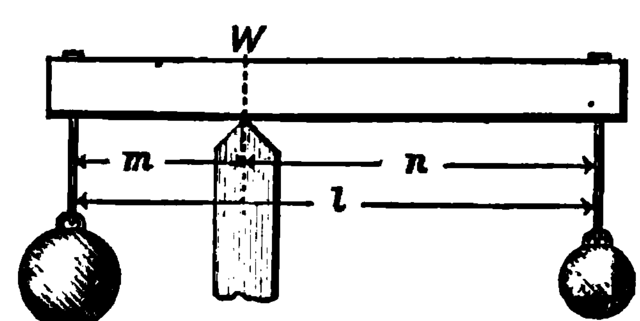

FIG. 6.

Here, the fulcrum W is not at the middle as in *Fig.* 4, but at a point which divides the lever into two unequal parts, m and n.

In accordance with the principle of the lever, the two parts m and n, when there is an equilibrium, are in proportion to the two weights P and R; or, the shorter arm is to the longer as the lesser weight is to the greater;* or,

$$m : n :: P : R$$

* For a demonstration of the lever principle see an article, by the author, in the *Mathematical Monthly*, published at Cambridge, U. S., vol. 1, 1858, page 77.

from which we have

$$Rm = Pn$$

$$R = P\frac{n}{m} \qquad (1.)$$

and

$$P = R\frac{m}{n} \qquad (2.)$$

As an example: suppose the lever to be 12 feet long, and so placed upon the fulcrum as to make the two arms, m and n, 4 and 8 feet respectively. Then, if the shorter arm have suspended from its end a weight, R, of 500 pounds, what weight, P, will be required at the end of the longer arm to produce equilibrium?

Formula (2.) is appropriate to this case. Therefore $P = R\frac{m}{n} = 500 \times \frac{4}{8} = 250$ pounds; equals the weight required on the longer arm.

From *Art.* **25** it is evident that the sum of the weights R and P is equal to the upward force or reaction at W.

Therefore, we have,

$$W = R + P$$

and

$$W - R = P$$

Substituting this value for P in formula (2.), we have

$$W - R = R\frac{m}{n}$$

$$W = R + R\frac{m}{n}$$

$$W = R\left(1 + \frac{m}{n}\right)$$

$$\frac{W}{1 + \frac{m}{n}} = R$$ and, multiplying by n,

$$\frac{Wn}{n + m} = R$$

and, since $n + m$ is equal to the whole length of the beam, or to l, therefore

$$R = W\frac{n}{l} \qquad (3.)$$

In a similar manner, it is found that

$$P = W\frac{m}{l} \qquad (4.)$$

28.—A Loaded Beam Supported at Each End.—In *Fig.* 7 a weight W, is carried by a beam resting at its ends upon two supports. Here we have, with the pressures in reversed order, similar conditions with those shown in *Fig.* 6. Here, also, it will be observed that the weight W is equal to the sum of the upward resistances R and P (*Arts.* **25** and **26**)—neglecting for the present the weight of the beam itself—and that the upward resistance at R may be found by formula (*3.*); while that at P is found by formula (*4.*).

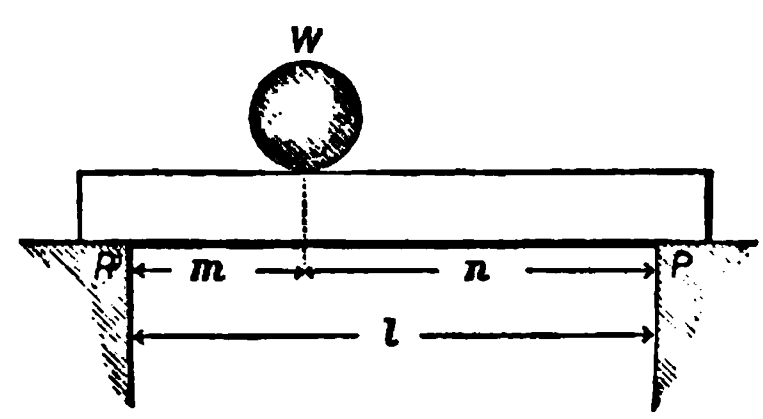

FIG. 7.

For example: suppose the weight W, *Fig.* 7, to be 800 pounds; and that it be located five feet from one end of the beam and eight feet from the other end.

Here $W = 800$, $m = 5$, $n = 8$ and $l = 13$.

To find the pressure at R, we have, by formula (*3.*),

$$R = W\frac{n}{l} = 800 \times \frac{8}{13} = 492\tfrac{4}{13} \text{ pounds.}$$

To find the pressure at P, we have, by formula (*4.*),

$$P = W\frac{m}{l} = 800 \times \frac{5}{13} = 307\tfrac{9}{13} \text{ pounds.}$$

To verify the rule, we find that

$$492\tfrac{4}{13} + 307\tfrac{9}{13} = 800 \text{ pounds} = W.$$

Either one of these upward pressures, or reactions, being found, the other may be determined by subtracting the first from *W*.

From the above, we see that the portion of a weight borne by one support is equal to the product of the weight into its distance from the other support, divided by the length between the two supports.

29.—A Bent Lever.—In *Fig.* 8 let *P C G* be a rigid bar, shaped to a right angle at *C*, and free to revolve on *C* as a centre. Let *R* and *H* be two weights attached by cords to the points *P* and *G*, the cords passing over the pulleys *D* and *E*. Let the weights be so proportioned as to produce an equilibrium.

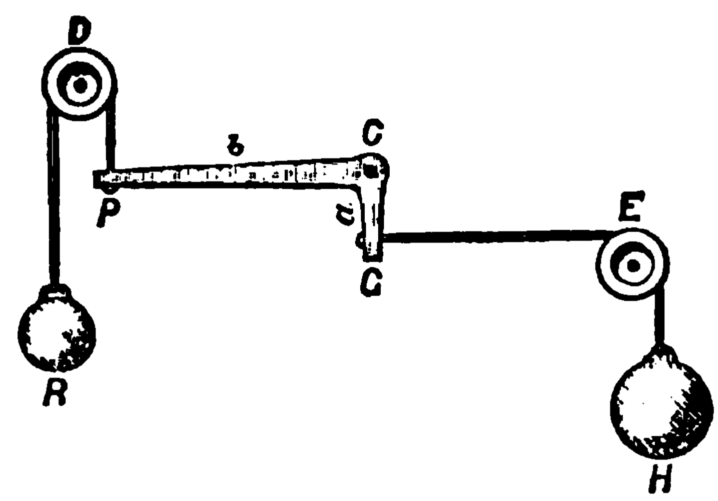

FIG. 8.

Here *P C G* is what is termed a bent lever, and the arms *a* and *b* are in proportion to the weights *R* and *H*; or,

$$a : b :: R : H \qquad \text{and}$$

$$H = R \frac{b}{a}$$

30.—Horizontal Strains Illustrated by the Bent Lever.—To apply the principle of the bent lever let a beam *R E* (*Fig.* 9) be laid upon two points of support, *R* and *E*, and be loaded at the middle with the weight *W*. The action of this weight upon the beam is similar in its effect to that taking place in the bent lever of *Fig.* 8, producing horizontal strains, which compress the fibres at the top of the beam and extend those at the bottom. (*Art.* **23**).

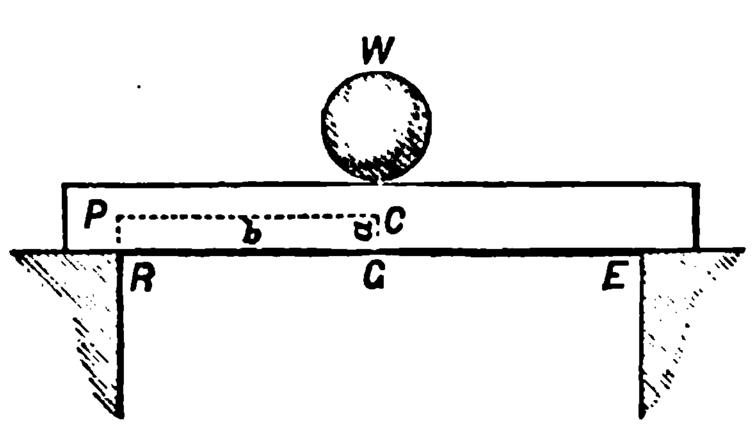

FIG. 9.

Let the line $P C$ represent the line of division between the compressed and the extended fibres. Then $P C G$ may be taken to represent the bent lever of *Fig.* 8; for the upward pressure or reaction at R, moving the arm of lever $P C$, which turns on the point C, as a centre, acts upon the point G, through the arm of lever $C G$, moving the point G horizontally from E, and thus extending the fibres in the line $G E$.

Now, if H represents this horizontal strain along the bottom of the beam, and R the vertical strain at P—both being due to the action of the weight W; if the arm $P C$ be called b, and $C G$ called a, then, as before,

$$a : b :: R : H \qquad \text{from which}$$

$$H = R \frac{b}{a}$$

For an application: let b in a given case equal 10 feet, a equal 6 inches, or 0·5 of a foot, and R equal 1200 pounds; what will be the horizontal strain in the fibres at the lower edge of the beam?

From the above formula,

$$H = R \frac{b}{a} = 1200 \times \frac{10}{0{\cdot}5} = 1200 \times 20 = 24{,}000$$

or the horizontal strain equals 24,000 pounds.

31.—Resistance of Fibres in Proportion to the Depth of Beam.—From the proportion in the last article,

$$a : b :: R : H \qquad \text{we have}$$

$$Ha = Rb \qquad \text{and dividing by } a\,b \text{ we have}$$

$$\frac{H}{b} = \frac{R}{a}$$

For any given material, the power of the fibres to resist tension is limited, and, since this power is represented by H,

therefore H is limited. In any given length of beam, b, which is dependent upon the length, is also given; hence $\frac{H}{b}$ becomes a fixed quantity; and since $\frac{H}{b} = \frac{R}{a}$, therefore $\frac{R}{a}$ is a fixed quantity. But R and a may vary individually, provided that the *quotient* of R divided by a be not changed. So, then, if R be increased, a must also be increased, and in a like proportion; if R be doubled, a must be doubled; if one be trebled, the other must be trebled; or, in whatever proportion one is increased or diminished, the other must be increased or diminished in like proportion. Therefore R and a are in direct proportion.

Take a as equal to one half of the depth of the beam, or $\frac{d}{2}$, and R as equal to one half the weight at the middle of the beam, or $\frac{W}{2}$.

Then, since a is in proportion to R, d is in proportion to W, or the depth of the beam must be in proportion to the weight.

This result is the same as that arrived at in *Art.* **23**; that the power of the fibres at the bottom to resist extension is in proportion to the depth of the beam.

CHAPTER III.

DESTRUCTIVE ENERGY AND RESISTANCE.

ART. 32.—**Resistance to Compression—Neutral Line.**—We have shown the manner in which the fibres at the convex side of a beam contribute to its strength by their resistance to *extension*. It may now be observed that the resistance to *compression* of the fibres at the concave side is but a counterpart of the resistance to *extension* of the fibres at the convex side.

Whatever resistance may be given out in one way at one side of the beam, a like amount of resistance will be called up in the other way at the other side. The one balances the other, like two weights at the ends of a lever (*Figs.* 4 *and* 6). If the powers of resistance to compression and extension be equal, as is the case in some kinds of wood, then one half of the fibres will be compressed while the other half are extended; and, should the beam be of rectangular section, the neutral line will occur at the middle of the height of the beam, and the condition of equilibrium will be as shown in *Fig.* 4.

If the capability to resist compression exceeds the resistance to extension, as in cast-iron, then the greater portion of the fibres will be employed in resisting tension, and the neutral line will be nearer to the concave side; an equilibrium represented by *Fig.* 6, in which the shorter arm of the lever may represent the portion of the fibres subjected to compression, and the longer arm those suffering tension, and where R, the heavier weight, may represent the power of any given number of fibres to resist compression, while P, the lesser

weight, represents the power of an equal number of fibres to resist tension.

In *Art.* **31** the power of the fibres at the convex side of a beam to resist extension was shown to be in proportion to the depth of the beam. This result was obtained by taking the position of the neutral line at the middle of the depth. The like result will be obtained even when the neutral line occurs at a point other than the middle. For, whatever be the proportionate distance of this line from the lower edge, that distance, for the same material, will always bear the same proportion to the depth of the beam.

33.—Elements of Resistance to Rupture.—Having now sufficient data for the purpose, the several elements of strength which have been developed may be brought together, and their sum taken as the total resistance to rupture.

First.—We have the rate of strength, or the weight in pounds required to break a *unit* of the given material one inch square and one foot long, when supported at each end (*Arts.* **12** *and* **13**). Let B represent this weight.

Second.—We have the strength in proportion to the area of cross-section, or to the product of the breadth into the depth (*Arts.* **16** *and* **17**). If b be put to represent the breadth, and d the depth, both in inches, then this element of strength may be represented by $b \times d$ or bd.

Third—and last, we have the strength due to the resistance of the fibres to a change in length, which has been shown to be in proportion to the depth (*Arts.* **22, 23** *and* **31**), and may therefore be represented by d.

Putting these three elements of strength together, and representing by R the total resistance, we have,

$$R = B \times bd \times d \qquad \text{or}$$

$$R = Bbd^2 \qquad (5.)^*$$

and this is the total power of resistance to a cross strain.

34.—Destructive Energies.—It is requisite now to consider the destructive energies. It has been shown (*Art.* **27**) that the power of a weight, acting at the end of a lever, is in proportion to the length of the lever. This is seen in *Fig.* 6, where a small weight acting at the end of the longer arm produces as great an effect as the larger weight upon the shorter arm. This principle may be stated thus: The moment of a weight is equal to the product of the weight into the length of the arm of leverage at which it acts.

If n (*Fig.* 6) be the arm of leverage, and P the weight acting at its end, then the moment of P is equal to the weight P multiplied by the length of the lever n; or,

$$\text{Moment} = Pn.$$

Let S represent the weight which it is found on trial is required to break a lever or rod of given material, one inch square, and projecting one foot from a wall into which it is firmly imbedded; the weight being suspended from the free end of the lever. Then, since the moment equals the weight into its arm of leverage, as above stated, which arm in this case equals unity, we have

$$S \times 1 = Pn$$

* Strictly speaking, the whole power of a beam to resist rupture is due to the resistance of the fibres to compression and extension,—as will be shown in speaking of the resistance to bending—and it is usual to obtain the amount of this power by a more direct method; arriving at the total resistance by one operation, and this based upon a consideration of the resistance offered by each fibre to a change of length, and taking the sum of these resistances; but it is thought that the method here pursued is better adapted to securing the object had in view in writing this work.

or the power of resistance of such a rod equals S, the weight required to break it.

Having this index of strength, S, and knowing (*Art.* **33**) that the resistance to breaking is in proportion to the breadth and the square of the depth, then for levers larger than one inch square, and longer than one foot, when the destructive energy equals the resistance, we have

$$Pn = Sbd^2 \qquad (6.)$$

that is, for the moment, or destructive energy, we have P, the weight in pounds, multiplied by n, the length in feet; and for the resistance, we have S, the index of strength for the sectional area of one inch square, multiplied by the breadth of the lever, and by the square of its depth; the breadth and depth both being in inches.

35.—Rule for Transverse Strength of Beams.—This formula, (*6.*), gives a rule for the transverse strength of levers. From it we may derive a rule for the transverse strength of beams supported at both ends.

We know, for example, from *Arts.* **25** and **26**, that the strains in a lever are the same as in a beam which is twice the length of, and loaded at the middle with twice the weight supported at the end of the lever. Therefore, when P is equal to the half of W, the weight at the middle of a beam (*Fig.* 5), and n is equal to the half of l, the length of the beam, we have

$$Pn = \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4} \qquad \text{and since, } (\textit{form. 6}),$$

$$Pn = Sbd^2 \qquad \text{by substitution we have}$$

$$\frac{Wl}{4} = Sbd^2 \qquad (7.) \qquad \text{or}$$

$$Wl = 4Sbd^2 \qquad (8.)$$

in which Wl equals the moment or destructive energy of a weight at the middle of a beam, and $4Sbd^2$ equals the resist-

ance of the beam. But this resistance was found (*Art.* **33**) to be equal to Bbd^2; therefore,

$$4Sbd^2 = Bbd^2$$

hence $$Wl = Bbd^2 \qquad (9.)$$

This is the required rule for the strength of beams supported at each end. In it W equals the pounds laid on at the middle of the beam, l the length of the beam in feet, b and d the breadth and depth respectively of the beam in inches, and B the weight in pounds at the middle required to break a unit of material (*Art.* **12**) of like kind with that in the beam, when strained in a similar manner.

It may be observed here that from

$$Bbd^2 = 4Sbd^2 \qquad \text{as above, we have}$$
$$B = 4S$$

or, the weight at the middle required to break a unit of material, when supported at each end, is equal to four times the weight required to break it when fixed at one end only, and the weight suspended from the other.*

*** Professor Moseley, in his "Engineering and Architecture," puts S to represent the index of strength, but his definition of this index shows it to be not the same as that for which S is put in this work. While, with us, S represents the resistance to rupture of a unit of material (one inch square and one foot long), fixed at one end and loaded at the other; in his work (*Art.* 408, p. 521, Mahan's Moseley, New York, 1856), S is placed to represent the "*resistance in pounds opposed to the rupture of each square inch at the surface exposed to a tensile strain.*"**

To compare the two, let M be put for the S of Prof. Moseley. Then his expression (*Art.* 414, p. 528) for rectangular beams,

$$P = \frac{1}{6} S \frac{bc^2}{a} \qquad \text{becomes}$$

$$P = M \frac{bc^2}{6a} \qquad \text{in which}$$

P is the weight at one end of a beam, which is fixed at the other end, and c is the depth and a the length, both in inches. If for c we put d and for a we put n, representing feet instead of inches, so that $a = 12\,n$, then

36.—Formulas Derived from this Rule.—From the general formula, (*9.*), of *Art.* **35**, any one of the five quantities named may be found, the other four being given.

$$P = M \frac{bd^2}{72n} \qquad \text{and}$$

$$72\ Pn = Mbd^2$$

Now we have found (*form.* 6), that

$$Pn = Sbd^2$$

Multiplying this by 72 gives

$$72\ Pn = 72 Sbd^2$$

Comparing this value of $72\ Pn$ with that from Prof. Moseley, as above, we have

$$Mbd^2 = 72 Sbd^2$$

from which $M = 72S$

or M, the S of Prof. Moseley, is equal to 72 times the S of this work.

We also find that Prof. Rankine (Applied Mechanics, *Arts.* **294** and **296**) similarly designates the index of strength; or, as he and Prof. M. both term it, "the modulus of rupture." Prof. R. defines it the same as Prof. M.; except, that instead of limiting it to the tensile strain, he applies it equally to that element, tension or compression, which first overcomes the strength of the beam.

Prof. Rankine further defines it (p. 634) to be "*eighteen times the load which is required to break a bar of one inch square, supported at two points one foot apart, and loaded in the middle between the points of support.*" Now the bar here described is identical with the unit of material adopted in this work (*Arts.* **12** and **13**); to designate the strength of which we have used the symbol B. To compare the two, we have, as above found,

$$M = 72S$$

and also, (*Art.* **35**)

$$B = 4S$$

Multiplying the latter equation by 18, we have

$$18B = 72S \qquad \text{or}$$

$$18B = M \qquad \text{or}$$

as defined by Prof. Rankine, M, the S of Prof. Moseley, is equal to 18 times the value of B, the index of strength as used in this work. Hence the values of S, as given for various materials by Profs. Moseley and Rankine, are 18 times the values of B in this work for the same materials. Owing, however, to a considerable variation in materials of the same name, this relation will be found only approximate.

For example,

$$B = \frac{Wl}{bd^2} \qquad (10.)$$

$$b = \frac{Wl}{Bd^2} \qquad (11.)$$

$$d = \sqrt{\frac{Wl}{Bb}} \qquad (12.)$$

$$W = \frac{Bbd^2}{l} \qquad (13.)$$

$$l = \frac{Bbd^2}{W} \qquad (14.)$$

In these formulas B is the *breaking* weight in pounds applied at the *middle.* The value of B (*Arts.* **33** and **35**) is given for the *length* in *feet,* and the *breadth* and *depth* in *inches.*

QUESTIONS FOR PRACTICE.

37.—What kind of strain is a floor beam subjected to?

38.—In a beam subjected to a transverse strain, how does the breadth contribute to its strength?

39.—How does the depth contribute to its strength?

40.—What are the elements of resistance, and what is the expression for this resistance?

41.—When a beam supported at each end carries a load at its middle, what is the amount of pressure sustained by the two points of support, taken together?

42.—What portion of the load is upheld by each support?

43.—If the load be not at the middle, what is the sum of the pressures upon the two points of support?

44.—In the latter case, what proportions do the parts borne at the two points of support bear to each other?

45.—What expression represents that borne by the *near* support.

46.—What expression represents the pressure upon the *remote* support?

47.—If a beam, 12 feet long between bearings, carries a load of 15,000 pounds, at a point 4 feet from one bearing, what portion of this load is borne by the near support?

And what is the pressure upon the remote support?

48.—When a beam is subjected to transverse strain at its middle, what constitutes the destructive energy tending to rupture?

49.—When the destructive energy and the resistance are in equilibrium, what expression represents the conditions of the case?

50.—What is the breaking load of a Georgia pine beam, 15 feet long between the bearings; the breadth being 4 inches, the depth 10, and the load at the middle?

51.—How many times as strong as when laid on the flat is a beam when set on edge?

CHAPTER IV

THE EFFECT OF WEIGHT AS REGARDS ITS POSITION.

ART. **52.—Relation between Destructive Energy and Resistance.**—In a beam, laid upon two bearings, and sustaining a load at the middle, we have discovered certain relations between the load and the beam.

The load has a tendency to destroy the beam, while the beam has certain elements of resistance to this destructive power.

The destructive energy exerted by the load is equal to the product of half the load multiplied by half the length of the beam. The power of resistance of the beam is equal to the product of the area of cross-section of the beam, multiplied by its depth and by the strength of the unit of material. At the moment of rupture, the destructive energy and the power of resistance are equal; or, as modified in *Art.* **35**,

$$Wl = Bbdd \qquad \text{or, as in formula } (9.),$$
$$Wl = Bbd^2$$

53.—Dimensions and Weights to be of Like Denominations with Those of the Unit Adopted.—In applying the above formula it is to be observed, that the length, breadth and depth, in any given case, are to be taken in like denominations with those of the unit of material adopted (*Art.* **33**). For example: if the unit of material be that of this work, then, in the application of the formula, the breadth and depth are to be taken in *inches*, and the length between bearings in *feet*.

It is also requisite that the weight be taken in like denomination with that by which the resistance of the unit of material was ascertained. If the one is in ounces, the other is also to be in ounces; if one is in pounds, the other must be in pounds; or, if in tons, then in tons.

The strength of the unit of material adopted for this work is given in pounds; therefore, in applying the rule, the weight given, or to be found, must necessarily be in pounds.

54.—Position of the Weight upon the Beam.—The location of the weight upon the beam now requires consideration.

Upon our unit of material, which is supported at each end, the load is understood to have been located at the middle of the length; so, in using formula (*9.*), the weight given, or sought, must be located at the middle of the length of the given beam.

55.—Formula Modified to Apply to a Lever. — By proper modifications this formula may also be applied to the case of a weight suspended from one end of a lever or projecting beam. To show the application, we proceed as follows:

In *Fig.* 10 one half of the load W is borne on each one of the supports A and B.

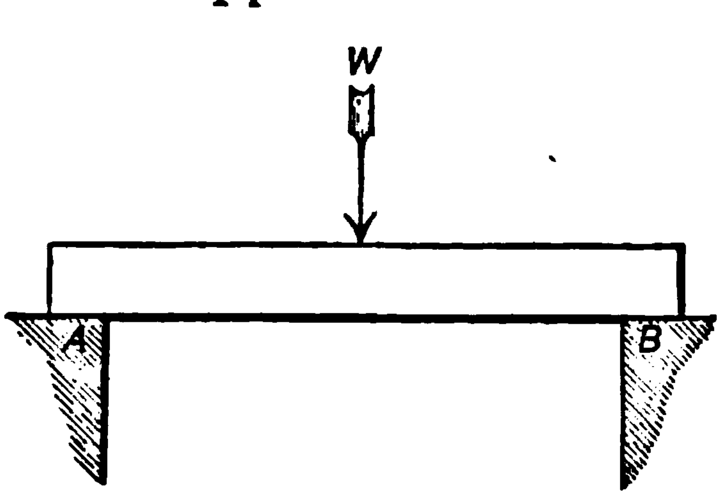

FIG. 10.

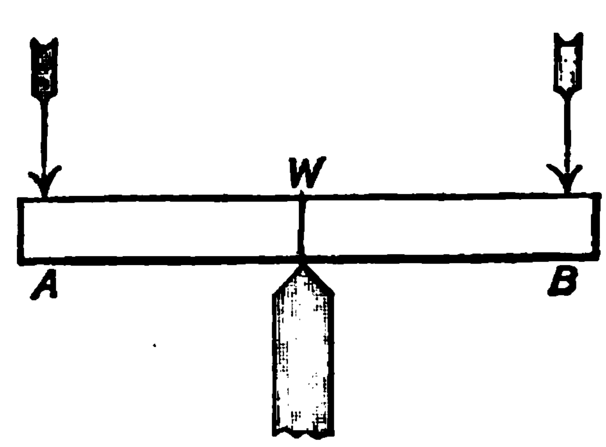

FIG. 11.

In *Fig.* 11 we have a beam of the same length, and subjected to the same forces, but in reversed order (*Art.* **26**).

While *Fig.* 10 represents a beam supported at both ends and loaded in the middle, one half of *Fig.* 11 may be taken to represent a lever projecting from a wall and loaded at the free end.

In these two cases the moment or destructive energy tending to break the beam is the same in each, and yet it is produced in *Fig.* 11 with only one half the weight, acting at the end of a lever only one half the length of the beam. We have, therefore,

$$\tfrac{1}{2}W \times \tfrac{1}{2}l = \tfrac{1}{4}Wl$$

or, in a lever, it requires but a quarter of the weight to produce a given destructive energy, that is required in a beam of equal length, laid upon two supports—that is to say, if two beams of like material, and of the same cross-section, be subjected to transverse strains, in like positions as to breadth and depth, one beam being supported at both ends and loaded in the middle, and the other one firmly fixed in a wall at one end and loaded at the other; and if the distance between the wall and the weight in this latter beam be equal to the distance between the bearings in the former; then but one quarter of the weight requisite to break the beam supported at both ends will be required to break the projecting one.

If the former requires 10,000 pounds to break it, then the latter will be broken by 2500 pounds.

The proportion between the weights is as 4 to 1. But suppose the weights upon the two beams are equal. In this case the lever will have to be made stronger, and its sectional area enlarged sufficiently to carry 4 times the weight. Hence we have, for beams fixed at one end and loaded at the other,

$$4Wl = Bbd^2$$

in which W is the weight suspended from the end of the lever, and l is the length of the lever; or, to correspond with

the symbols used in *Art.* **34**, where P equals the weight and n equals the length of the lever, we have

$$4Pn = Bbd^2 \qquad (15.)$$

56.—Effect of a Load at Any Point in a Beam.—The next case for consideration is that of the effect of a weight located at *any point* in the length of a beam, the beam being supported at both ends.

In *Arts.* **27** and **28** it was shown, in cases of this kind, that R, the portion of the whole weight borne at the nearer end, is (*form.* *3.*) equal to $W\frac{n}{l}$; and that P, the portion resting upon the more remote end, is (*form.* *4.*) equal to $W\frac{m}{l}$; where W equals the weight on the beam, R the portion of the weight carried to the near support, P the portion carried to the remote support, l the length of the beam, m the distance from the weight to the near support, and n the distance to the remote support.

As shown in *Art.* **34**, the effective power or moment of a weight is equal to the product of the weight into the arm of the lever, at the end of which it acts. In *Fig.* 6 the weight R may be taken to represent the reaction of the point of support R in *Fig.* 7; and the destructive effect at the point of the fulcrum W in *Fig.* 6, taken to be the same as that at the location of the weight W in *Fig.* 7, as the strains in the two pieces are equal; and hence, the moment of R, *Fig.* 6, is equal to the product of R into its arm of lever m, or equal to Rm.

Taking the value of R in formula (*3.*), and multiplying it by its arm of lever, m, we have

$$Rm = W\frac{n}{l}m = W\frac{mn}{l}$$

Again, taking the value of P in formula (*4.*), and multiplying it by its arm of lever, n, we have

$$Pn = W\frac{m}{l}n = W\frac{mn}{l}$$

The two results agree, as they should.

57.—Rule for a Beam Loaded at Any Point.—These formulas may be tested by taking the two extreme conditions, the load at the middle and at the end.

First: When the load is at the middle

$$m = n = \tfrac{1}{2}l$$

the destructive energy, as above, will be

$$D = W\frac{mn}{l} = W\frac{\frac{1}{2}l \times \frac{1}{2}l}{l} = W\frac{\frac{1}{4}l^2}{l} = W\tfrac{1}{4}l = \tfrac{1}{4}Wl$$

the same value as obtained in *Art.* **35.**

Second: When the weight is moved towards the nearer end, m becomes gradually shorter, and when the weight in its movement reaches the point of support, m becomes zero, and n equals l. The destructive energy will then be

$$D = W\frac{mn}{l} = W\frac{0 \times l}{l} = W0 = 0$$

as it ought to be, for the weight no longer exerts any *cross strain* upon the beam.

The destructive energy therefore of a weight, W, when laid at any point upon a beam, is

$$D = W\frac{mn}{l}.$$

When laid at the middle, it is as above shown,

$$W\frac{mn}{l} = \tfrac{1}{4}Wl$$

In formula (*7.*) we have

$$\tfrac{1}{4}Wl = Sbd^2$$

therefore, by substitution,

$$W\frac{mn}{l} = Sbd^2$$

Multiplying by 4, we have

$$4W\frac{mn}{l} = 4Sbd^2$$

and since, by *Art.* **35**,

$$4S = B$$

we have

$$4W\frac{mn}{l} = Bbd^2 \qquad (16.)$$

a rule for the resistance of a beam when the weight is located at any point in its length.

58.—Effect of an Equally Distributed Load.—Let the effect of an equally distributed weight now be considered.

Formula (*16.*) gives the effect of a weight at any point of a beam—that is, the effect of the weight at the point where it is located; but what effect *at the middle* of the beam is produced by a weight *out* of the middle?

When a weight is hung at the end of a projecting lever, its effective energy, at any given point of the length of the lever, is equal to the product of the weight multiplied into the distance of that point from the weight (*Art.* **34**).

In *Arts.* **27** and **28** we have the effect of the weight W upon its points of support. For the remote end, in *Fig.* **7**, this is $P = W\frac{m}{l}$. This is the reaction, or power acting upward

at the point of support P. We have, *Arts.* **56** and **57**, the moment or destructive energy due to this reaction equal to

$$Pn = W\frac{m}{l}n = W\frac{mn}{l}$$

but if, instead of the whole distance n, we take only a part of it, or say to the middle of the beam, or $\frac{1}{2}l$, we have, instead of Pn,

$$P \times \tfrac{1}{2}l = W\frac{m}{l} \times \tfrac{1}{2}l = \tfrac{1}{2}W\frac{ml}{l} = \tfrac{1}{2}Wm$$

or, we have, for M, the effect at the middle due to a weight placed at any point,

$$M = \tfrac{1}{2}Wm$$

This result may be tested as in *Art.* **57**; for let $m = \frac{1}{2}l$, then $M = \frac{1}{2}Wm$ becomes

$$M = \tfrac{1}{2}W \times \tfrac{1}{2}l = \tfrac{1}{4}Wl$$

which is a quarter of the weight at the middle into the whole length, as shown in *Art.* **55**.

Again, taking the other extreme; when m becomes zero, then $M = \frac{1}{2}Wm$ becomes

$$M = \tfrac{1}{2}W \times 0 = 0$$

which is evidently correct, for when the weight is moved from over the clear bearing on to the point of support it ceases to exert any cross strain whatever upon any point of the beam.

From the above, we conclude that the effect produced at the middle of a beam, by a weight located at any point of its length, is equal to the product of half the weight into its distance from its nearest point of support.

This result would be true of a second weight, and a third, and of any number of weights. If the weights R, P, Q, etc. (*Fig.* 12), be located on a beam, at distances from their near-

est point of support equal to m, r, s, etc., their joint effect at the middle of the beam will be

$$\tfrac{1}{2}Rm + \tfrac{1}{2}Pr + \tfrac{1}{2}Qs + \text{etc.}$$

or

$$\tfrac{1}{2}(Rm + Pr + Qs + \text{etc.})$$

59.—Effect at Middle from an Equally Distributed Load.— We may now ascertain the effect produced at the middle of a beam by an equally distributed load.

Let a beam, $A\,B$ (*Fig.* 12), of homologous material, and of equal sectional area throughout its length, be divided into any number of equal parts. The weight of any one of these parts will equal that of any other part, and therefore we have in this beam a case of an equally distributed load.

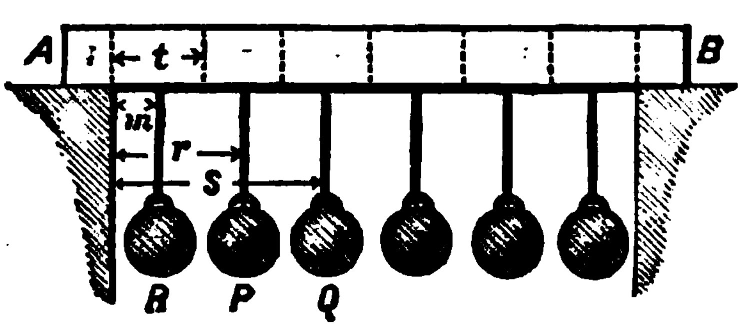

FIG. 12.

Now, suppose the weight of each of these parts to be concentrated at its centre of gravity, and represented by a ball, as R, P, or Q, suspended from that centre of gravity. Let t equal the length of each of the parts into which the beam is divided, then $m = \frac{1}{2}\,t$, $r = \frac{3}{2}\,t$ and $s = \frac{5}{2}\,t$, and, since $M = \frac{1}{2}\,Wm$, we have for the effect of the weight R, at the middle of the beam, $M = \frac{1}{2}\,R\,\frac{1}{2}\,t$; for the effect of P, $M = \frac{1}{2}\,P\,\frac{3}{2}\,t$; and for the effect of Q, $M = \frac{1}{2}\,Q\,\frac{5}{2}\,t$; etc., for all the weights on one half of the beam.

If these results be doubled (for the effects of the weights on the other half would equal these), we shall have the total effect at the middle of the beam of all the weights. When,

as in this case, the beam is divided into six parts, we have for the total effect at the middle,

$$M = \frac{1}{2}\,tR + \frac{3}{2}\,tP + \frac{5}{2}\,tQ$$

Now if we put the symbol U to represent a uniformly distributed load, we have

$$R = P = Q = \frac{U}{6}$$ therefore

$$M = \frac{1}{2}\,t\frac{U}{6} + \frac{3}{2}\,t\frac{U}{6} + \frac{5}{2}\,t\frac{U}{6}$$

$$M = \frac{Ut}{6}\left(\frac{1}{2} + \frac{3}{2} + \frac{5}{2}\right) = \frac{Ut}{6} \times \frac{9}{2} = \frac{3}{4}\,Ut$$

In this case t equals $\frac{l}{6}$, therefore

$$M = \frac{3}{4}\,U\frac{l}{6} = \frac{1}{8}\,Ul$$

in which U equals the whole weight uniformly distributed over the beam.

We have seen (*Art.* **35**) that $\frac{1}{4}Wl$ is the destructive energy of a weight concentrated at the centre of the beam. We now see, as above, that this same effect is produced by $\frac{1}{8}Ul$. We therefore have

$$\tfrac{1}{8}Ul = \tfrac{1}{4}Wl$$ or, multiplying by 4,

$$\tfrac{1}{2}U = W$$

or, when the effects of the two loads upon a beam are equal, one half of U, the distributed load, will equal the load W, concentrated at the middle.

60.—Example of Effect of an Equally Distributed Load. —Let R, P, Q, etc., each equal 20 pounds; or the whole load U equal $6 \times 20 = 120$ pounds. Let the whole length, 12 feet, be divided into six equal parts, and the equal loads be suspended from the centre of each of these parts. Then from the nearer point of support, A, the distance m to R is one foot; the distance r to P is three feet; and the distance s to

Q is five feet; and, since R, P, and Q are each equal to 20, and (*Art.* **58**)

$$M = \tfrac{1}{2} Wm \qquad \text{therefore}$$
$$M = \tfrac{1}{2} Rm + \tfrac{1}{2} Pr + \tfrac{1}{2} Qs$$
$$M = \tfrac{1}{2} \times 20\,(m + r + s)$$
$$M = 10\,(1 + 3 + 5) = 10 \times 9 = 90$$

The like effect, 90 pounds, is had from the three weights upon the other half of the beam. Adding these, we have 180 pounds. This is the destructive energy exerted at the middle of the beam by the six weights, or by U, the 120 pounds equally distributed along the beam. As a test of this, let it now be shown what weight concentrated at the middle of the beam would produce the like effect. In *Art.* **35** we have for the destructive energy, $D = \tfrac{1}{4} Wl$, from which $W = \dfrac{D}{\tfrac{1}{4} l}$, and since, as above, $D = 180$ and $l = 12$, we have $W = \dfrac{180}{3} = 60$ pounds. This is the weight concentrated at the middle. Above, we had U, the equally distributed weight, equal to 120 pounds, or twice 60. Therefore $2W = U$. Thus, as before, it is seen that an equally distributed weight produces an effect at the middle equal to that produced by one half the weight if concentrated at the middle.

61.—Result also Obtained by the Lever Principle.—This result may also be obtained by an application of the lever principle. In *Fig.* 13 a double lever is loaded with weights, producing strains similar to those in a beam such as *Fig.* 12. Here the arm of lever at which R acts is five feet, that of P three feet, and Q one foot; therefore,

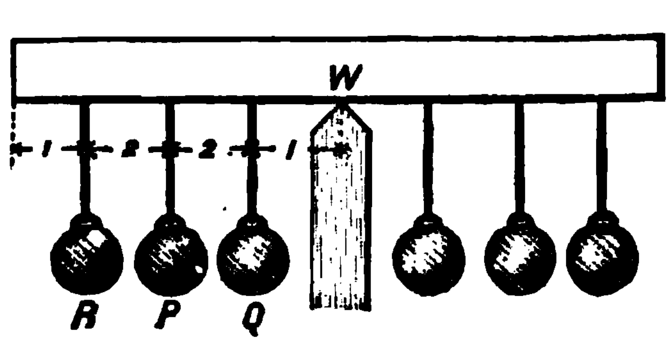

FIG. 13.

$$\begin{aligned} R \times 5 &= 20 \times 5 = 100 \\ P \times 3 &= 20 \times 3 = 60 \\ Q \times 1 &= 20 \times 1 = 20 \\ & \qquad\qquad \overline{180} \text{ pounds.} \end{aligned}$$

This is the whole energy, because the weights on the other side of the fulcrum do not add to the strain at W; they only balance the weights R, P, and Q.

The full effect, therefore, at the middle of the beam is 180 pounds, as before shown, and this effect is produced by $3 \times 20 = 60$ pounds equally distributed.

Now, what concentrated weight at the end of the lever would produce an equal effect?

Since the weight P, at the end of a lever, multiplied by n, the length of the lever, is the moment or destructive energy of the weight, therefore

$$Pn = 180 \qquad \text{the moment as above, or}$$

$$P = \frac{180}{n} = \frac{180}{6} = 30.$$

and this is one half of 60, the distributed weight which produced a like effect.

Hence we find that a given load, if concentrated at the middle of a beam, will have a destructive energy there equal to that of twice said load equally distributed over the length of the beam; or, in other words, an equally distributed load will need to be double the weight of a concentrated load to produce like effects upon any given beam.

In formula (*9.*) W represents the concentrated weight at the middle. If for W we substitute its equivalent $\frac{1}{2}U$, we have

$$\tfrac{1}{2}Ul = Bbd^2 \qquad (17.)$$

QUESTIONS FOR PRACTICE.

62.—A white pine beam, 6×9 inches, supported at each end, and set upon edge, is 12 feet long. What weight laid at 4 feet from one end would break it?

63.—What weight equally distributed over the length of the above beam would break it?

64.—What weight concentrated at the middle of the length of the same beam would break it?

65.—What weight would break this beam if suspended from one end of it, the other end being fixed in a wall?

CHAPTER V.

COMPARISON OF CONDITIONS—SAFE LOAD.

ART. **66.—Relation between Lengths, Weights and Effects.**—In the consideration of the effect of weights upon beams, we have deduced certain formulas applicable under various conditions. These rules will now be presented in such manner as to show by comparison: *first*, what relation the lengths and weights bear to each other when the effects are equal; and, *second*, the resulting effects when the lengths and weights are equal.

67.—Equal Effects.—Take the four *Figs.*, 14, 15, 16 and 17.

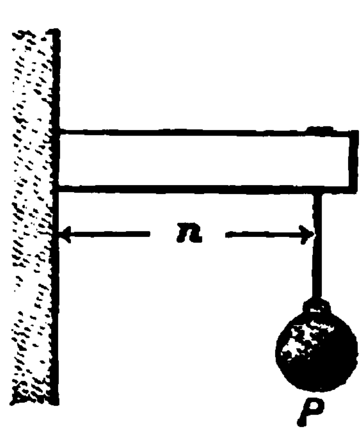

FIG. 14.

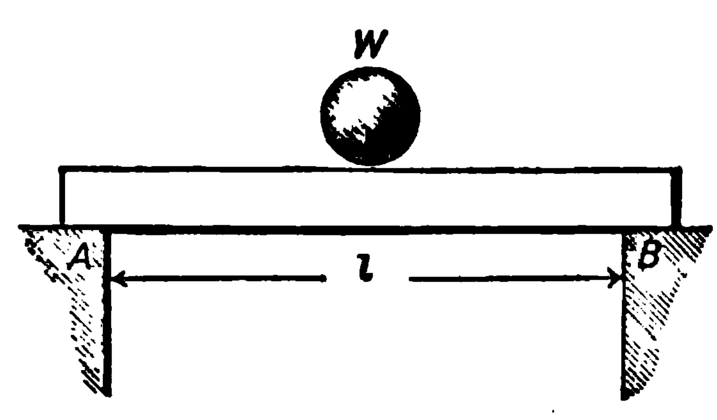

FIG. 16.

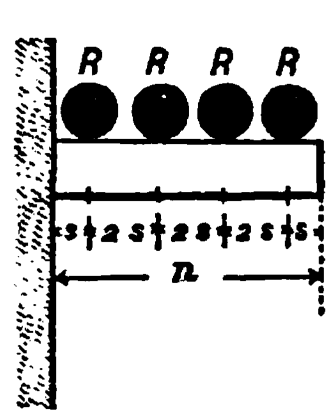

FIG. 15.

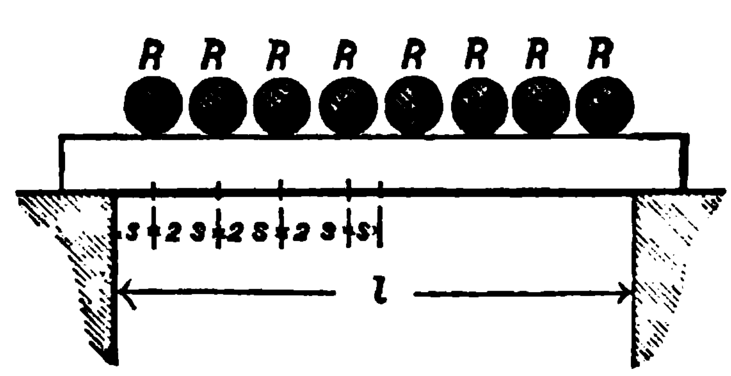

FIG. 17.

The lengths of the beams and the amounts of the weights with which they are loaded, are such as to produce equal

effects. For example, the dimensions are such that in all of the figures, $l = 2n$ and $s = \frac{n}{8} = \frac{l}{16}$; and the weights are so proportioned that $W = 2\ P = 4R$. By comparison, we find that in *Fig.* 14 the destructive energy is

$$Pn = \tfrac{1}{2}W \times \tfrac{1}{2}l = \tfrac{1}{4}Wl$$

In *Fig.* 15 the destructive energy is equal to the sum of the products of the several weights R, into their respective distances from the point of support; or,

$$Rs + R\,3s + R\,5s + R\,7s =$$
$$Rs\,(1 + 3 + 5 + 7) = 16\,Rs = 16 \times \tfrac{1}{4}W \times \tfrac{1}{16}l = \tfrac{1}{4}Wl$$

In *Fig.* 16 the destructive energy is

$$\tfrac{1}{2}W \times \tfrac{1}{2}l = \tfrac{1}{4}Wl$$

In *Fig.* 17 the destructive energy equals the sum of the products of the several weights R, into one half their respective distances from the nearest point of support (*Art.* **58**),

or,
$$2\,(\tfrac{1}{2}Rs + \tfrac{1}{2}R\,3s + \tfrac{1}{2}R\,5s + \tfrac{1}{2}R\,7s) =$$
$$2\,[\tfrac{1}{2}Rs\,(1 + 3 + 5 + 7)] =$$
$$2\,(\tfrac{1}{2}Rs\,16) = 16\,Rs = 16 \times \tfrac{1}{4}W \times \tfrac{1}{16}l = \tfrac{1}{4}Wl$$

When the load is at any point upon the beam, the destructive energy is $W\frac{mn}{l}$.

This case is a modification of *Fig.* 16, for, when

$$m = n = \tfrac{1}{2}l$$ we have,

$$W\frac{\tfrac{1}{2}l \times \tfrac{1}{2}l}{l} = W\frac{\tfrac{1}{4}l^2}{l} = \tfrac{1}{4}\,Wl$$

68.—Comparison of Lengths and Weights Producing Equal Effects.—We now see that, in order to produce equal effects, we must have the length and weight in *Fig.* 16 twice

those in *Fig.* 14; and the length and weights of *Fig.* 17 twice those of *Fig.* 15.

Again, we see that, while the lengths of *Figs.* 14 and 15 are the same, the weights of the latter are equal in amount to twice that of the former; and that the same proportions exist in *Figs.* 16 and 17.

69.—The Effects from Equal Weights and Lengths.—In regard to the second relation, as expressed in *Art.* **66.**

We have, in *Figs.* 18, 19, 20, and 21, examples showing the

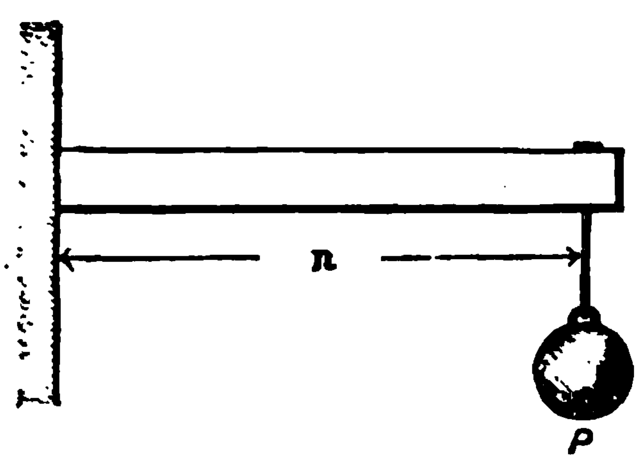

FIG. 18.

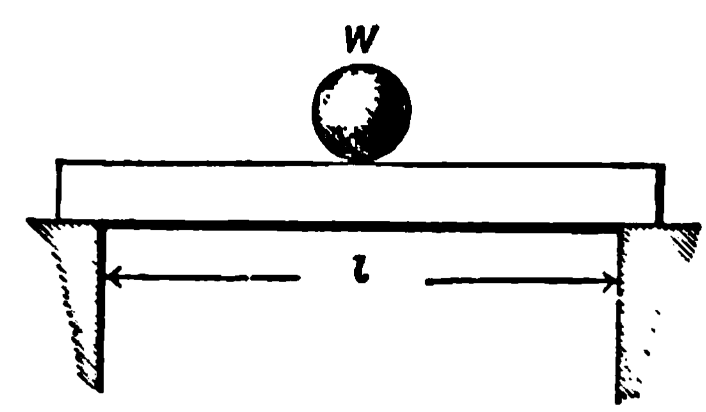

FIG. 20.

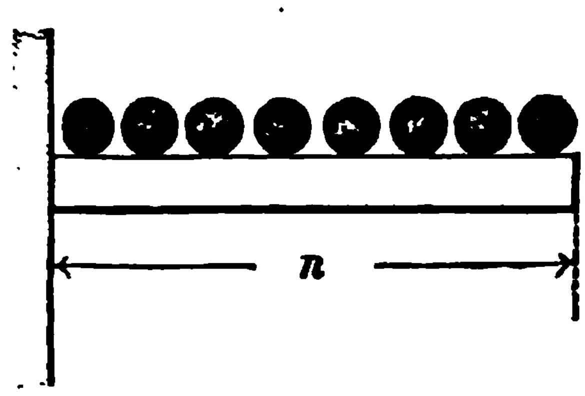

FIG. 19.

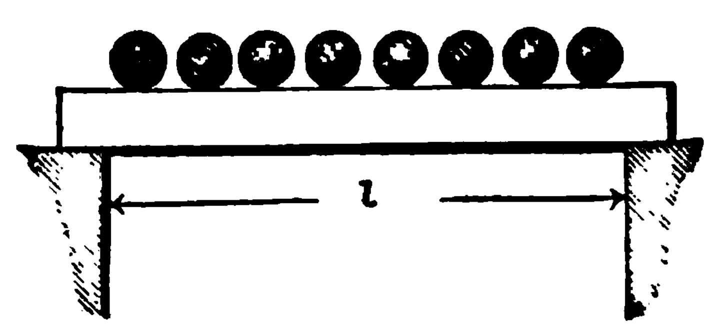

FIG. 21.

difference of effect when the load upon each beam is equal to the load upon either of the other beams, and the lengths of the beams are equal.

The destructive energy is

in *Fig.* 18, $D = Pn$

" 19, $D = \frac{1}{2}Un$

" 20, $D = \frac{1}{2}W \times \frac{1}{2}l = \frac{1}{4}Wl$

" 21, $D = \frac{1}{2} \times \frac{1}{2}U \times \frac{1}{2}l = \frac{1}{8}Ul$

70.—Rules for Cases in which the Weights and Lengths are Equal.—Putting these equal to the resistance for levers, we have (*Art.* **35**) for the case shown

$$
\begin{array}{lrl}
\text{in } Fig.\ 18, & Pn & = Sbd^2 \\
\text{"} \quad 19, & \tfrac{1}{2}Un & = Sbd^2 \\
\text{"} \quad 20, & \tfrac{1}{4}Wl & = Sbd^2 \\
21, & \tfrac{1}{8}Ul & = Sbd^2
\end{array}
$$

and, since (*Art.* **35**) $4S = B$, $S = \frac{1}{4}B$. If in the above we substitute this value for S, we shall have the following rules:

$$
\begin{array}{lrll}
\text{For case } 1, & 4Pn & = Bbd^2 & (15.) \\
\text{"} \quad \text{"} \quad 2, & 2Un & = Bbd^2 & (18.) \\
\text{"} \quad \text{"} \quad 3, & Wl & = Bbd^2 & (9.) \\
\text{"} \quad \text{"} \quad 4, & \tfrac{1}{2}Ul & = Bbd^2 & (17.) \\
\text{and in case } 5, & 4W\dfrac{mn}{l} & = Bbd^2 & (16.)
\end{array}
$$

this last being that of a load located at any point in the length of a beam (*Art.* **57**).

71.—Breaking and Safe Loads.—These rules show the relation of the load to the resistance. Before showing their applications, the proportion which exists between the *breaking load* and what is called the *safe load* will be considered.

72.—The above Rules Useful Only in Experiments.—The rules thus far shown have all been based upon the condition of equilibrium between the destructive power of the load and the resistance of the material; or, in other words, an equilibrium at the point of rupture. Hence they are chiefly useful in testing materials to their breaking point.

73.—Value of a, the Symbol of Safety. — To make the rules useful to the architect, it is requisite to know what portion of the breaking load should be trusted upon a beam. It is evident that the permanent load should not be so great as to injure the fibres of the beam.

The proportion between the safe and the breaking weights differs in different materials. The breaking load on a unit of material being represented by B, as before, let T represent the safe load, and a the proportion between the two; or, $T : B :: 1 : a = \frac{B}{T}$ then $T = \frac{B}{a}$. The values of a, for several kinds of building materials, have been found and recorded in Table XX., an examination of which will show that a, for many kinds of materials, is nearly equal to 3, a number which is in general use.*

74.—Value of a, the Symbol of Safety.—In the rules a may be taken as high as we please above the value given for a in the table; but *never lower* than the value there given. If a be taken at 4, then, as above, $T = \frac{B}{a} = \frac{1}{4}B$ equals the safe power of the unit of material, and we have $Wl = \frac{1}{4}Bbd^2$; or, $4Wl = Bbd^2$, as the proper rule for a beam supported at each end and loaded at the centre. In order, however, to

* This is the value as fixed by taking the *average* of the results of the tests of several specimens of the same kind of material, or material of the same name.

Owing to the large range in the results in any one material, it is not safe, in a general use of this symbol, to take it at the average given in the table. It should for ordinary use be taken higher.

When the kind of material in any special and important work is known, and tests can be made of several fair specimens of it, and from the results computations made of the values of a, then an average of these would be safe to use. For the ordinary woods in general rules, it is prudent to take the value of a at not less than 4.

make the rules general, we shall not adopt any definite number, as 4, but use the symbol a, the value of which is to be taken from the table in accordance with the kind of material employed, increasing its value at discretion. (See note, *Art.* **73**.)

75.—Rules for Safe Loads.—The rules, with this factor a introduced, will then be as follows:

Rule	1,	$4Pan = Bbd^2$	(*19.*)
"	2,	$2Uan = Bbd^2$	(*20.*)
"	3,	$Wal = Bbd^2$	(*21.*)
"	4,	$\frac{1}{2}Ual = Bbd^2$	(*22.*)
"	5,	$4Wa\frac{mn}{l} = Bbd^2$	(*23.*)

76.—Applications of the Rules.—In this form the rules are ready for use—applying them as below.

Rule 1 is applicable to all cases where a load is suspended from the end of a lever (*Fig.* 18), said lever being fixed at the other end in a horizontal position.

Rule 2 applies to cases where a load is equally distributed upon a lever fixed at one end (*Fig.* 19).

Rule 3 is applicable to a load concentrated at the middle of a beam supported at both ends (*Fig.* 20).

Rule 4 is applicable to equally distributed loads upon beams supported at both ends (*Fig.* 21).

Rule 5 is applicable to a load concentrated at any point upon a beam supported at both ends (*Fig.* 7).

77.—Example of Load at End of Lever.—To show the practical working of these rules, take, first, an example coming under rule 1, formula (*19.*),

$$4Pan = Bbd^2$$

Let it be required to find the requisite breadth and depth of a piece of Georgia pine timber, fixed at one end in a wall, and sustaining safely, at five feet from the wall, a weight of 1200 pounds; the ratio between the safe and breaking weights being taken as 1 to 4, and the value of B for Georgia pine being 850 (*Art.* 13).

78.—Arithmetical Exemplification of the Rule.—The first thing, in applying a rule, is to distinguish between the known and the unknown factors of an equation, by so transposing them that those which are known shall stand upon one side, and the unknown upon the other side of the equation. In rule 1, formula (*19.*), as above, the known factors are 4, P, a, n and B; therefore we transpose, so that

$$\frac{4Pan}{B} = bd^2$$

Substituting the known quantities for the symbols of the first member, we have

$$\frac{4 \times 1200 \times 4 \times 5}{850} = bd^2$$

79.—Caution in Regard to *a*, the Symbol of Safety.—The working of this problem is interrupted to remark that students are liable to err in estimating the value of a, making it a fraction instead of a whole number. Thus, if the proportion between the safe and the breaking weights be as 1 to 4, they, starting with the idea that the safe weight is to be one fourth of the breaking weight, make a equal to $\frac{1}{4}$, instead of 4. This is a serious error, as the result would be a destructive energy of only one sixteenth (for $\frac{1}{4} : 4 :: 1 : 16$) of the true amount, and consequently the resultant resistance of the timber would be but one sixteenth of what it should

be, and in practice it would be found that the beam would *break down* with only one fourth of the amount considered the *safe weight.*

To farther explain the value of a, let W equal the breaking weight, and T the safe weight; the proportion being as 4 to 1. Then $T = \frac{1}{4}W$, or $4T = W$. Now, in formula (*9.*) ($Wl = Bbd^2$), in order to preserve equality, it is requisite, in removing the symbol W denoting the breaking weight, that we substitute its equal, or $4T$. So when, in the new formula for safe weight, W is understood to represent *not* the breaking but the *safe* weight, $4T$ becomes $4W$, and we have $4Wl = Bbd^2$; therefore the symbol a is to be not a fraction but a whole number.

Returning from this digression to the expression at the end of *Art.* **78**, and reducing it, we have

$$bd^2 = \frac{96000}{850} = 112{\cdot}94$$

Here we have the value of the breadth multiplied by the square of the depth, but neither the one nor the other is as yet determined.

80.—Various Methods of Solving a Problem.—There are at least three ways of procedure by which to determine the value of each of these factors. The breadth and depth may be required to be equal; the breadth may be required to bear a certain proportion to the depth; or, one of the factors may be fixed arbitrarily.

First. If the timber is to be square, then b will equal d,

$$bd^2 = d^3, \text{ and } d = \sqrt[3]{112{\cdot}94} = 4{\cdot}83$$

that is, the dimensions required are 4·83, or, say 5 inches square.

Second. Let the breadth be to the depth in the proportion of 6 to 10, then

$$b : d :: 6 : 10 \qquad \text{or}$$

$$10b = 6d \qquad \text{or}$$

$$b = 0{\cdot}6d \qquad \text{Then}$$

$$112{\cdot}94 = bd^2 = 0{\cdot}6d \times d^2 = 0{\cdot}6d^3$$

$$\frac{112{\cdot}94}{0{\cdot}6} = d^3 = 188{\cdot}23 = \overline{5.73}^3 \qquad \text{that is}$$

$$d = 5{\cdot}73, \text{ and } b = 5{\cdot}73 \times 0{\cdot}6 = 3{\cdot}44$$

The timber should be therefore 3·44 inches broad, and 5·73 inches deep; or, $3\frac{1}{2} \times 5\frac{3}{4}$ inches.

Third. The breadth or depth may be determined arbitrarily, or be controlled by circumstances. Let the breadth be fixed, say at 3 inches, then

$$112{\cdot}94 = bd^2 = 3d^2$$

$$\frac{112{\cdot}94}{3} = d^2 = 37{\cdot}65$$

$$d = 6{\cdot}14$$

The dimensions should be $3 \times 6{\cdot}14$, or, say, $3 \times 6\frac{1}{8}$ inches.

Again, let the depth be fixed, say at 6 inches, then

$$112{\cdot}94 = bd^2 = b \times 6^2$$

$$\frac{112{\cdot}94}{36} = b = 3{\cdot}14$$

thus giving as the dimensions of the beam $3{\cdot}14 \times 6$, or, say $3\frac{1}{4} \times 6$ inches.

We have now these four answers to the question of *Art.* **77**, namely:

If the beam be square, the side of the square must be 5 inches.

If the breadth and depth be in the proportion of 6 to 10, the breadth must be $3\frac{1}{2}$ and the depth $5\frac{3}{4}$ inches.

If the breadth be fixed at 3 inches, then the depth must be 6¼ inches.

If the depth be fixed at 6 inches, then the breadth must be 3¼ inches.

81.—Example of Uniformly Distributed Load on Lever. —Take an example coming under rule 2, formula (*20.*),

$$2Uan = Bbd^2$$

Let the conditions be similar to those given in *Art.* **77**, except that the weight is to be equally distributed, instead of being concentrated at the end. What are the required dimensions of breadth and depth?

The formula transposed becomes,

$$\frac{2Uan}{B} = bd^2$$

As the known factors are all the same as in the last example, except the numerical co-efficient, which here is only one half of its former value, it follows that bd^2 in this case must be equal to one half of bd^2 in the previous case; or,

$$\frac{112{\cdot}94}{2} = 56{\cdot}47 = bd^2$$

Now to apply this result:

First. If the timber be square,

$$56{\cdot}47 = d^3 = \overline{3{\cdot}84}^3$$

Second. If the breadth and depth are to be as 6 to 10,

$$56{\cdot}47 = 0{\cdot}6\ d^3$$

$$\frac{56{\cdot}47}{0{\cdot}6} = d^3 = 94{\cdot}12 = \overline{4{\cdot}55}^3$$

and $$b = 4{\cdot}55 \times 0{\cdot}6 = 2{\cdot}73$$

Third. If the breadth be fixed at 2 inches, then

$$56{\cdot}47 = bd^2 = 2d^2$$
$$\frac{56{\cdot}47}{2} = d^2 = 28{\cdot}24$$
$$d = 5{\cdot}31$$

Fourth. If the depth be fixed at 5 inches, then

$$56{\cdot}47 = bd^2 = b \times 5^2$$
$$b = \frac{56{\cdot}47}{25} = 2{\cdot}26$$

The four answers are, therefore, $3\frac{7}{8}$ square—$2\frac{3}{4} \times 4\frac{5}{8}$—$2 \times 5\frac{3}{8}$ and $2\frac{1}{4} \times 5$; and the beam may be made of the dimensions named in either of these four cases and be equally strong.

82.—Load Concentrated at Middle of Beam.—In an example under rule 3, the value of bd^2 in the formula $Wal = Bbd^2$, would be just one quarter of that required by rule 1.

83.—Load Uniformly Distributed on Beam Supported at Both Ends.—In cases under rule 4, the values of bd^2 would be only one eighth of those under rule 1; and, in general, the five rules given are so related that when the result of computations under any one of them has been obtained, the result in any other one may be found by proportion, in comparing the two rules applicable.

QUESTIONS FOR PRACTICE.

84.—What breadth and depth are required for a white pine beam, of sufficient strength to carry safely 3000 pounds equally distributed over its length, the beam being 12 feet long and supported at each end? The breadth is to be one half of the depth, and the factor of safety a equals 4.

85.—What would be the size if square?

86.—What would be the depth if the breadth be fixed at 3 inches?

87.—What would be the breadth if the depth were fixed at 6 inches?

CHAPTER VI.

APPLICATION OF RULES—FLOORS.

ART. **88.—Application of Rules to Construction of Floors.**—Having completed the investigation of the strength of beams to resist rupture so far as to obtain formulas or rules applicable to the five principal cases of strain, we will now show the application of these rules to the solution of such problems as occur in the construction of floors. As these rules, however, are founded simply upon the resistance to rupture, the size of a beam determined by them will be found to be much less than by rules hereafter given; and the beam, although perfectly safe, will yet be found so small as to be decidedly objectionable on account of its excessive deflection. Owing to this, floor beams in all cases should be computed by the rules founded upon the resistance to flexure, as in Chapter XVII.

89.—Proper Rule for Floors.—Floor beams are usually subjected to equally distributed loads. For this, formula (*22.*) is appropriate, as it "is applicable to equally distributed loads upon beams supported at both ends." It is

$$\tfrac{1}{2}Ual = Bbd^2$$

90.—The Load on Ordinary Floors, Equally Distributed.—The load upon ordinary floors may be considered as being equally distributed; at least when put to the severest test—a densely crowded assemblage of people. For this load all floors should be prepared.

91.—Floors of Warehouses, Factories and Mills.—The floors of stores and warehouses, factories and mills, are required to sustain even greater loads than this, but in all the load may be treated as one equally distributed.

92.—Rule for Load upon a Floor Beam.—Each beam in a floor is subjected to the strain arising from the load upon so much of the floor as extends on each side half way to the next adjoining beam; or, that portion of the floor which is measured by the length of the beam and by the distance apart from centres at which the beams are laid. Denote the distance apart, in feet, at which the beams are placed (measuring from the centres of the beams) by c. Then cl will equal the surface of the floor carried by one of the beams.

If the load in pounds upon each superficial foot of the floor be expressed by f, then the total load upon a floor beam will be cfl. This is an equivalent for U, the load.

By substituting for U its value cfl in the formula

$$\tfrac{1}{2}Ual = Bbd^2 \qquad \text{we have}$$

$$\tfrac{1}{2}acfl^2 = Bbd^2 \qquad (24.)$$

which is a rule for the load upon a floor beam.

93.—Nature of the Load upon a Floor Beam.—Before this formula can be used, the value of f must be determined.

This symbol represents a compound weight, comprising the weight of the materials of construction and that of the superimposed load.

The weight of the materials of construction is also in itself a compound load. A part of this load—the floor plank and ceiling (the latter being either of boards or plastering)—will be a constant quantity in all floors; but the floor beam

will vary in weight as the area of its cross-section. In all cases of wooden beams, however, the weight of the beam is so small, in proportion to the general load, that a sufficiently near approximation to its weight may be assigned in each case before the exact size of the beam be ascertained.

94.—Weight of Wooden Beams.—For example, in floors for dwellings, the beams will vary from 3×8 to 3×12, according to the length of the beam. If the timber be white pine (the weight of which is about 30 pounds per cubic foot, or $2\frac{1}{2}$ pounds per superficial foot, inch thick), the 3×8 beam will weigh 5 pounds, and the 3×12 beam $7\frac{1}{2}$ pounds; or, as an average, say $6\frac{1}{4}$ pounds per *lineal* foot for all white pine beams for dwellings. For spruce, the average weight is about the same. Hemlock, which is a little heavier, may be taken at 7 pounds; and Georgia pine (seldom used in dwellings) should be put at about 9 pounds per *lineal* foot.

95.—Weight in Stores, Factories and Mills to be Estimated.—For stores, factories and mills the weight is greater, and is to be estimated.

96.—Weight of Floor Plank.—The weight of the floor plank, if of white pine or spruce, is about 3 pounds; or, if of Georgia pine, about $4\frac{1}{2}$ pounds per *superficial* foot.

97.—Weight of Plastering.—The weight of plastering varies from 7 to 11 pounds, and is, on the average, about 9 pounds, including the lathing and furring, per superficial foot.

98.—Weight of Beams in Dwellings.—The weights of beams, given in *Art.* **94**, are for the *lineal* foot, but it is requisite that this be reduced so as to show the weight per *square foot superficial* of the floor. When the distance from

centres at which the floor beams are placed is known, the weight per lineal foot divided by the distance between centres in feet will give the desired result.

Thus, let the distance from centres of white pine floor beams be 16 inches, or $1\frac{1}{3}$ feet. Then $6\frac{1}{2} \div 1\frac{1}{3} = 4\frac{7}{8}$ pounds.

As the average distance from centres in dwellings differs little from 16 inches, the weight of beams may be safely taken at 5 pounds per superficial foot for white pine and spruce.

99.—Weight of Floors in Dwellings.—In summing up we have, for the weight of the floor plank, 3 pounds; for the plastering, 9 pounds, and for the beams, 5 pounds; and the sum of these items, 17, or, in round numbers, say 20 pounds is the total weight of the materials of construction upon each superficial foot of the floor of ordinary dwellings; and this is large enough to cover the weight per superficial foot, even when a heavier kind of timber, such as Georgia pine, is used.

100.—Superimposed Load.—We have now to consider the superimposed weight, or the load to be carried upon the floor.

101.—Greatest Load upon a Floor.*—Mr. Tredgold, in speaking of bridges, says (*Treatise on Carpentry, Art.* 273): "The greatest load that is likely to rest upon a bridge at one time would be that produced by its being covered with people." Again he says: "It is easily proved that it is about the greatest load a bridge can possibly have to sustain, as well as that which creates the most appalling horror in the case of failure." The floors of churches, theatres, and other

* The substance of the following discussion of the load per foot upon a floor was read by the author before the American Institute of Architects, and published in the *Architects' and Mechanics' Journal*, New York, in April, 1860.

assembly rooms, and also those of dwellings, are all liable to be covered with people at some time (although not usually), to the same compactness as a bridge. Therefore, to find the greatest strain to which floor timbers of assembly rooms and dwellings are subjected, it will be requisite, simply, to weigh the people; or, to find an answer to the question in the experiments of those who have weighed them.

102.—Tredgold's Estimate of Weight on a Floor.—Mr. Tredgold, in the article quoted, says: "Such a load is about 120 lbs. per foot;" and again, at page 283 of his *Treatise on the Strength of Iron*, he says: "The weight of a superficial foot of a floor is about 40 lbs. when there is a ceiling, counter-floor, and iron girders. When a floor is covered with people, the load upon a superficial foot may be calculated at 120 lbs. Therefore 120 + 40 = 160 lbs. on a superficial foot is the least stress that ought to be taken in estimating the strength for the parts of a floor of a room."

103.—Tredgold's Estimate not Substantiated by Proof.—Mr. Tredgold's most excellent works on construction have deservedly become popular among civil engineers and architects. With very few exceptions, the whole of the valuable information advanced by him has stood the test of the experience of the last fifty years; and notwithstanding that many other works, valuable to these professions, have since appeared, his works still remain as standards. Statements made by him, therefore, should not be dissented from except upon the clearest proof of their inaccuracy; and only after obtaining *ample* proof is the statement here ventured that Mr. Tredgold *was* in error when he fixed upon 120 pounds per foot as the weight of a crowd of people.

In the writings of Mr. Tredgold, his positions are generally sustained by extensive quotations and references; but

in this case, so important, he gives neither reference, data from which he derives the result, nor proof of the correctness of his statement. This proof must be sought elsewhere.

104.—Weight of People—Sundry Authorities.—In the year 1848, an article appeared in the *Civil Engineer and Architects' Journal*, containing information upon this subject. From this article we learn that upon the fall of the bridge at Yarmouth, in May 1845, Mr. James Walker, who was employed by government to investigate the matter, stated in evidence before the coroner, that his estimate of the load upon the bridge was based upon taking the weight of people at an average of 7 stone (98 pounds) each; and admitted that this was a large estimate, rather higher, perhaps, than it ought to be; yet he did so because it was customary to estimate them at this weight; and further, that he calculated that six people would require a square yard for standing room. At this rate there would be two persons in every three feet, and the weight would be 65 pounds per foot.

Herr Von Mitis, who built a steel suspension bridge over the Danube, at Vienna, estimated 15 men, each weighing 115 Vienna pounds, to a square fathom of Vienna. This, in English measurement and weight, would be equal to 39 men in every hundred square feet, and nearly 55 pounds per foot.

Drury, in his work on suspension bridges, lays down an arbitrary standard of two square feet per man of 10 stone weight. This equals 70 pounds per superficial foot.

In testing new bridges in France, it is usual for government to require that 200 kilogrammes per square metre of platform shall be laid on the bridge for 24 hours. This is equal to 41 pounds per foot.

The result of combining the above four instances is an average of $57\frac{3}{4}$ pounds per foot.

But we have a more accurate estimate, founded upon trustworthy data. Quetelet, in his *Treatise on Man,* gives the average weight of males and females of various ages as follows:—

Average weight of males at	5, 10 and	15	years,	61·53
" " "	20 "	25	"	135·59
" " "	30, 40 "	50	"	140·21
Average weight of females at	5, 10 and	15	years,	57·50
" " "	20 "	25	"	116·33
" " "	30, 40 "	50	"	121·80
			6)	632·96
	Total average weight in lbs. =			105·5

105.—Estimated Weight of People per Square Foot of Floor.—The weight of men, women and children, therefore, is 105·5 pounds each, on the average. This may be taken as quite reliable as to the weight of people. Now as to the space occupied by them.

It is known among military men that a body of infantry closely packed will occupy, on the average, a space measuring $15 \times 20 = 300$ square inches each. At this rate, 48 men would occupy 100 square feet, and if a promiscuous assembly should require the same space each, then there would be a load of 50·64 pounds upon each square foot. In military ranks, however, the men would weigh more. Taking the weight of males from 20 to 50 years, in the above table—this being the probable range of the ages of soldiers—the average is found to be 137·9; a weight of 66 pounds upon each superficial foot of floor; and this weight may be taken as the greatest which can arise from a crowd of people.

106.—Weight of People, Estimated as a Live Load.— But this is simply the *weight*, no allowance being made for any increase of strain by reason of the movement of the people upon the floor. We will now consider the increase made in consequence of the agitation of the weight through walking and other movements.

In walking, the body rises and falls, producing in its fall a strain additional to that due to its weight when quiet.

The moving force of a falling body is known to be equal to the square root of $64\frac{1}{3}$ times the space fallen through in feet, multiplied by the weight of the body in pounds. By this rule, knowing the weight and the height of fall, we may compute the force.

The weight in the present case, 66 pounds, is known, but the height of fall is to be ascertained. This height is not that of the rise and fall of the *foot*, but of the *body*; the latter being less than the former. The elevation of body varies considerably in different persons, as may be seen by observing the motions of pedestrians. Some rise and fall as much as half an inch at each step, while others deviate from a right line but slightly. If, in the absence of accurate observation, the rise be assumed at a quarter of an inch, as a fair average, then the moving force of the 66 pounds, computed by the above rule, would be 76.4 pounds. This would be the *moving force* at the moment of contact, and the *effect* produced would be equal to this, provided that the falling body and the floor were both quite inelastic; but owing to the presence of an elastic substance on the soles of the feet, and at the joints of the limbs, acting as so many cushions, the force of the blow upon the floor is much diminished. The elasticity of the floor also diminishes the effect of the force to a small degree. Hence the increase of over ten pounds, as found above, would be much diminished, probably one half, or, say to six pounds.

107.—Weight of Military.—This six pounds would be the increase per foot superficial. To make this effect general over the whole surface of the floor, it is requisite that the weight over the whole surface fall at the same instant; or, that the persons covering the floor should all step at once, or with regular military step. It will be found that this is the severest test to which a floor of a dwelling or place of assembly can be subjected. In promiscuous stepping the strain would be much less, scarcely more than the quiet weight of the people.

108.—Actual Weights of Men at Jackson's and at Hoes' Foundries.—The above results, it must be admitted, are derived from data—with reference to the height of fall, and to the lessening effect of the elastic intervening substances,—which are in a measure assumed, and hence are not quite conclusive. They need the corroboration of experiment.

To test them, I experimented, in April, 1860, at the foundry of Mr. James L. Jackson in this city. He kindly placed at my service his workmen and his large scale. The scale had a platform of $8\frac{1}{2} \times 14$ feet. It was of the best construction, and very accurate in its action. Eleven men, taken indiscriminately from among the workmen of the foundry, stood upon the platform. Their combined weight while standing quietly was 1535 pounds, being an average of 139·55 pounds per man. This is but a pound and a half more than was derived from the tables of Quetelet. It is quite satisfactory in substantiating the conclusion there drawn.*

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* In May, 1876, since the above was written, by the courtesy of Messrs. R. Hoe & Co., of this city, who placed at my disposal their platform scale and men, I was enabled, by a second experiment, to ascertain the weight of men and the space they occupy. Selecting twenty-six stalwart men from their smith shop, they were found to weigh 3955 pounds, and to occupy upon the platform a space $7 \times 7\frac{1}{2} = 52\frac{1}{2}$ square feet, or $75\frac{1}{3}$ pounds per superficial foot. This is a

109.—Actual Measure of Live Load.—After ascertaining the quiet weight of the men, they commenced walking about the platform, stepping without order, and indiscriminately. The effect of this movement upon the scale was such as to make it register 1545 pounds; an increase of only ten pounds, or less than one per cent. They were then formed in a circle and marched around the platform, stepping simultaneously or in military order. The effect upon the scale produced by this movement was equal to 1694 pounds, an increase of 159 pounds, or over ten per cent. This corroborated the results of the computation before made most satisfactorily; ten per cent of the weight per foot, 66 pounds, being 6·6 pounds.

As a final trial, the men were directed to use their utmost exertion in jumping, and were urged on in their movements by loud shouting. The greatest consequent effect produced was 2330 pounds, an increase of 795 pounds, or about 52 per cent.

110.—More Space Required for Live Load.—This seems a much more severe strain than the former, but we must consider that men engaged in the violent movements necessary to produce this increase of over 50 per cent need more standing room. Packed closely, occupying only 15 × 20 inches (the space allowed per man in computing the weight per foot to be 66 pounds), it would not be possible to move the limbs sufficiently for jumping. To do this, at least twice as much space would be required. But, to keep within the limits of safety, let only one half more space be allowed. In this case the 66 pounds would be the weight on a foot

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larger average than found at Mr. Jackson's, or than any previous weight on record, and is accounted for by the fact that these were muscular men, weighing about 12¼ pounds each more than the heaviest hereinbefore noticed, and much heavier than it were reasonable to expect in assemblages generally.

and a half, or there would be but 44 pounds on each foot of surface.

Add to this the 50 per cent for the effects of jumping, or 22 pounds, and the sum, 66 pounds, is the total effect of the most violent movements on each foot of the floor; the same as for the weight of men standing quietly, but packed so much more closely.

111.—No Addition to Strain by Live Load.—The greatest effect, then, that it appears possible to produce by an assembly on a floor, is from the regular marching of a body of men, closely packed; and amounts to $66 + 6{\cdot}6 = 72{\cdot}6$ pounds per superficial foot.

This result would show the necessity of providing for ten per cent additional to the weight of the people. This in general is not needed, for the conditions of the case generally preclude the possibility of obtaining this additional strain upon the floor. The strain of 66 pounds is only obtained by crowding the people closely together in the whole room. To obtain the ten per cent additional strain, they must be set to marching; but there is no space in which to march, unless they march out of the room, and in doing this the strain is not increased, for the weight of those who pass out is fully equal to the stress caused by the act of marching.

Were both ends of the room quite open, or were it a long hall, as a bridge, through which the people could march solid, the throng being sufficiently numerous to keep the floor constantly full, then the ten per cent would need to be added, but not in ordinary cases of floors of rooms.

112.—Margin of Safety Ample for Momentary Extra Strain in Extreme Cases.—It may be argued still, that, although the room be full and marching can only be effected by some of the people leaving the floor, yet this additional strain *will* be

obtained in consequence of the exertion made in the act of taking the very first step, before any have left the room. To this we reply that the strain thus produced would not endanger the safety of the floor, because this strain, when compared with the ultimate strength of the beams sustaining it, would be quite small, and its existence be but momentary. Beams made so strong as not to break with less than from three to five times the permanent load would certainly not be endangered by the addition for a moment of only ten per cent of that load.

113.—Weight Reduced by Furniture Reducing Standing Room.—Hence, for all ordinary cases, no increase of strength need be made for the effects of motion in a crowd of people upon a floor, and therefore the amount before ascertained, 66 pounds, or, in round numbers, say 70 pounds, may be used in the computations as the full strain to which the beams may be subjected. Indeed, the cases are rare where the strain will even be as much as this. When we consider the space occupied in dwellings by furniture, and in assembly rooms by seats, the presence of these articles reducing the standing room, the average weight per foot superficial will be found to be very much less.

114.—The Greatest Load to be provided for is 70 Pounds per Superficial Foot.—As a conclusion, therefore, floor beams computed to safely sustain 70 pounds per superficial foot, or to break with not less than three or four times this, will be quite able to bear the greatest strain to which they may be subjected in the floors of assembly rooms or dwellings; and especially so when the precaution of attaching them to each other by bridging* is thoroughly performed, thereby ena-

* The subjects of Floor Beams and of Bridging are farther treated in Chapters XVII. and XVIII.

bling the connected series of beams to sustain the concentrated weight of a few heavier persons or of some heavy article of furniture.

115.—Rule for Floors of Dwellings.—We now have, by including the weight of the materials of construction as shown in *Art.* **99**, the total weight per superficial foot, as follows:—

$$f = 70 + 20 = 90$$

for the floors of dwellings. With this value of f, formula (*24.*),

$$\tfrac{1}{2}\,acfl^2 = Bbd^2 \qquad \text{becomes}$$

$$\tfrac{1}{2}\,ac\,90\,l^2 = Bbd^2 \qquad \text{or, when } a = 4$$

$$180\,cl^2 = Bbd^2 \qquad (25.)$$

116.—Distinguishing Between Known and Unknown Quantities.—This formula may now be applied in determining problems of floor construction in dwellings, in which the safe strength is taken at one fourth of the breaking strength.

In distinguishing between known and unknown quantities, we will find generally that B and l are known, while c, b and d are unknown.

From formula (*25.*) therefore, we have, by grouping these quantities,

$$\frac{180\,l^2}{B} = \frac{bd^2}{c} \qquad (26.)$$

117.—Practical Example.—Formula (*26.*) is a general rule for the strength of floor beams of dwellings.

As an example under this rule, let it be required to find the sectional dimensions and the distance from centres of the beams of a floor of a dwelling; the span or length between bearings being 20 feet, and the material, spruce.

Here $B = 550$ and $l = 20$; and from formula (*26.*)

$$\frac{180 \times 20^2}{550} = \frac{bd^2}{c} = 130{\cdot}9$$

118.—Eliminating Unknown Quantities.—We have here the numerical value of a quotient, arising from a division of the product of the breadth and square of the depth, by the distance from centres at which the beams are to be placed.

Two of the three unknown quantities are now to be assigned a value, before the third can be determined. Circumstances will indicate which two may be thus eliminated. In some cases the breadth and depth of the timber are fixed, and the distance from centres is the unknown quantity; in others, the distance from centres and the depth may be the fixed quantities, and the breadth be the factor to be found; or, the distance and the breadth be fixed upon, and the depth be the quantity sought for. Generally, the breadth and depth are assigned according to the requirements of the case, or simply as a trial to ascertain the scope of the question, and the distance from centres is the dimension left to be determined by the formula.

119.—Isolating the Required Unknown Quantity.—In the solving of a question of either kind, the formula must first be transposed so as to remove all of the factors, except the one sought for, to the same side of the equation; thus,

$$\frac{bd^2}{c} = 130{\cdot}9 \qquad \text{becomes either}$$

$$b = \frac{130{\cdot}9\,c}{d^2} \qquad \text{or}$$

$$d^2 = \frac{130{\cdot}9\,c}{b} \qquad \text{or}$$

$$c = \frac{bd^2}{130{\cdot}9}$$

Assuming the value of any two of the factors, we select the proper formula and proceed with the test for the third factor.

120.—Distance from Centres at Given Breadth and Depth. For example, fix the breadth and depth at 3 and 9 inches. Then to find c, the above expression,

$$c = \frac{bd^2}{130 \cdot 9} \qquad \text{becomes}$$

$$c = \frac{3 \times 9^2}{130 \cdot 9} = \frac{243}{130 \cdot 9} = 1 \cdot 86$$

The value of c being in feet, this gives about 1 foot 10 inches, or 22 inches.

121.—Distance from Centres at Another Breadth and Depth.—The above result may be considered too great, and beams of less size and nearer together be more desirable. If so, assume a less size, say 3×8; we then have

$$c = \frac{3 \times 8^2}{130 \cdot 9} = \frac{192}{130 \cdot 9} = 1 \cdot 47$$

This gives c equal to about $17\frac{1}{2}$ inches.

122.—Distance from Centres at a Third Breadth and Depth.—With the object in view of economy of material, let another trial be had, fixing the size at $2\frac{1}{2} \times 9$. In this case

$$c = \frac{2\frac{1}{2} \times 9^2}{130 \cdot 9} = \frac{202 \cdot 5}{130 \cdot 9} = 1 \cdot 55$$

This gives for c about $18\frac{1}{2}$ inches. The answers then to this problem are,

for	3×9	inches,	22	inches	from centres,
"	3×8	"	$17\frac{1}{2}$	"	"
and "	$2\frac{1}{2} \times 9$	"	$18\frac{1}{2}$	"	"

These trials may be extended to any other proportions thought desirable, fixing first the breadth and depth, and then determining the corresponding value of c. (See precaution, *Art.* **88**.)

123.—Breadth, the Depth and Distance from Centres being Given.—Again, it may be desirable to assume a value for c, and then to ascertain the proper corresponding breadth and depth. In this case, one of the two unknown factors, b and d, must also be assumed. Let us fix upon $c = 1{\cdot}5$ and $d = 8$, then the formula in *Art.* **119**,

$$b = \frac{130{\cdot}9\,c}{d^2} \text{ becomes } b = \frac{130{\cdot}9 \times 1{\cdot}5}{64} = 3{\cdot}07$$

or, say 3 inches for the breadth.

124.—Depth, the Breadth and Distance from Centres being Given.—If the breadth be assumed, say at $2\frac{1}{2}$, then, with $c = 1.5$, to find the depth we have (*Art.* **119**),

$$d^2 = \frac{130{\cdot}9\,c}{b} = \frac{130{\cdot}9 \times 1{\cdot}5}{2{\cdot}5} = 78{\cdot}54$$
$$d = 8{\cdot}86 = 8\tfrac{7}{8} \text{ inches.}$$

Thus, when placed at 18 inches from centres, we have, in the one case 3×8 inches, and in the other $2\frac{1}{2} \times 8\frac{7}{8}$ inches.

125.—General Rules for Strength of Beams.—Any other case of wooden beams for dwellings may be treated in a similar manner, using formula (*25.*),

$$180\,cl^2 = Bbd^2$$

Beams of any material for any building may be determined by the general formula (*24.*),

$$\tfrac{1}{3}acfl^2 = Bbd^2$$

in all cases regarding the caution given in *Art.* **88.**

QUESTIONS FOR PRACTICE.

126.—In the floor of a dwelling, composed of 3×9 inch beams 16 feet long, how far from centres should spruce beams be placed?

127.—How far if of hemlock?

128.—How far if of white pine?

129.—In the floor of a dwelling, composed of $2\frac{1}{2} \times 10$ inch beams 19 feet long, how far from centres should spruce beams be placed?

130.—How far if of hemlock?

131.—How far if of white pine?

132.—In a floor of 4×12 inch beams 23 feet long, and required to carry 150 pounds per superficial foot (including material of construction), how far from centres should spruce beams be placed, the factor of safety being 4?

133.—How far if of hemlock?

134.—How far if of white pine?

135.—How far if of Georgia pine?

CHAPTER VII.

GIRDERS, HEADERS AND CARRIAGE BEAMS.

ART. **136.—A Girder Defined.**—By the term girder is meant a heavy timber set on posts or other supports, and serving, as a substitute for a wall, to carry a floor.

137.—Rule for Girders.—A girder sustaining a tier of floor beams carries an equally distributed load; the same per superficial foot as that which is carried by the floor beams.
aining the size of the girder formula (*24.*) is appli-
nely,

$$\tfrac{1}{2}acfl^2 = Bbd^2$$

-Distance between Centres of Girders.—In apply-
ormula to girders, it is to be observed that c repre-
distance between centres of girders, where there
r more, set parallel; or, the distance from centre of
one of the walls of the building, if the girder be
nidway between the two walls; or, an average of
distances, if not midway. As an example of the
e,—in a building 30 feet wide, the centre of a girder
t from one wall and 18 feet from the other. Here

$$c = \frac{12 + 18}{2} = 15$$

139.—Example of Distance from Centres.—What is the required size of a Georgia pine girder placed upon posts set 15 feet apart, the centre of the girder being 12 feet from one wall and 18 feet from the other; the load per foot superficial of floor, including the weight of the materials of construction, being 100 pounds, and the value of a being taken at 4?

140.—Size of Girder Required in above Example.—By transposing formula (*24.*) we have

$$\frac{\frac{1}{2}acfl^2}{B} = bd^2$$

and if the breadth be to the depth in the proportion of, say 7 to 10, then (*Art.* **80**)

$$\frac{\frac{1}{2}acfl^2}{B} = 0{\cdot}7d^3$$

$$\frac{0{\cdot}5 \times 4 \times 15 \times 100 \times 15^2}{0{\cdot}7 \times 850} = d^3 = 1134{\cdot}45$$

$$d = \sqrt[3]{1134{\cdot}45} = 10{\cdot}43$$

and $b = 0{\cdot}7 \times 10{\cdot}43 = 7{\cdot}30$.

Therefore the girder should be $7{\cdot}3 \times 10{\cdot}43$; or, to avoid fractions, say 8×11 inches.

141.—Framing for Fireplaces, Stairs and Light-wells.—We will now consider the subject of framing around openings in floors, for fireplaces, stairs and light-wells.

142.—Definition of Carriage Beams, Headers and Tail Beams.—*Fig.* 22 may be taken for a representation of a stairway opening in a floor; AB and CD being the walls of the

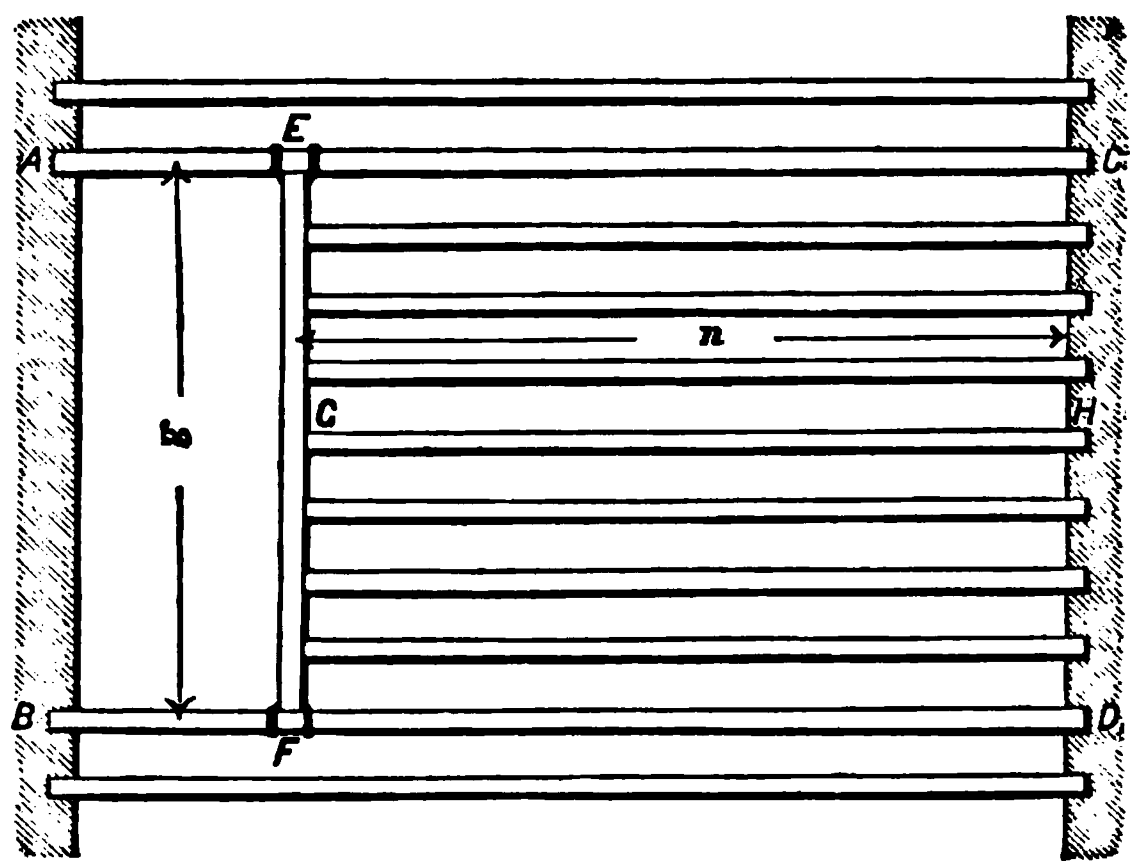

FIG. 22.

building, AC and BD the *carriage beams* or *trimmers*, EF the *header*, and the beams which reach from the header to the wall CD, such as GH, the *tail beams*.

143.—Formula for Headers—General Considerations.— First, the *headers*.

The load upon the header EF is equally distributed, therefore formula (*22*.) is applicable.

$$\tfrac{1}{2}Ual = Bbd^2$$

The header carries half the load upon the tail beams, or the load upon a space equal to the length of the header by half the length of the tail beams. Let g represent the length of the header, n the length of the tail beams, and f the load per foot superficial; then U, the load upon the header, equals

$$g\frac{n}{2}f = \tfrac{1}{2}fng = U$$

and, as g here represents l, the length, therefore,

$$Ul = \tfrac{1}{2}fng^2$$

and formula (*22.*) becomes

$$\tfrac{1}{2}a\tfrac{1}{2}fng^2 = Bbd^2$$

$$\tfrac{1}{4}afng^2 = Bbd^2$$

144.—Allowance for Damage by Mortising.—This last formula should be modified so as to allow for the damage done to the header by the mortising for the tenons of the tail beams. This cutting of the header ought to be confined as nearly as possible to the middle of its height, so that the injury to the wood may be at the place where the material is subject to the least strain.

If this is properly attended to, it will be a sufficient modification to make the depth of the header one inch more than that required by the formula. Thus, when the depth by the formula is required to be 9 inches, make the actual depth 10; or, for d^2 substitute $(d-1)^2$, d being the actual depth. The rule, thus modified, will determine a header of the requisite strength with a depth one inch less than the actual depth. This will compensate for the damage caused by mortising.

The expression in the last article then becomes

$$\tfrac{1}{4}afng^2 = Bb(d-1)^2$$

145.—Rule for Headers.—Generally, the depth of a header is equal to the depth of the floor in which it occurs. Hence, when the depth of the floor beams has been determined, that of the header is fixed. There remains then only the breadth to be found.

We have, for the breadth of a header (from *Art.* **144**)

$$b = \frac{afng^2}{4B(d-1)^2} \qquad (27.)$$

(See precaution in *Art.* **88.**)

146.—Example.—In a tier of nine inch beams, what is the required breadth of a white pine header at the stairway of a dwelling, the header being 12 feet long, and carrying tail beams 16 feet long; the factor of safety being 4?

In formula (27.), making $a = 4$, $f = 90$, $n = 16$, $g = 12$, $B = 500$ and $d = 9$, the formula becomes

$$b = \frac{4 \times 90 \times 16 \times 12^2}{4 \times 500 \times 8^2} = 6{\cdot}48$$

The breadth of the header should be $6\frac{1}{2}$, or say 7 inches, and its size 7×9 inches.

147.—Carriage Beams and Bridle Irons.—A carriage beam, or trimmer, in addition to the load of an ordinary beam, is required to carry half the load of the header which hangs upon it for support. As this is a concentrated load at the point of connection, all mortising at this point to receive the header should be carefully avoided, and the requisite support given with a bridle iron, as in *Fig.* 23.

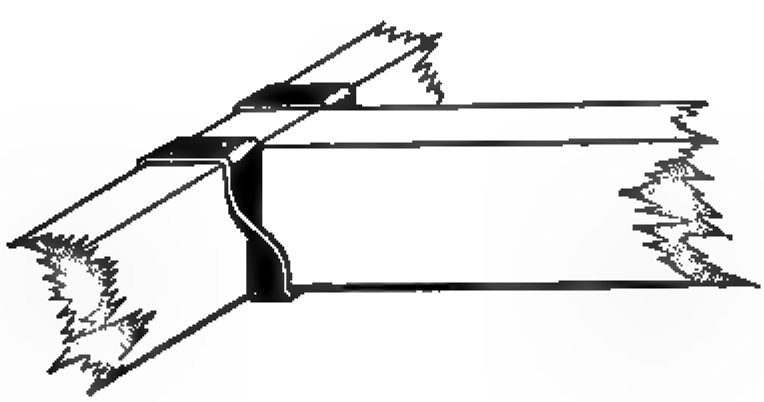

Fig. 23.

148.—Rule for Bridle Irons.—In considering the strain upon a bridle iron, we find that it has to bear half the load upon the header, and, as the iron has two straps, one on each ... header, each strap has to bear only a quarter of ... pon the header.

... ve seen (*Art.* 143) that the load upon the header ... ng, where g represents the length of the header, n ... of the tail beams, both in feet, and f the load per

superficial foot. The load upon each strap of the bridle iron will, therefore, be equal to

$$\tfrac{1}{4} \times \tfrac{1}{2} fng = \tfrac{1}{8} fng$$

Good refined iron will carry safely from 9000 to 15,000 pounds to the square inch of cross-section. Owing, however, to the contingencies in material and workmanship, it is prudent to rate its carrying power, for use in bridle irons, at not over 9000 pounds.

If the rate be taken at this, and r be put to represent the number of inches in the cross-section of one strap of the bridle iron, then $9000r$ equals the pounds weight which the strap will safely bear; and when there is an equilibrium between the weight to be carried and the effectual resistance, we shall have

$$\tfrac{1}{8} fng = 9000r$$

from which

$$r = \frac{fng}{72000} \qquad (28.)$$

149.—Example.—For an example, let $f = 100$, $n = 16$, and $g = 12$; then

$$r = \frac{100 \times 16 \times 12}{72000} = 0{\cdot}266$$

If the bridle iron were made of $\tfrac{1}{4}$ by $1\tfrac{1}{2}$ inch iron ($\tfrac{1}{4} \times 1\tfrac{1}{2} = 0{\cdot}375$) the size would be ample. For such a header they are usually made heavier than this, yet this is all that is needed. It is well to have the bridle iron as broad as possible, in order to give a broad bearing to the wood, so that it shall not be crushed.

150.—Rule for Carriage Beam with One Header.—To return to the carriage beam, or trimmer. The weight to be carried upon a carriage beam is compounded of two loads; one the ordinary or distributed load upon a floor beam, as

shown in formula (*24.*); the other a concentrated load from the header. Of the former a carriage beam is required to carry one half as much as an ordinary beam; or, the load which comes upon the space from its centre half way to the adjacent common beam. This is the half of that shown in formula (*24.*), or

$$\tfrac{1}{4}acfl^2 = Bbd^2$$

The symbol W in formula (*23.*) represents the load from the header, and is equal (*Art.* **143**) to $\tfrac{1}{2}fng$. The carriage beam carries half this load, or $\tfrac{1}{4}fng$; hence

$$\tfrac{1}{4}fng = W \qquad \text{or, by formula (23.),}$$

$$4Wa\frac{mn}{l} = 4a\tfrac{1}{4}fng\frac{mn}{l} = afg\frac{mn^2}{l}$$

Combining this with the formula for the diffused load, we have

$$\tfrac{1}{4}acfl^2 + afg\frac{mn^2}{l} = Bbd^2 \qquad \text{or}$$

$$af\left(\tfrac{1}{4}cl^2 + gn^2\frac{m}{l}\right) = Bbd^2 \qquad (29.)$$

This is a rule for the resistance to rupture in carriage beams having one header. (See *Art.* **241**, and caution in *Art.* **88**.)

151.—Example.—As an example, let it be required to show the breadth of a white pine carriage beam 20 feet long, carrying a header 10 feet long, with tail beams 16 feet long, in a floor of 10-inch beams, which are placed 15 inches from centres; and where the load per superficial foot is 100 pounds, and the factor of safety is 4.

Transposing formula (*29.*) we have

$$b = af\,\frac{\left(\tfrac{1}{4}cl^2 + gn^2\frac{m}{l}\right)}{Bd^2}$$

in which $a=4$, $f=100$, $c=15$ inches $=1\frac{1}{4}$ feet, $l=20$, $g=10$, $n=16$, $m=l-n=20-16=4$, $B=500$ and $d=10$. Therefore,

$$b=4\times 100\times\frac{\left(\frac{1}{4}\times 1\frac{1}{4}\times 20^2+10\times 16^2\times\frac{4}{20}\right)}{500\times 10^2}$$

$$b=400\times\left(\frac{125+512}{50000}\right)=5{\cdot}096$$

The breadth required is 5·096, or say 5 inches. The trimmer should be 5×10 inches.

152.—Carriage Beam with Two Headers.—For those cases in which the opening in the floor (*Fig.* 25) occurs at or near the middle (instead of being at one side, as in *Fig.* 22), two headers are required; consequently the carriage beam, in addition to the load upon an ordinary beam, has to carry *two concentrated loads.*

To obtain a rule for this case the effect produced upon a beam by two concentrated loads will first be considered.

153.—Effect of Two Weights at the Location of One of Them.—The moment of *one* weight upon a beam is (*Art.* **56**) $W\frac{mn}{l}$. This is the effect *at* the point of location of the weight. A second weight, at another point, will produce a strain at the location of the first weight. To find this strain, let two weights, W and V (*Fig.* 24) be located upon a beam resting upon two supports, A and B. Let the distance from W to the support which may be reached without passing the other weight, be represented by m, and the distance to the other support by n. From V let the distances to the supports be designated respectively by s and r; s and n being distances from the same support.

The letters W and V, representing the respective weights, are to be carefully assigned as follows:—Multiply

one of the weights by its distance from one support, and the product by the distance from the other. Treat the other weight in the same manner; and that weight which, when so multiplied, shall produce the greater product is to be called W.

For example, in *Fig.* 24 let the two weights equal 8000 and

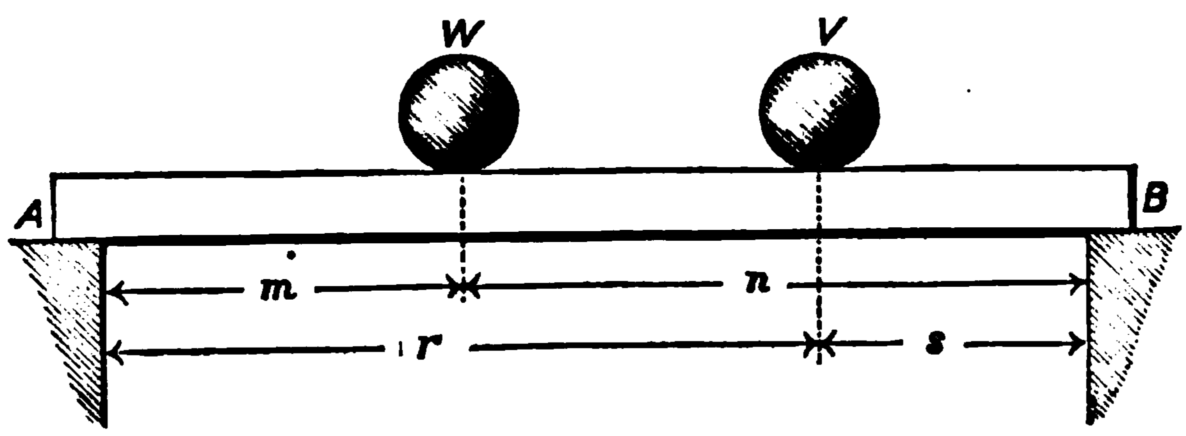

FIG. 24.

6000, $l = 20$, the distances from the 8000 weight to the supports equal 4 and 16, and those from the 6000 weight equal 5 and 15.

Then $$8000 \times 4 \times 16 = 512000$$
and $$6000 \times 5 \times 15 = 450000$$

The former result being the greater, the former weight, 8000, is to be called W, and the latter V.

The moment or effect of the weight W at its location is equal, as before stated, to $W\frac{mn}{l}$. The effect of the weight V at the point W will (*Art.* **27**) be equal to the portion of V borne at A, multiplied by the arm of lever m (*Arts.* **34** and **57**). The portion of V sustained by A is (*Arts.* **27** and **28**), $V\frac{s}{l}$; hence the effect of V at W will be $V\frac{s}{l} \times m = V\frac{ms}{l}$.

Adding the two effects, we have

$$W\frac{mn}{l} + V\frac{ms}{l} = \frac{m}{l}(Wn + Vs)$$

This is the total effect produced at W by the two weights.

In like manner it may be shown that the total effect at V is

$$V\frac{rs}{l} + W\frac{ms}{l} = \frac{s}{l}(Vr + Wm)$$

These are the moments or total effects at the two points of location. The first, when modified by the factor of safety a, gives

$$a\frac{m}{l}(Wn + Vs) = Sbd^2 = \frac{B}{4}bd^2$$

(see *Art.* **35**) from which we have

$$4a\frac{m}{l}(Wn + Vs) = Bbd^2 \qquad (30.)$$

for the dimensions at W. Then, also,

$$4a\frac{s}{l}(Vr + Wm) = Bbd^2 \qquad (31.)$$

for the dimensions at V.

(See caution in *Art.* **88**.)

154.—Example.—When the beam is to be of equal cross-section throughout its length, as is usually the case, then formula (*30.*), giving the larger of the two results, is to be used.

For example, let a weight of 8000 pounds be placed at 3 feet from one end of a beam 12 feet long between bearings, and another weight of 3000 pounds at 5 feet from the other end.

Then, as directed in *Art.* **153**,

$$8000 \times 3 \times 9 = 216000$$
$$3000 \times 5 \times 7 = 105000$$

The weight of 8000 pounds having given the larger product, it is to be designated by W, and the other weight by V.

Making $a=4$, we have for the greater effect (*form. 30.*),

$$4a\frac{m}{l}(Wn+Vs)=Bbd^2$$

$$4\times4\times\frac{3}{12}\times(\overline{8000\times9}+\overline{3000\times5})=Bbd^2=348000$$

and with $B=500$, and $b=0{\cdot}7d$, we have

$$B\times0{\cdot}7d\times d^2=348000$$

$$d^3=\frac{348000}{500\times0{\cdot}7}=994{\cdot}29$$

$$d=9{\cdot}98$$

$$b=0{\cdot}7\times9{\cdot}98=6{\cdot}99$$

or the beam should be $6{\cdot}99\times9{\cdot}98$, or 7×10 inches.

155.—Rule for Carriage Beam with Two Headers and Two Sets of Tail Beams.—Let the rules of *Art.* **153** be applied to the case of a carriage beam with two concentrated loads, as in *Fig.* 25.

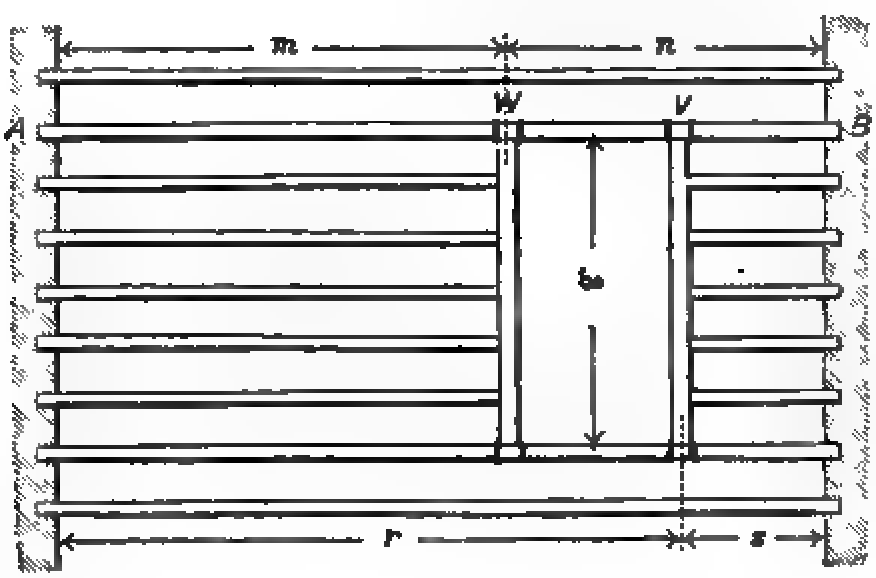

FIG. 25.

the opening in the floor is midway between the two sets of tail beams are of equal length; or, nd $n=r$; therefore $mn=sr$. The weights are also nerefore $Wmn=Vrs$; or, the strains at the headers

are equal. By moving the opening from the middle, the weight at the header carrying the longer tail beams is increased; so also the product of the distances to the supports is increased; therefore the letter W is to be put at that header which carries the longer tail beams, for then the product Wmn will exceed the product Vrs.

The weight at W is equal to the load upon one end of the header which is lodged there for support. This is equal to (*Arts.* **143** and **150**) $\frac{1}{4}fgm$ (m being the length of the tail beams sustained by this header), or $W = \frac{1}{4}fgm$.

In like manner it may be shown that $V = \frac{1}{4}fgs$.

By substituting these values of W and V in formula (*30.*) we have

$$\frac{4am}{l}(\tfrac{1}{4}fgmn + \tfrac{1}{4}fgs^2) = Bbd^2$$

$$\frac{afgm}{l}(mn + s^2) = Bbd^2$$

In addition to this load, the carriage beam is required to carry half the load upon a common beam, or half that shown at formula (*24.*), or $\frac{1}{4}acfl^2$. The expression for the full effect at W therefore is

$$Bbd^2 = \frac{afgm}{l}(mn + s^2) + \tfrac{1}{4}acfl^2$$

$$Bbd^2 = af\left[m(mn + s^2)\frac{g}{l} + \tfrac{1}{4}cl^2\right] \qquad (32.)$$

In like manner we find for the full effect at V,

$$Bbd^2 = af\left[s(rs + m^2)\frac{g}{l} + \tfrac{1}{4}cl^2\right] \qquad (33.)$$

(See caution in *Art.* **88.**)

These two formulas (*32.* and *33.*) give the sizes of the carriage beam at W and V respectively, but when the beam is made equal in size throughout its length, as is usual, the larger expression (*form. 32.*) is to be used.

156.—Example.—What is the required breadth of a Georgia pine carriage beam 25 feet long, carrying two headers 12 feet long, so placed as to provide an opening between them 5 feet wide; the tail beams being 15 feet long on one side of the opening and 5 feet long on the other; the floor beams being 14 inches deep and placed 18 inches from centres; the load per superficial foot being 150 pounds, and the factor of safety being 4?

Taking m to represent the longer tail beams, we have $a = 4$, $f = 150$, $m = 15$, $n = 10$, $s = 5$, $g = 12$, $l = 25$, $c = 18$ inches $= 1\frac{1}{2}$ feet, $B = 850$ and $d = 14$.

Formula (*32.*) now becomes

$$850 \times b \times 14^2 = 4 \times 150 \left[15(\overline{15 \times 10} + 5^2)\frac{12}{25} + \overline{\tfrac{1}{4} \times 1\tfrac{1}{2} \times 25^2} \right]$$

$$b = \frac{4 \times 150}{850 \times 196} \left[15\left(\overline{15 \times 10} + 25\right)\frac{12}{25} + \overline{\tfrac{1}{4} \times 1\tfrac{1}{2} \times 625} \right] = 5{\cdot}38$$

showing that the breadth should be 5·38. The beam may be made $5\frac{1}{2} \times 14$ inches.

157.—Rule for Carriage Beam with Two Headers and One Set of Tail Beams.—The preceding discussion, and the rules derived therefrom, are applicable to cases in which the two headers include an opening between them. When the headers include a series of tail beams between them, leaving an opening at each wall (*Fig. 26*), then the loads at W and V are equal; for the total load is that which is upon the one series of tail beams, and is carried in equal portions at the ends of the two headers—a quarter of the whole load at each

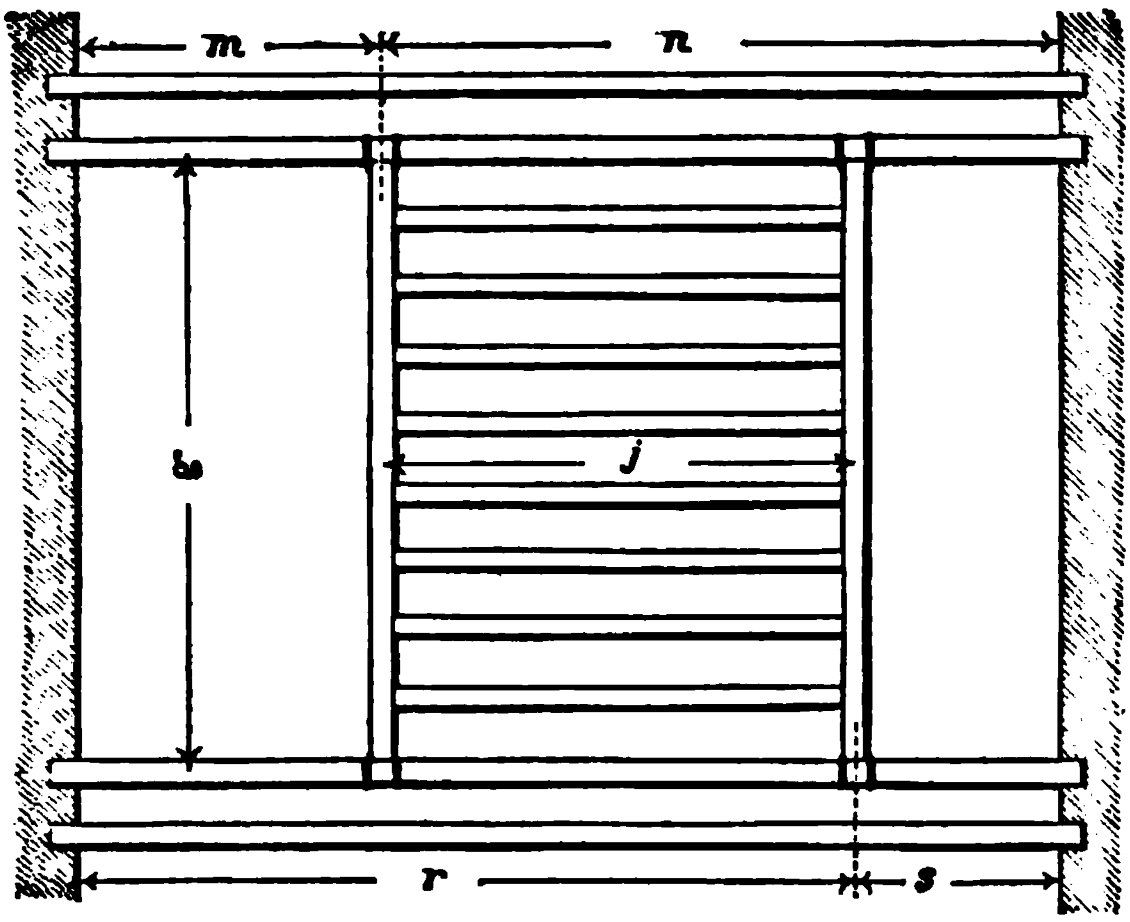

FIG. 26.

end of each header. If by j we represent the length of the tail beams, we have $W = V = \frac{1}{4} jfg$, and from formula (*30.*) we have, for the effect at W,

$$\frac{4aWm}{l}(n+s) = \frac{ajfgm}{l}(n+s)$$

Add to this half the load upon a common beam, $\frac{1}{4}acfl^2$ (*Art.* **92**), and we have, as the full effect at W,

$$\frac{ajfgm}{l}(n+s) + \frac{1}{4}acfl^2 = af\left[\frac{jg}{l}m(n+s) + \frac{1}{4}cl^2\right]$$

and, for the size of the beam at W,

$$af\left[\frac{jg}{l}m(n+s) + \frac{1}{4}cl^2\right] = Bbd^3 \qquad (34.)$$

Similarly, we find for the size of the beam at V,

$$af\left[\frac{jg}{l}s(r+m) + \frac{1}{4}cl^2\right] = Bbd^3 \qquad (35.)$$

These are identical, except that $s(r+m)$ in (*35.*) occupies the place of $m(n+s)$ in (*34.*). (See caution in *Art.* **88.**)

As in *Art.* **153**, care must be taken to designate by the proper symbols the weights and their distances. In that article the proper designation was found by putting the letter W to that weight which when multiplied into its distances m and n would give the greater product. Here, as the weights are equal, the comparison may be made simply between the two rectangles mn and rs. Of these, that will give the greater product which appertains to the weight located nearer the middle of the beam; this weight, therefore, is to be designated by W, and will be found at that header which is at the side of the wider opening. The distances m and n appertain to the weight W. The symbols being thus carefully arranged, formula (*34.*) gives the larger result, and is to be used when the beam is to be of equal sectional area throughout.

158.—Example.—To show the application of this rule, let it be required to find the size of a carriage beam in a tier of beams 12 inches deep and 16 inches from centres, with a weight per superficial foot of 100 pounds. In this case what should be the breadth of a white pine carriage beam 20 feet long between bearings, carrying two headers 12 feet long each, with one series of tail beams 10 feet long between them, so located as to leave an opening 6 feet wide at one wall and 4 feet at the other; the factor of safety being 4 ? Here we have the two distances m and s equal to 6 and 4, and putting m for the larger we have $a=4$, $f=100$, $j=10$, $g=12$, $l=20$, $m=6$, $n=14$, $s=4$, $c=1\frac{1}{8}$, $B=500$ and $d=12$.

Transposing formula (*34.*) to find b, we obtain

$$b=\frac{af}{Bd^2}\left[\frac{jg}{l}m(n+s)+\tfrac{1}{2}cl^2\right]$$

$$b = \frac{4 \times 100}{500 \times 12^2} \times \left[\overline{\frac{10 \times 12}{20} \times 6(14+4)} + \overline{\tfrac{1}{4} \times 1\tfrac{1}{3} \times 20^2} \right] = 4 \cdot 34$$

The breadth is required to be 4·34 inches, and the size of carriage beam, say $4\frac{1}{2} \times 12$ inches. (See caution, *Art.* **88**.)

QUESTIONS FOR PRACTICE.

159.—A building, 26 feet wide between the walls, has a tier of floor beams 12 inches deep and 14 inches from centres, supported at 16 feet from one of the walls by a girder resting upon posts set 15 feet apart. Upon that side of the building where the girder is 16 feet distant from the wall a stair opening occurs, extending 14 feet along the wall, and 6 feet wide. The floor is required to carry 150 pounds per foot superficial, including the weights of the materials of construction, with a factor of safety of 4. The girder, trimmers and header all to be of Georgia pine.

NOTE.—The resulting answers to the following questions will be smaller than if obtained under rules in Chapter XVII. (See *Art.* **88**.)

160.—What must be the breadth and depth of the girder, the breadth being equal to 55 hundredths of the depth?

161.—What should be the breadth of the carriage beams?

162.—What should be the breadth of the header?

163.—What should be the area of cross-section of the bridle iron?

164.—Another opening 6 feet wide in the same tier of beams, has headers 10 feet long, with tail beams on one side 6 feet long and on the other side 4 feet long. What should be the breadth of the carriage beams?

165.—What ought the breadth of the floor beams of the aforesaid floor to be on the 16 feet side of the girder, if of white pine?

166.—In the same tier of beams there is still another pair of carriage beams. These carry two headers 16 feet long, and the two headers carry between them one series of tail beams 8 feet long, thus forming two openings, one at the girder 3 feet wide and the other at the wall 5 feet wide. What should be the breadth of these carriage beams?

CHAPTER VIII.

GRAPHICAL REPRESENTATIONS.

ART. 167.—Advantages of Graphical Representations.— In the discussion of the subject of rupture by cross-strains, rules have been given by which the effect in certain cases has been ascertained; for example, that at the middle of a beam which rests upon two supports; that at the wall in the case of a lever inserted in the wall; and that at any given point in the length of a beam or lever.

These rules are perhaps sufficiently manifest; but when it becomes desirable to know the effect of the load in a new location, or under other change of conditions, an entirely new computation is needed.

To obviate the necessity for this labor, and to fix more strongly upon the mind the rules already given, the method of representing strains graphically, or by diagrams, is useful, and will now be presented.

168.—Strains in a Lever Measured by Scale.—In *Fig.* 27 we have a lever AB, or half beam, in which the destructive energy or moment of the weight P, suspended from the free end B, is equal to the product of the weight into the arm of leverage at the end of which it acts (*Art.* **34**); or $D = \frac{1}{2}lP$.

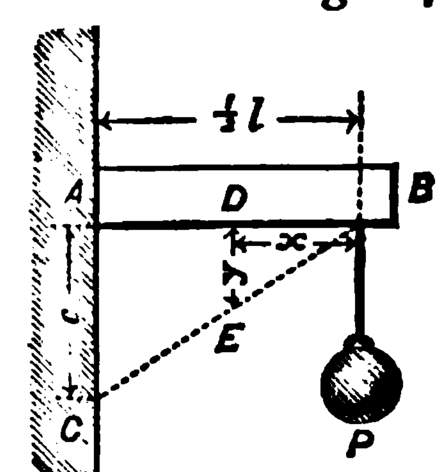

FIG. 27.

From A drop the vertical line $AC = c$, make it by any convenient scale equal to $\frac{1}{2}lP$, and join C and B. The tri-

angle ABC forms a scale upon which the strain produced at any point in AB may be obtained, simply by measurement; for, at any point, D, the ordinate $DE\ (=y)$, drawn parallel with the line AC, is equal (measured by the same scale) to the strain at the point D. In the two homologous triangles ABC and DBE, we have this proportion:

$$\tfrac{1}{2}l : c :: x : y = \frac{cx}{\tfrac{1}{2}l}$$

By construction $c = \tfrac{1}{2}lP$, therefore

$$y = \frac{\tfrac{1}{2}lPx}{\tfrac{1}{2}l} = Px$$

equals the weight into the arm of lever at the end of which it acts; or $Px = y$ is the destructive energy or moment of the weight P at the point D.

In this equation $(Px = y)$ since P is constant, the value of y is dependent upon that of x, for however x may be varied, y will vary in like manner. If x be doubled, y will be doubled; if x be multiplied or divided by any number, y will require to be multiplied or divided by the same number.

We conclude then that we may assign any value to x desirable, or select any point in AB for the location of D, from D draw an ordinate DE, parallel with the line AC, and measuring the ordinate by the same scale by which c was projected, find the strain or destructive energy exerted upon the beam at the selected point D.

169.—Example—Rule for Dimensions.—For example, let $P = 100$ and $l = 20$, then $AB = \tfrac{1}{2}l = 10$, and

$$\tfrac{1}{2}lP = 10 \times 100 = 1000$$

Now from a scale of equal parts (say tenths of an inch, or any other convenient dimensions), lay off c equal to ten of the divisions of the scale; then each division represents 100

pounds and $c = 1000 = \frac{1}{4}lP$. Draw the line CB, and from any point D draw the ordinate y. Suppose that y, measured by the same scale, is found to equal $7\frac{1}{4}$; then the strain at D equals $7\frac{1}{4} \times 100 = 725$ pounds.

If $y = 6$, then the strain at D equals 600 pounds; and so of any other ordinate, its measure will indicate the strain in the beam at the end of that ordinate.

We have, therefore [as in *Art.* **34**, formula (*6.*)]

$$Px = Sbd^2$$

and, with a the factor of safety, and putting for S its equivalent $\frac{1}{4}B$ (*Art.* **35**),

$$Pax = \tfrac{1}{4}Bbd^2$$

or,

$$4Pax = Bbd^2 \qquad (36.)$$

It is to be observed that the b and d of this formula are those required at D, the location of the ordinate y.

When x equals the length of the lever AB, equals $\frac{1}{2}l$, we have

$$4Pa\tfrac{1}{2}l = Bbd^2$$
$$2Pal = Bbd^2$$

and if P be taken as $\frac{1}{2}W$, W being the load at the centre of a *whole* beam, we have

$$2 \times \tfrac{1}{2}Wal = Bbd^2$$
$$Wal = Bbd^2$$

the same as formula (*21.*).

170.—Graphical Strains in a Double Lever.—In *Fig.* 28 we have a beam AB resting upon a point at the middle C, and carrying the two equal loads R and P suspended from the ends.

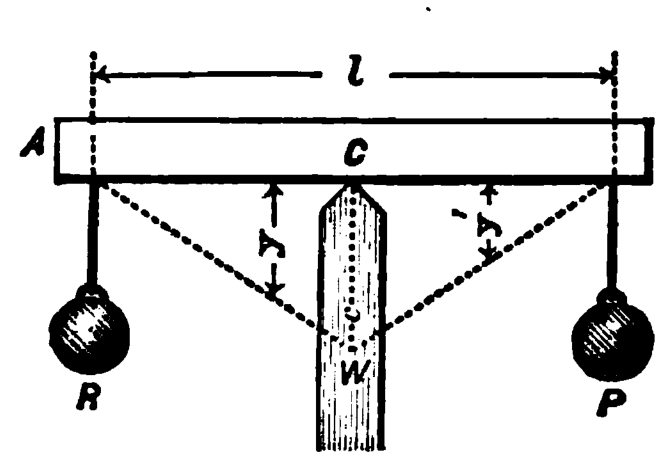

FIG. 28.

The half of this beam, or CB, is under the same conditions of strain as the beam AB in *Fig.* 27, and since the weights R and

P are equal, and C is at the middle of AB, the one half of the beam, or AC, is strained alike with the other half CB. Therefore a strain at any point in the length of the beam is measured by an ordinate from that point to the line AWB, and formula (*36.*) is applicable to this case also, conditioned that x does not exceed $\frac{1}{2}l$.

171.—Graphical Strains in a Beam.—In *Fig.* 29 we have a beam AB, resting upon two supports A and B, and loaded at middle with the weight W, one half of which, R, is borne upon A, and the other half, P, is supported by B.

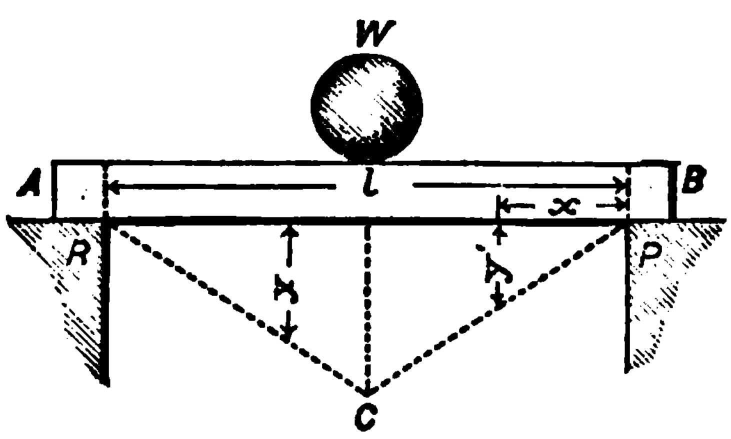

FIG. 29.

This beam has the same strains as that of *Fig.* 28, therefore (see *Art.* **26**) the same formula (*36.*) is applicable, namely:

$$4Pax = Bbd^2$$

$P = \frac{1}{2}W$, and by substitution

$$4 \times \tfrac{1}{2} Wax = Bbd^2$$

$$2Wax = Bbd^2 \qquad (37.)$$

a rule applicable to this case, conditioned that x shall not exceed $\frac{1}{2}l$.

When $x = \frac{1}{2}l$ then we have

$$2Wa\tfrac{1}{2}l = Bbd^2$$

$$Wal = Bbd^2$$

the same as given in formula (*21.*).

Again, if x be diminished until it shall reach zero, then

$$2Wax = 0$$

or the strain is nothing. This is evidently correct, as the effect of the weight, in producing cross-strain, disappears at

the edge of the bearing. We are not to be permitted, however, in shaping the beam to its exact requirements, to remove all material at and upon the bearing wall, for there is another strain, known as the *shearing strain*, for which provision is to be made at the end of the beam.

This strain we will now consider.

172.—Nature of the Shearing Strain.—The nature of the shearing strain, as well as of the cross-strain, is very clearly shown in *Fig.* 30, a diagram suggested by a similar one in "Unwin's Wrought-Iron Bridges and Roofs, London, 1869."

In this figure a semi-beam, AB, fixed in a wall at A, is cut through at CD, and the severed piece, CB, is held in place by means of a strut at D and a link at C, which resist the compression and tension due to the cross-strain arising from the weight P; and by the weight R (equal to P) suspended over a pulley E, which prevents the severed beam from sinking, or resists the shearing strain.

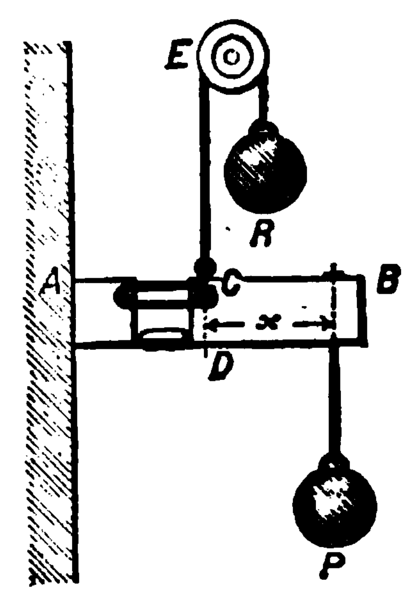

FIG. 30.

As the link C and strut D are both acting in a horizontal direction, they can have no effect in resisting a vertical strain, consequently the weight P must be entirely sustained by the counter-weight R, and as the action of the latter is directly opposite to that of the former, it must be equal to it in amount.

In the above arrangement we may see that were the strut D removed, the beam CB, under the action of the weight P, would revolve upon C as a centre, closing the gap at the bottom; hence the strut D is compressed.

In like manner, if the link at C were removed, the

weight P would cause the beam to revolve on D, making wider the opening at the top, and showing that the link C is in tension. If the tension at C be represented by t, the compression at D by c, and the depth CD by d, then

$$td = cd = P \times CB$$

Disregarding the weight of the beam, the shearing strain at CD equals the weight P. As this strain is wholly independent of the distance between C and B, the beam may be cut at any point in its length with a like result as to the amount of the shearing strain. At every point we shall have $R = P$, or the shearing strain equal to the weight.

If the weight of the beam be included in the consideration, the shearing strain at any point will equal the weight P plus the weight of so much of the beam as extends beyond the point at which the shearing strain is considered.

Let CD be the cross-section at which it is required to find the shearing strain; let x equal the distance from this cross-section to B, in feet; and let e represent the weight per foot lineal of the beam; then the weight of the piece CB will equal ex, and the shearing strain at CD will equal $P + ex$, or the destructive energy is

$$D = P + ex$$

173.—Transverse and Shearing Strains Compared.—Before this formula can be available, it is needed to know the resistance of the different materials to this kind of force. Experiments have been made upon wrought-iron which show that its shearing resistance is about seventy-five per cent of its resistance to tension. If, in the absence of the experiments necessary to establish the resistance to shearing in materials generally, it be assumed that they bear the

same proportion to their tensile resistance as is found in wrought-iron, this shearing strength may be put equal to

$$R = \tfrac{3}{4}Tbd$$

in which T equals the absolute resistance to tension per square inch of cross-section.

The resistance of certain woods to tension may be found in Table XX.

When $D = R$ we have

$$P + ex = \tfrac{3}{4}Tbd$$

This gives bd, or the area of cross-section, equal only to the destructive energy. In this case rupture would ensue. We therefore introduce the factor of safety, a, and have

$$a(P + ex) = \tfrac{3}{4}Tbd \qquad (38.)$$

The portion of T considered safe is from one sixth to one ninth. We then have $a = 6$ to $a = 9$.

As an example: Suppose a semi-beam (as AB, *Fig.* 30) of white pine to be 10 feet long, and loaded at the end with $P = 10{,}000$ pounds; what would be the required area of cross-section at the wall?

Here the weight of the beam is so small in comparison with the load P that it may be neglected in the computation. Throwing it out of the formula, we have

$$Pa = \tfrac{3}{4}Tbd \qquad (39.)$$

Let $a = 9$ and $T = 12000$; then

$$10000 \times 9 = \tfrac{3}{4} \times 12000 \times bd$$

$$\frac{10000 \times 9}{\tfrac{3}{4} \times 12000} = bd = 10$$

To compare this requirement with that for the cross-strain, we make use of the formula for this strain, (*19.*),

$$4Pan = Bbd^2$$

and, making $a = 4$, have

$$4 \times 10000 \times 4 \times 10 = 500 \times bd^2$$

$$\frac{4 \times 10000 \times 4 \times 10}{500} = bd^2 = 3200$$

and, making $d = 16$, have

$$b \times 16^2 = 3200$$

$$b = \frac{3200}{16^2} = 12 \cdot 5$$

therefore the area will be $12\frac{1}{2} \times 16 = 200$ square inches.

This is the area required at the wall, but at the end B, the point of attachment of the weight, we have seen (*Fig.* 27) that the destructive energy in cross-strain is zero. Were this the only effect produced by the weight P, the beam might be tapered here to a point. Owing, however, to the shearing effect of the weight, we find, as above, a requirement of material equal to 10 inches in area, or the beam $12\frac{1}{2}$ inches wide would require to be eight tenths of an inch thick; and the rope supporting the weight should be so attached as to have a bearing across the whole width of the piece.

174.—Rule for Shearing Strain at Ends of Beams.— The shearing strains at the two supports upon which a beam is laid are together equal to the weight of the beam and the load laid upon it. If the beam be of equal cross-section throughout its length, and the load upon the beam be located at the middle, or symmetrically about the middle, then the

weight of the beam and its load will be sustained half upon each support. In this case, the shearing strain at the two supports will be equal, and each equal to half the total load. Putting W for the load upon the beam, and el for the weight of the beam, then for the shearing strain at each end of the beam we have

$$\tfrac{1}{2}W+\tfrac{1}{2}el=D$$

Putting this equal to the safe resistance [see formula (*38.*), *Art.* **173**] we shall have

$$a\left(\tfrac{1}{2}W+\tfrac{1}{2}el\right)=\tfrac{2}{3}Tbd$$

$$\tfrac{1}{2}a\,(W+el)=\tfrac{2}{3}Tbd$$

$$\frac{2a}{3T}(W+el)=bd \qquad (40.)$$

When the load is not at the middle nor symmetrically disposed about the middle, the portion borne upon each support may be found by formulas (*3.*) and (*4.*), *Art.* **27**. The shearing strain at each support is equal to the reaction of the support or to the load it bears.

175.—Resistance to Side Pressure.—Beyond the foregoing considerations, there is still another of some importance. Care should be taken that the surfaces of contact of the wall and the beam are of sufficient area to be unyielding. Usually the wall composed of brick or stone is so firm that there need be no apprehension of its failure, and yet it is well to *know* that it is safe. It should, therefore, be carefully considered, to see that the given surface is sufficiently large for the given material to carry safely the weight proposed to be distributed over it. In calculations for heavy roof trusses this precaution is particularly necessary.

The upper surface of the joint, or under side of the beam,

requires especial attention. This is usually of timber, and parallel with the fibres of the material. The pressure upon the surface tends to compress these fibres more compactly together by closing the cells or pores which occur between the fibres. When pressed in this way, timber is much more easily crushed, as may readily be supposed, than when the pressure is applied at the ends of the fibres in a line parallel with their direction.

The resistance to side pressure approaches the resistance to end pressure in proportion to the hardness of the material.

By experiments made by the author some years since, to test the side resistance, results of which are recorded in the *American House Carpenter*, page 179, it appears that the hardest woods, such as lignum-vitæ and live oak, will resist about $1\frac{1}{2}$ times the pressure endwise that they will sidewise; ash, $1\frac{3}{4}$ times; St. Domingo mahogany, twice; Baywood mahogany, oak, maple and hickory, about 3 times; locust, black walnut, cherry and white oak, about $3\frac{1}{2}$ times; Georgia pine, Ohio pine and whitewood, about 4 times; chestnut, 5 times; spruce and white pine, 8 times; and hemlock, 9 times. Their resistance to side pressure is in proportion to the solidity of the material, or inversely in proportion to the size of the pores of the wood.

In the above classification, the comparison is not that of the *absolute* resistance of the several kinds of wood to side pressure. It is only a comparison of the results of the two pressures on the same wood. Whitewood, classed above orgia pine, resists sidewise only as much, absolutely, pine. Its power of resistance to end pressure is the f any of the woods, being but one half that of white

average effectual resistance to side pressure per nch of surface, p, for

Spruce	= 250	pounds.
White pine	= 300	"
Hemlock	= 300	"
Whitewood	= 300	"
Georgia pine	= 850	"
Oak	= 950	"

Under these pressures only a slight impression is made, and the woods may be safely trusted with these respective amounts.

176.—Bearing Surface of Beams upon Walls.—The surface of the beam in contact with the wall must be sufficient in extent to insure that it shall not be exposed to more pressure than is above shown to be safe. If b equal the breadth of the beam, h the length of the bearing surface, and p the resistance per inch, as above, then the total resistance equals

$$R = bhp$$

The destructive energy for one end of the beam is, as before (*Art.* **174**),

$$D = \tfrac{1}{2}W + \tfrac{1}{2}el$$

When there is equilibrium, then $R = D$, or

$$\tfrac{1}{2}(W+el) = bhp \qquad (41.)$$

Owing to the deflection of the beam by the load upon it, its extreme ends may be slightly raised from off the bearing surface, and in consequence the pressure be concentrated at the edge of the wall. No serious effect will ensue from this, for if the pressure be greater than the timber can resist at the edge, the fibres will be crushed there, but only sufficiently so to allow the surface of contact to extend towards

the end of the beam, until it is so enlarged as to effectually resist any further crushing.

Beams which are likely to be depressed considerably should have their ends formed so that their under surface will coincide throughout with the wall surface when the greatest load shall have been put upon them.

177.—Example to Find Bearing Surface.—Let a white pine carriage beam 6 inches wide, 24 feet long between bearings, and weighing 15 pounds per lineal foot, be loaded with 12,000 pounds, equally distributed over its length. What should be the length of the bearing upon each wall?

By transposition, formula (*41.*) becomes

$$\frac{W+el}{2bp} = h$$

In this case, $W = 12{,}000$, $e = 15$, $l = 24$, $b = 6$, and $p = 300$; then

$$\frac{12000 + \overline{15 \times 24}}{2 \times 6 \times 300} = h = 3{\cdot}43$$

or the end of the beam must extend upon the wall, say $3\frac{1}{2}$ inches. The usual bearing for floor beams, which is 4 inches, would in this case be amply sufficient.

Where the concentrated weight is so large in comparison with the weight of the beam, the latter weight may be neglected without any serious result; for had we considered the 12,000 pounds only, in the above example, the value of h would have been 3·33, only a tenth of an inch shorter than the former result.

178.—Shape of Side of Beam, Graphically Expressed.—As will be observed, we have digressed from the principal subject. This became necessary in order to explain the apparently anomalous result of leaving the beam without any

support at the ends. For it was seen that in an application of the formula for cross-strains the requirement of material gradually lessened towards the ends of the beam, until at the very edge of the bearings it entirely disappeared.

To prevent the beam, with its load, from falling as a dead weight between the bearings; or, to provide against the *shearing* strain, as well as against the crushing of the material upon its bearings, we have turned aside so far as seemed to be needed. And before returning to the main subject, it may be well here to show that the line CB in *Figs.* 27 and 29, limiting the ordinates of cross-strain in the lever and beam, does not show, as might be supposed, the shape of the depth of a lever or beam having a cross-section of equal strength throughout its length. A short consideration of the relation between the strains at given points in the length will show the true shape.

By construction, c, *Fig.* 27, is equal to $\frac{1}{2}lP$, and from this we have shown (*Art.* **168**) that

$$y = Px$$

and when the destructive energy and the resistance are equal

$$\frac{1}{2}lP = Sbd^2 \qquad \text{and}$$
$$Px = Sbd_{,}^2 \qquad \text{from which}$$
$$c : y :: Sbd^2 : Sbd_{,}^2 \qquad \text{and when}$$

S and b are constant

$$c : y :: d^2 : d_{,}^2$$

or, the ordinates are in proportion to the *squares* of the depths, and not directly as the *depths* themselves.

From these ordinates, however, the shape of the side of the lever may be directly found by taking their square roots. For let AB in *Fig.* 31 be the upper edge of the lever, and

CB the line limiting the ordinates of cross strain. Then, if AD be made equal to the square root of AC, and, correspondingly, $d_{,}$, $d_{,,}$, $d_{,,,}$, etc., be each made respectively equal to the square root of the ordinate upon which it lies, and if a line be drawn through the ends of $d_{,}$, $d_{,,}$, $d_{,,,}$, etc., this line, DEB, will limit the shape of the lever.

FIG. 31.

This curve line is a semi-parabola, with its vertex at B and its base vertical at AD. By construction, each ordinate y is in proportion to x, its distance from B, or (since y equals d^2) d^2 is in proportion to x, a property of the parabola. Hence to obtain the shape of the lower edge of the lever, any method of describing a parabola may be used, making AD, its base, equal to (*form.* 19.)

$$d = \sqrt{\frac{4Pan}{Bb}}$$

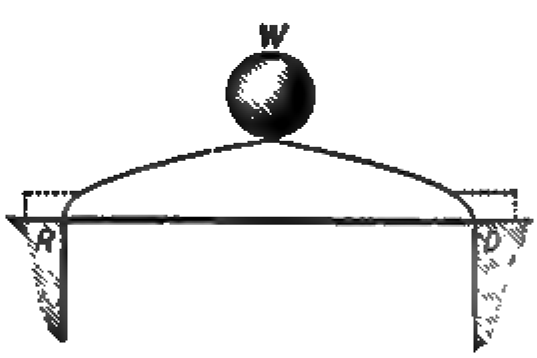

FIG. 32.

As a whole beam is in like condition with two semi-beams, as to the cross strains, therefore the shape of a whole beam of equal strength throughout its length is that given by two semi-parabolas placed base to base, as in *Fig.* 32.

QUESTIONS FOR PRACTICE.

179.—In a semi-beam, or lever, 10 feet long, fixed in a wall, and loaded at the free end with 3672 pounds, what is the destructive energy at the wall?

180.—Make a graphic representation of the above by a horizontal scale of one foot to the inch, and a vertical scale of 1000 foot-pounds to the inch. What is the height *CA* of the triangle of cross-strains, in terms of the scale selected?

181.—Measuring horizontal distances from the free end, what are the lengths, by the scale, of the respective ordinates at the several distances of 5, 6, 7, 8 and 9 feet; and what the amount of cross-strain corresponding thereto at these several points in the beam?

182.—What will be the required depth at the wall, and at 9 and 8 feet respectively from the free end; the lever being of Georgia pine, 6 inches broad, and the factor of safety 4?

183.—In a white pine beam, 4 inches broad, 16 feet long between bearings, and loaded at the middle with 3250 pounds, what should be the respective depths at the several distances of 3, 5, 7 and 8 feet from one end, the factor of safety being 4?

184.—A white pine semi-beam, 12 feet long and 4 inches broad, is loaded with 693 pounds at the free end, including the effect of the weight of the beam itself. The factor of safety is 4, the beam is of constant breadth and depth

throughout its length, and its weight is 30 pounds per cubic foot.

What is its required depth at the wall?

What is the weight suspended from the end of the beam?

What is the shearing strain at the wall?

What is the shearing strain at 5 feet from the wall?

185.—A beam of Georgia pine, 4 inches broad and 20 feet long, is loaded at the middle with $9644\frac{2}{3}$ pounds. The beam is 17 inches high at the middle, and tapered in parabolic curves to each end. The material of the beam is estimated at 48 pounds per cubic foot. What is the weight of the beam?

186.—What is the shearing strain at each wall?

With a factor of safety of 9, how high is the beam required to be at the ends to resist the shearing strain safely?

187.—How far upon each wall is the beam required to extend, in order to prevent crushing of the material?

CHAPTER IX.

STRAINS REPRESENTED GRAPHICALLY.

ART. **188.—Graphic Method Extended to Other Cases.**— In *Figs.* 27, 28 and 29, with a given maximum strain upon a semi-beam, or upon a full beam, we have a ready method of finding the strain at any given point in the length.

This simple method of ascertaining the strain at any point, graphically, is based upon a principle which is applicable to strained beams under conditions other than those given, as will now be shown.

189.—Application to Double Lever with Unequal Arms.— In *Figs.* 28 and 29 the load upon the beam is at the middle. But it may be shown that the triangle of strains is applicable in cases where the load is not at the middle.

Let R and P, *Fig.* 33, represent two unequal weights,

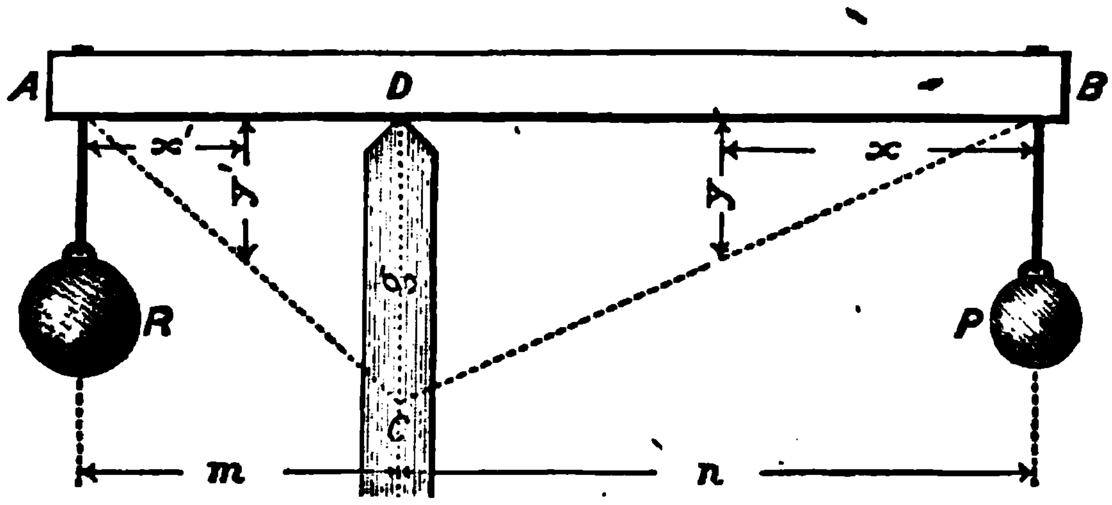

FIG. 33.

suspended from the ends of a balanced lever AB. From the law of the lever, we have (*Art.* **27**)

$$Rm = Pn$$

If CD, called g, be made of a length to represent Pn, then will it also represent Rm; for $Rm = Pn$. Hence, since the triangle BCD is the triangle of strains, in which an ordinate, y, showing the strain at any given point in DB, may be drawn, therefore the triangle ACD will give ordinates, y', measuring the strains at the points in AD, from which they may be drawn; or, since

$$Pn : g :: Px : y$$

$$y = \frac{g}{n} x \qquad (42.)$$

so also

$$Rm : g :: Rx' : y'$$

$$y' = \frac{g}{m} x' \qquad (43.)$$

190.—Application to Beam with Weight at Any Point.— In *Fig.* 34, AB represents a beam supported at each end, carrying a load W at a point nearer to A than to B. This

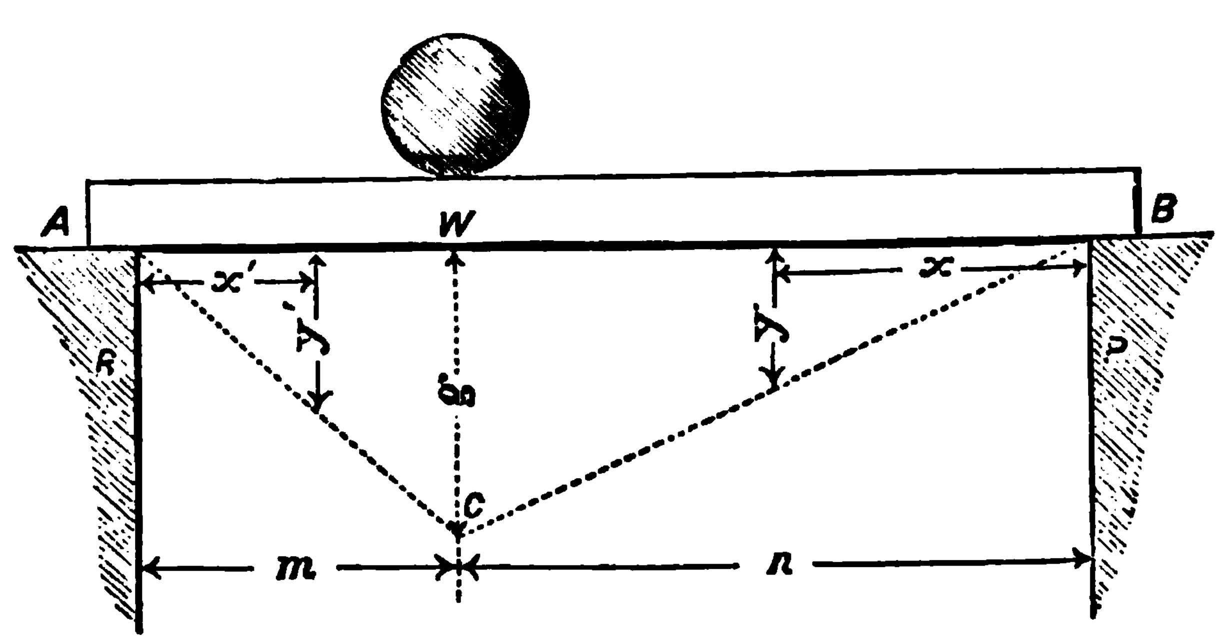

FIG. 34.

beam is strained in all respects like that in *Fig.* 33, except that the strains are in reversed order. Therefore an ordinate, y, drawn across the triangle BCW, will indicate the strain at the point of its location. So an ordinate, y', across the triangle ACW, will indicate the strain at its point of

location. Or, generally, the two triangles ACW and BCW limit the ordinates which measure the strains at any point in the length of the beam. Thus when

$$g = Pn = Rm \qquad \text{we have}$$

$$y = Px \quad \text{and} \quad y' = Rx'$$

and since $\quad P = W\frac{m}{l} \quad \text{and} \quad R = W\frac{n}{l}$ (*Art.* **27**)

we have
$$y = W\frac{m}{l}x \qquad (44.)$$

$$y' = W\frac{n}{l}x' \qquad (45.)$$

Now, since $Rm = Pn = g$, equals the destructive energy of the weight at its location, therefore any ordinate across the triangles ACW and BCW equals, when measured by the same scale, the destructive energy at the location of that ordinate, and when the resistance is equal to the destructive energy we have for the strain at any point to the right of the weight

$$W\frac{m}{l}x = Sbd^2$$

Putting for S its equivalent $\frac{1}{4}B$ (*Arts.* **35** and **57**) to agree with the unit of dimensions, we have, for the safe weight,

$$4Wa\frac{m}{l}x = Bbd^2 \qquad (46.)$$

which, with x at its maximum equal to n, is identical with formula (*23.*).

For the safe weight at any point to the left of the weight we have

$$4Wa\frac{n}{l}x' = Bbd^2 \qquad (47.)$$

191.—Example.—As an example in the application of these expressions, let it be required to find the strains at

various points in the length of a white pine beam, the maximum strain being given.

Let the beam be 10 feet long and loaded with 2000 pounds at a point three feet from the left-hand end.

What is the strain at the location of the weight? What are the several strains at 2, 4 and 6 feet from the right-hand end and at 2 feet from the left-hand end?

Take first the strains to the right.

Here, by formula (*44.*), $y = W\frac{m}{l}x$, and with x at its maximum we have

$$y = 2000 \times \frac{3}{10} \times 7 = 4200$$

In *Fig.* 35, make the length between the bearings A and B by any scale, equal to 10 feet, and CW, or g, equal to 42 units of any other scale. Then each of these units will

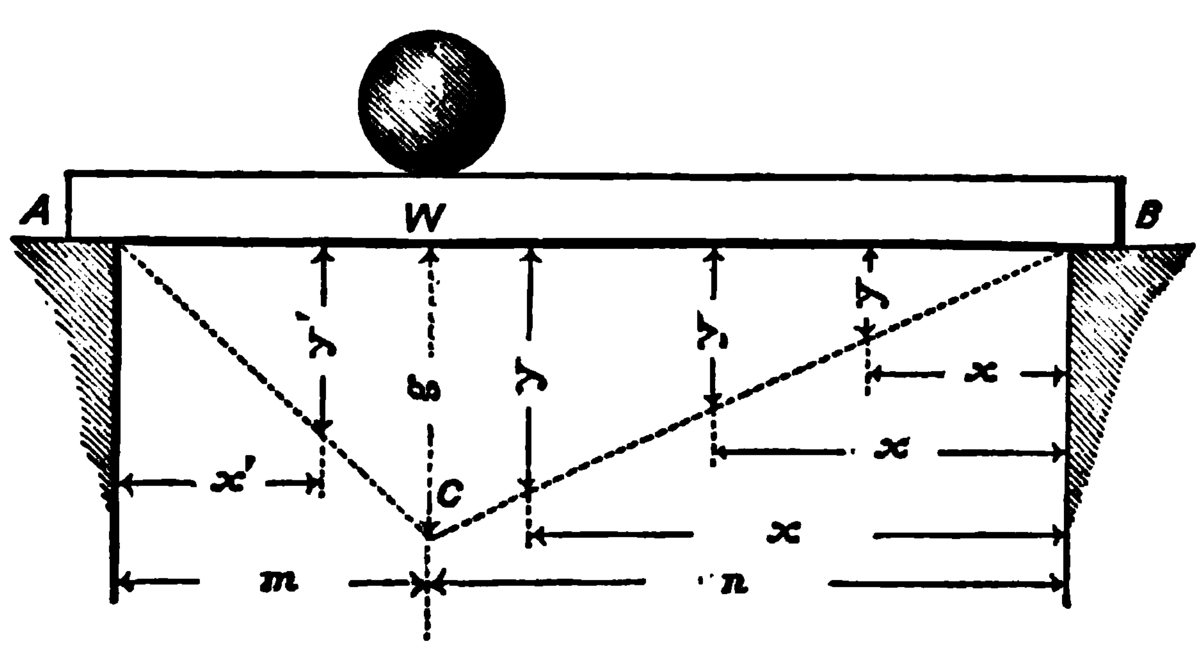

FIG. 35.

represent 100 pounds of strain. The number of units in the length of the ordinates, y, at the several distances, x equal to 2, 4 and 6 feet, and of $x' = 2$ feet, will give, when multiplied by 100, the strains at these several points. Thus it will be found that,

	at 2	feet	from B,	$y = 12$,	and	$12 \times 100 = 1200$;
	" 4	"	" B,	$y = 24$,	"	$24 \times 100 = 2400$;
	" 6	"	" B,	$y = 36$,	"	$36 \times 100 = 3600$;
and	" 2	"	" A,	$y' = 28$,	"	$28 \times 100 = 2800$.

Now, if it be required to find the proper depth of the beam at these several points, we take, for the right-hand end, formula (*46.*),

$$4Wa\frac{m}{l}x = Bbd^2$$

in which W represents 2000 pounds, the weight upon the beam, and in which $W\frac{m}{l}x$ will give the strain at each ordinate; and by transposition have

$$d^2 = \frac{4Wam}{Bbl}x \qquad (48.)$$

and if $a = 4$, $B = 500$ and $b = 3$, we have

$$d^2 = \frac{4 \times 2000 \times 4 \times 3}{500 \times 3 \times 10}x = 6{\cdot}4\,x$$

and therefore

when	$x = 2$	then	$d^2 = 6{\cdot}4 \times 2 = 12{\cdot}8$	and	$d = 3{\cdot}58$
"	$x = 4$	"	$d^2 = 6{\cdot}4 \times 4 = 25{\cdot}6$	"	$d = 5{\cdot}06$
"	$x = 6$	"	$d^2 = 6{\cdot}4 \times 6 = 38{\cdot}4$	"	$d = 6{\cdot}20$
"	$x = n = 7$	"	$d^2 = 6{\cdot}4 \times 7 = 44{\cdot}8$	"	$d = 6{\cdot}69$

For the left-hand end we use formula (*47.*)

$$4Wa\frac{n}{l}x' = Bbd^2$$

$$d^2 = \frac{4Wan}{Bbl}x' \qquad (49.)$$

$$d^2 = \frac{4 \times 2000 \times 4 \times 7}{500 \times 3 \times 10} x' = 14 \cdot 93\, x'$$

and hence,

when $x' = 2$ then $d^2 = 14 \cdot 93 \times 2 = 29 \cdot 9$ and $d = 5 \cdot 47$

" $x' = m = 3$ " $d^2 = 14 \cdot 93 \times 3 = 44 \cdot 8$ " $d = 6 \cdot 69$

This last result agrees with the last from the right-hand end, as it should, for they are both for the same location. The above results are all obtained by computations, but the value of d^2, at as many points as may be desired, can be obtained by scale, in a similar way with the ordinates for the destructive energy; but this scale, for the purpose of obtaining the depths, must be made with the principal ordinate, g, equal to the requirement

$$d^2 = \frac{4Wamn}{Bbl} \qquad (50.)$$

(see *form. 23.*), and then the square root of each ordinate drawn across the scale will be the required depth at its location.

For example: Make g, by any convenient scale, equal to $44 \cdot 8$ as above required; then the several values of d^2 at 2, 4 and 6 feet may be found by measuring the ordinates drawn at these several distances from B.

The square root of each ordinate will equal the depth of the beam there. The results obtained by measurements, although not exact to the last decimal, are yet sufficiently exact for all practical purposes. If it be required to find the exact dimension, this may be done by computation, as shown, and the diagram will then serve the very useful purpose of checking the result against any serious error in the calculation.

192.—Graphical Strains by Two Weights.—The value of graphic representations is manifest where two or more weights are carried at as many points upon a beam.

In *Fig.* 36 we have a beam carrying two weights A' and B'.

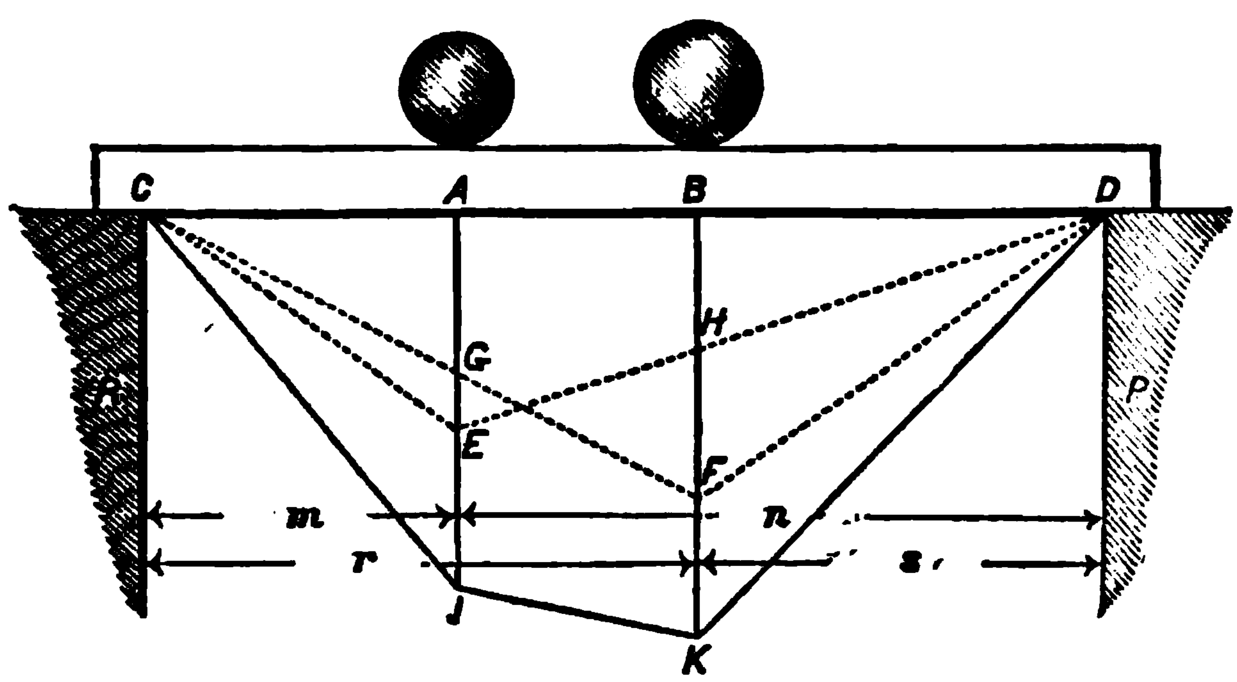

FIG. 36.

The destructive energy of the weight A', at its location, is equal to (*Art.* **56**)

$$D' = A'\frac{mn}{l}$$

and the destructive energy of the weight B', at *its* location, is equal to

$$D'' = B'\frac{rs}{l}$$

Make AE equal to $A'\frac{mn}{l}$ by any convenient scale. By the same scale make BF equal to $B'\frac{rs}{l}$. Draw the lines CE and DE, CF and DF.

Now, while AE represents the effect of the weight A' at the point A, so also AG measures (*Art.* **190**) the effect, at the same point, of the weight B'; therefore make EJ equal to AG, then AJ is the total effect at A of both weights.

In like manner (*FK* being made equal to *BH*), *BK* measures the total effect at *B*. Draw the line *CJKD*, and by dropping a vertical ordinate from any point in the beam *CD* to this line, we have the total strain in the beam at that point.

193.—Demonstration.—The above may be proved, as follows:

First. Let the ordinate occur between the two weights as *LM*, *Fig.* 37.

Extend the lines *CF*, *DE* and *JK*, till they meet at *R* and *S*, and draw *CR* and *DS*.

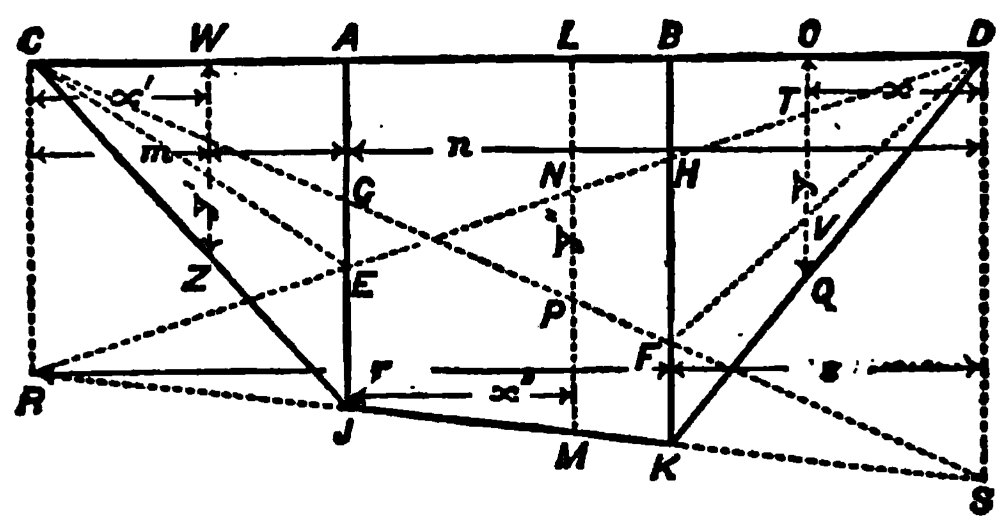

FIG. 37.

Now the effect of *B'* at *B*, is measured by *BF*, and at *L* by *LP* (*Art.* 189). Also the effect of *A'* at *A*, is measured by *AE*, and at *L* by *LN*. The joint effect of *A'* and *B'* at *L*, is thus $LP+LN$, and if it can be shown that *PM* equals *LN*, then

$$LP+LN=LP+PM=LM$$

equals the joint effect of the two weights *A'* and *B'*, at *L*.

In two triangles of equal base and altitude, two lines drawn parallel to the respective bases, and at equal altitudes, are equal; from which, conversely, if two triangles of equal base have equal lines drawn parallel to the base, and

at equal altitudes, then the altitudes of the two triangles are equal. In the present case we have $AE = GJ$; for $AG = EJ$ by construction; and if, to each of these equals we add the common quantity GE, the sums will be equal, or

$$AG + GE = GE + EJ$$
$$AE = GJ$$

The two triangles ADE and GSJ are therefore standing upon equal bases, AE and GJ.

Moreover, at equal distances, AB, from the line of bases AJ, and parallel with it, we have the two lines BH and FK, made equal by construction. Consequently, the two triangles have equal altitudes. Hence all lines drawn across them, parallel with and at equal distances from the base, are equal, and therefore LN and PM, having these properties, are equal, and $LM = LP + LN$ equals the true measure of the strain induced at L by the weights A' and B'; or, in general, any vertical ordinate drawn across $AJKB$ will measure the total strain caused by the two weights at the location of the ordinate.

194.—Demonstration—Rule for the Varying Depths.— *Second.* Let the ordinate occur at one end, between B and D, as OQ, *Fig.* 37.

Here we have OT for the strain caused by A', and OV for the strain caused by B'; or the total strain equals $OT + OV$.

Now if VQ can be proved equal to OT, we shall have

$$OT + OV = VQ + OV = OQ$$

equal to the total strain at O.

We have the two triangles BDH and FDK, with bases

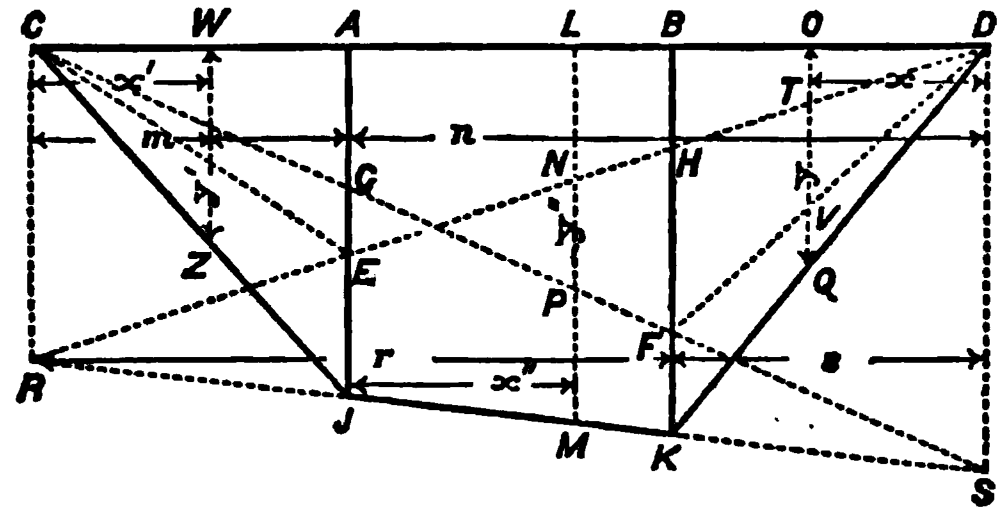

FIG. 37.

BH and *FK*, made equal by construction, and with equal altitudes *BD*, and we have the two lines *OT* and *VQ* drawn parallel with, and at equal altitudes (*BO*) from the base; consequently *OT* and *VQ* are equal, and *OQ* measures the total strain of the two weights at *O*; or, in general, any vertical ordinate drawn across *BDK* will measure the total strain at the location of the ordinate.

Since it may be shown in like manner that any vertical ordinate drawn across *ACJ* will measure the total strain at its location, therefore we conclude that a vertical ordinate from *any* point in the beam *CD* to the line *CJKD* will show the total strain in the beam at that point.

In practice, the scale of strains *CJKD* may be constructed as just shown, in detail, but more directly by obtaining the points *J* and *K* in the following manner:

We have for the joint effect of the two weights at the location of one of them, *A*, (see *Art.* **153**)

$$D = \frac{m}{l}(Wn + Vs)$$

which becomes, on changing *W* and *V* into *A'* and *B'*,

$$D = \frac{m}{l}(A'n + B's) \qquad (51.)$$

equals the length ot the ordinate *AJ*.

In like manner we have

$$D' = \frac{s}{l}(B'r + A'm) \qquad (52.)$$

for the length of the line *BK*.

The points *J* and *K* are to be obtained by these expressions. The scale is then completed by connecting these points and the ends of the beam by the line *CJKD*. The strain at any point in the beam may then be readily measured, sufficiently near for all practical purposes.

If, however, the exact strain is desired, this may be obtained as follows:

Putting g for AJ, p for BK, and h for AB, we have for the several ordinates

$$s : p :: x : y$$

$$y = \frac{p}{s}x. \qquad (53.)$$

$$m : g :: x' : y'$$

$$y' = \frac{g}{m}x' \qquad (54.)$$

$$h : p-g :: x'' : y''-g$$

$$h(y''-g) = x''(p-g)$$

$$hy''-hg = x''(p-g)$$

$$hy'' = x''(p-g) + hg$$

$$y'' = \frac{p-g}{h}x'' + g \qquad (55.)$$

If it be required to know the *depth of the beam* at every point, to accord with the strain there, then, instead of making the two principal ordinates as above shown, find their lengths thus:

By formulas (*30.*) and (*51.*) make AJ equal to

$$d^2 = \frac{4a\frac{m}{l}(A'n + B's)}{Bb} \qquad (56.)$$

and by formulas (*31.*) and (*52.*) make BK equal to

$$d^2 = \frac{4a\frac{s}{l}(B'r + A'm)}{Bb} \qquad (57.)$$

Draw the line $CJKD$, and then an ordinate drawn across this scale at any point will give the *square* of the depth at that point. The square root of this length will be the required depth there.

195.—Graphical Strains by Three Weights.—In *Fig.* 38 we have a graphical representation of the strains resulting from *three* weights.

to the
' B' at
ie same

scale. Connect J, K and L each with the ends of the beam E and D. Make JF equal to $AM+AN$, KG equal to $BO+BP$, and LH equal to $CQ+CR$.

Join E, F, G, H and D, and this line will be the boundary of any vertical ordinate from any point in ED, which, by the same scale as used for AJ, etc., will measure the strain at the location of the ordinate.

In this diagram, the points F, G and H may be found directly, as follows:

To find F, we have (*Art.* **153**) $A'\frac{mn}{l}$ for the effect of A', $B'\frac{ms}{l}$ for B', and so, in like manner, we may have $C'\frac{mv}{l}$ for that of C'. Added together, these will equal

$$AF = \frac{m}{l}(A'n + B's + C'v) \qquad (58.)$$

To find G, we have $A'\frac{ms}{l}$ for A', $B'\frac{rs}{l}$ for B', and $C'\frac{rv}{l}$ for C'; which together give

$$BG = \frac{A'ms + B'rs + C'rv}{l} \qquad (59.)$$

To find H, we have $A'\frac{mv}{l}$ for A', $B'\frac{rv}{l}$ for B', and $C'\frac{tv}{l}$ for C'; which added, will equal

$$CH = \frac{v}{l}(A'm + B'r + C't) \qquad (60.)$$

If it be desirable, the strains may, as in the last figure, be computed; for putting g for AF, p for BG, k for CH, h for AB, and q for BC, we have, for an ordinate between C and D,

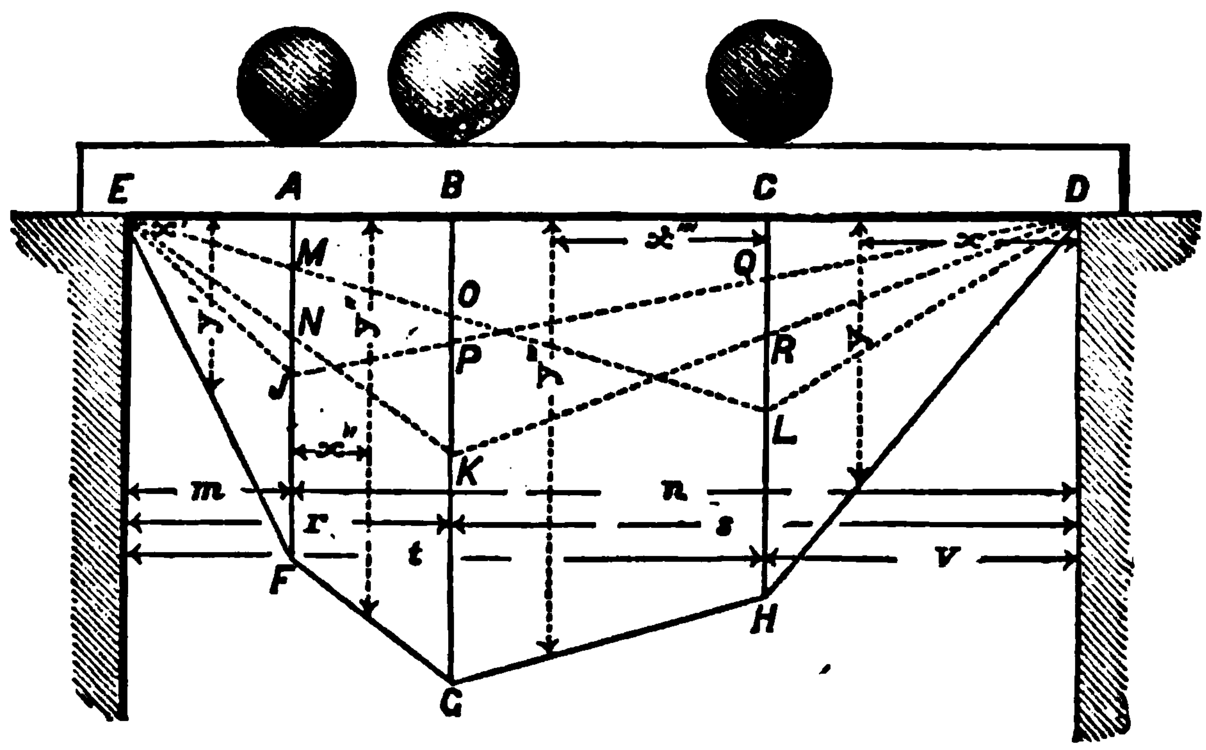

FIG. 38.

$$v : k :: x : y$$

$$y = \frac{k}{v} x \tag{61}$$

For an ordinate between E and A we have

$$m : g :: x' : y'$$

$$y' = \frac{g}{m} x' \tag{62.}$$

For an ordinate between A and B we have, as in *Fig.* 37,

$$y'' = \frac{p-g}{h} x'' + g \tag{63}$$

and for ordinates occurring between B and C we have

$$y''' = \frac{p-k}{q} x''' + k \tag{64.}$$

These expressions give the strains at any point, due to the three weights.

In like manner, we may find the strain at any point in a beam, arising from any number of weights.

To obtain the squares of the depths at various points by scale, make AF equal to

$$d^2 = \frac{4a\frac{m}{l}(A'n + B's + C'v)}{Bb} \qquad (65.)$$

Make BG equal to

$$d^2 = \frac{4a\frac{A'ms + B'rs + C'rv}{l}}{Bb} \qquad (66.)$$

Make CH equal to

$$d^2 = \frac{4a\frac{v}{l}(A'm + B'r + C't)}{Bb} \qquad (67.)$$

The square roots of ordinates upon this scale will give the depths required at their several locations.

196.—Graphical Strains by Three Equal Weights Equably Disposed.—Let us now consider the effect of equal weights, equably disposed.

In *Fig.* 39 we have three equal weights, L, placed at equal distances apart upon a beam, ED, the distance from either wall to its nearest weight being one half that between any two of the weights; or,

$$EA = CD = \tfrac{1}{2}AB = \tfrac{1}{2}BC = \tfrac{1}{6}l$$

The line $EFGHD$ is obtained as directed for *Fig.* 38. It may also be obtained analytically, thus:

First. The line AF, or the effect at A of the three weights, equals the sum of the three lines AJ, AO and AN.

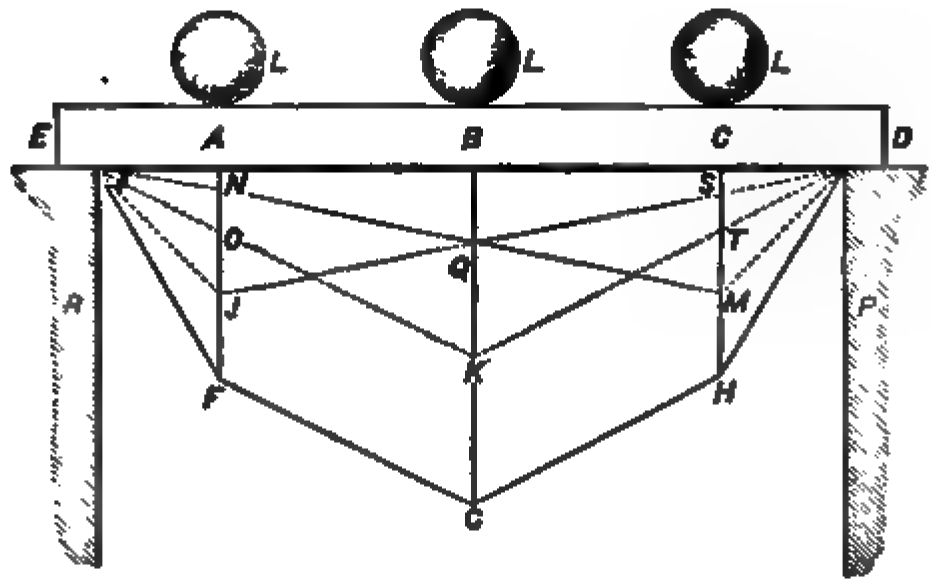

FIG. 39.

Let $EA = CD = t$, and $AD = h$, then $t + h = l$, and (*Art.* 56)

$$AJ = L\frac{t \times h}{l} = \tfrac{3}{3}L\frac{th}{l}$$

as per *Art.* 195.

$$AO = L\frac{BD \times EA}{l} = L\frac{\frac{2}{3}h \times t}{l} = \tfrac{2}{3}L\frac{th}{l}$$

$$AN = L\frac{CD \times EA}{l} = L\frac{\frac{1}{3}h \times t}{l} = \tfrac{1}{3}L\frac{th}{l}$$

$$AN = (\tfrac{3}{3} + \tfrac{2}{3} + \tfrac{1}{3})L\frac{th}{l} = \tfrac{6}{3}L\frac{th}{l} = AF$$

ine BG, or the effect at B of the three to the sum of the line BK and twice the

$BD = h$, and $t + h = l$; then

$$\frac{t \times h}{l}$$

$$\frac{EA \times BD}{l} = L\frac{\frac{1}{2}t \times h}{l} = \tfrac{1}{2}L\frac{th}{l}$$ and

$$= L\frac{th}{l} + \overline{2 \times \tfrac{1}{2}L\frac{th}{l}} = \tfrac{4}{2}L\frac{th}{l} = BG$$

Third. The effect at C produced by the three weights is equal to that at A.

We have, then,

for the total effect at A, $\quad AF = \frac{6}{3} L \frac{th}{l}$

" " " B, $\quad BG = \frac{8}{3} L \frac{th}{l}$

" " " C, $\quad CH = \frac{6}{3} L \frac{th}{l}$

197.—Graphical Strains by Four Equal Weights Equably Disposed.—When there are four equal weights, as in *Fig.* 40, similarly disposed as in *Fig.* 39, the effect at A is,

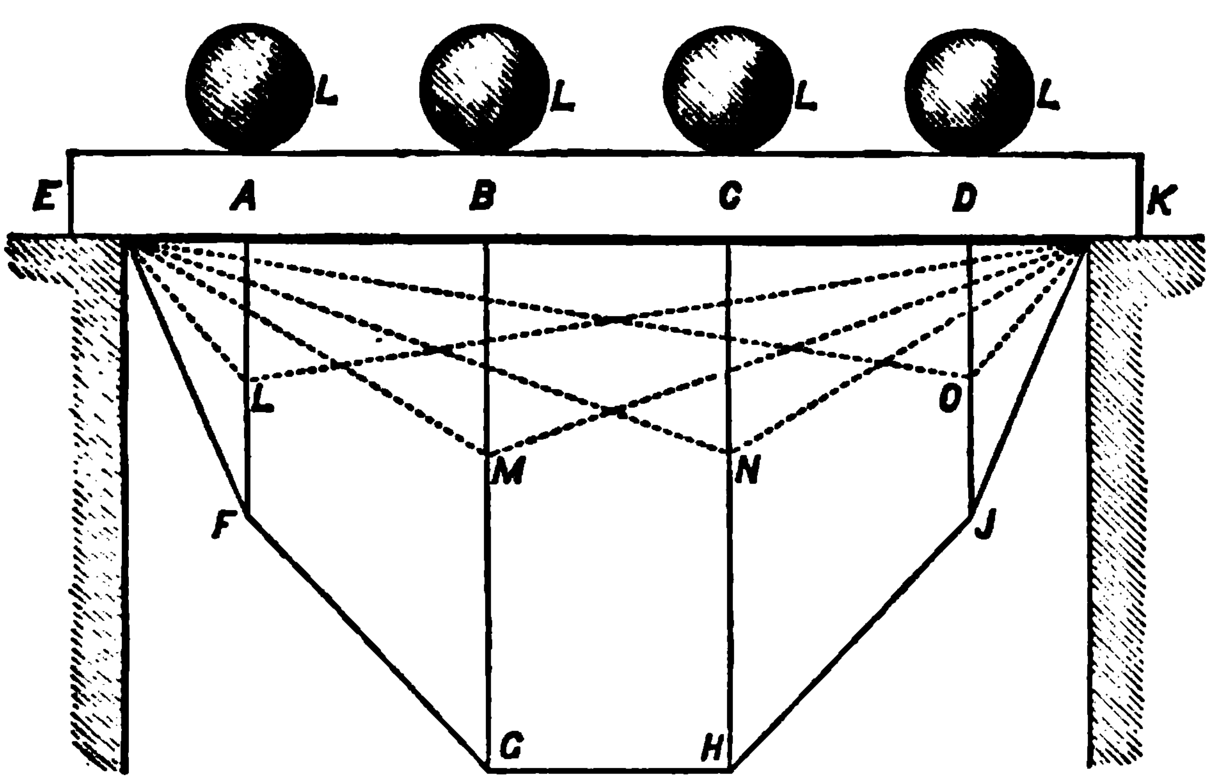

FIG. 40.

from load at A, $\quad L\frac{h \times t}{l} = \frac{7}{7} L \frac{ht}{l}$

" " B, $\quad L\frac{\frac{5}{7} h \times t}{l} = \frac{5}{7} L \frac{ht}{l}$

" " C, $\quad L\frac{\frac{3}{7} h \times t}{l} = \frac{3}{7} L \frac{ht}{l}$

" " D, $\quad L\frac{\frac{1}{7} h \times t}{l} = \frac{1}{7} L \frac{ht}{l}$

ct at A, of the four weights, is

$$+\tfrac{3}{5}+\tfrac{2}{5}+\tfrac{1}{5})L\frac{ht}{l} = \tfrac{10}{5}L\frac{ht}{l} = AF$$

B is,

oad at	A,	$L\frac{\frac{3}{5}t\times h}{l} = \frac{3}{5}L\frac{ht}{l}$
"	B,	$L\frac{t\times h}{l} = \frac{6}{5}L\frac{ht}{l}$
"	C,	$L\frac{\frac{4}{5}h\times t}{l} = \frac{4}{5}L\frac{ht}{l}$
"	D,	$L\frac{\frac{2}{5}h\times t}{l} = \frac{2}{5}L\frac{ht}{l}$

ct at B, of the four weights, is

$$+\tfrac{6}{5}+\tfrac{4}{5}+\tfrac{2}{5})L\frac{ht}{l} = \tfrac{15}{5}L\frac{ht}{l} = BG$$

t C is equal to that at B, and the effect at at at A.

ical Strains by Five Equal Weights Equably

en there are five equal weights, as in *Fig.* 41, ed as those in *Fig.* 39, the effect at A is,

oad at	A,	$L\frac{h\times t}{l} = \frac{5}{6}L\frac{ht}{l}$
"	B,	$\frac{4}{6}h\times t\frac{L}{l} = \frac{4}{6}L\frac{ht}{l}$
"	C,	$\frac{3}{6}h\times t\frac{L}{l} = \frac{3}{6}L\frac{ht}{l}$
"	D,	$\frac{2}{6}h\times t\frac{L}{l} = \frac{2}{6}L\frac{ht}{l}$
"	M,	$\frac{1}{6}h\times t\frac{L}{l} = \frac{1}{6}L\frac{ht}{l}$

L L L L L

FIG. 41.

or the total effect at A, of all the weights, is

$$\left(\tfrac{5}{5}+\tfrac{4}{5}+\tfrac{3}{5}+\tfrac{2}{5}+\tfrac{1}{5}\right)L\frac{ht}{l}=\tfrac{15}{5}L\frac{ht}{l}=AF$$

The total effect at B is,

from load at A,	$\tfrac{1}{2}t\times h\dfrac{L}{l}=\tfrac{1}{2}L\dfrac{ht}{l}$
" " B,	$h\times t\dfrac{L}{l}=\tfrac{4}{4}L\dfrac{ht}{l}$
" " C,	$\tfrac{3}{4}h\times t\dfrac{L}{l}=\tfrac{3}{4}L\dfrac{ht}{l}$
" " D,	$\tfrac{2}{4}h\times t\dfrac{L}{l}=\tfrac{2}{4}L\dfrac{ht}{l}$
" " M,	$\tfrac{1}{4}h\times t\dfrac{L}{l}=\tfrac{1}{4}L\dfrac{ht}{l}$

or the total effect at B, of all the weights, is

$$(\tfrac{1}{3}+\tfrac{7}{7}+\tfrac{5}{7}+\tfrac{3}{7}+\tfrac{1}{7})L\frac{ht}{l} = \tfrac{55}{21}L\frac{ht}{l} = BG$$

The total effect at C is,

from load at		A,	$\tfrac{1}{6}t \times h\frac{L}{l} = \tfrac{1}{6}L\frac{ht}{l}$
"	"	B,	$\tfrac{3}{6}t \times h\frac{L}{l} = \tfrac{3}{6}L\frac{ht}{l}$
"	"	C,	$h \times t\frac{L}{l} = \tfrac{5}{6}L\frac{ht}{l}$
"	"	D,	$\tfrac{3}{6}h \times t\frac{L}{l} = \tfrac{3}{6}L\frac{ht}{l}$
"	"	M,	$\tfrac{1}{6}h \times t\frac{L}{l} = \tfrac{1}{6}L\frac{ht}{l}$

or the total effect at C, of all the weights, is

$$(\tfrac{1}{6}+\tfrac{3}{6}+\tfrac{5}{6}+\tfrac{3}{6}+\tfrac{1}{6})L\frac{ht}{l} = \tfrac{13}{6}L\frac{ht}{l} = CH$$

The effects produced at D and M are, respectively, like those at B and A.

199.—General Results from Equal Weights Equably Disposed.—In looking over the results here obtained, it will be seen that in each case the effect is equal to $gL\frac{ht}{l}$, in which g is put for the numerical coefficient, L for any one of the equal weights with which the beam is loaded, t and h the respective distances from the point at which the strain is being measured to the ends of the beam, and l for the length of the beam. All of these are simple quantities except the coefficient g, and this it will be shown is subject to a certain law and may be stated in general terms.

200.—General Expression for Full Strain at First Weight. —The coefficient g is a fraction, having its numerator and denominator both dependent upon the number of weights upon the beam.

Let us first consider the value of the *numerator* in measuring the effect of the weights at A, the location of the first weight from the left.

With three weights, g, the coefficient, was $\frac{1}{5}+\frac{3}{5}+\frac{5}{5}=\frac{9}{5}$, the numerators being $1+3+5=9$.

With four weights, g was equal to $\frac{1+3+5+7}{7}=\frac{16}{7}$, the numerators being $1+3+5+7=16$.

With five weights, g was equal to $\frac{1+3+5+7+9}{9}=\frac{25}{9}$, and the numerators $1+3+5+7+9=25$.

In general, we shall find that the numerator of the fraction g, is in all cases equal to the sum of an arithmetical progression comprising the odd numbers 1, 3, 5, etc., to n terms; n being put to represent the number of weights upon the beam, the first term being unity, and the last being $2n-1$.

To find the sum of this progression, we have

$$S=\frac{(a+l)n}{2}$$

in which $S=$ the sum, $a=$ the first term, $l=$ the last term, and $n=$ the number of terms; or

$$S=\frac{\overline{1+(2n-1)}n}{2}=\frac{n+2n^2-n}{2}=n^2$$

Hence, the numerator of the coefficient of the expression showing the effect of any number of weights at the location, A, of the first weight, is equal to the square of the number of weights; thus, when there are

2 weights,	$n = 2$,	and the numerator		$= 2^2 =$	4
3 "	$n = 3$,	"	"	$= 3^2 =$	9
4 "	$n = 4$,	"	"	$= 4^2 =$	16
5 "	$n = 5$,	"	"	$= 5^2 =$	25
6 "	$n = 6$,	"	"	$= 6^2 =$	36

and so for any number of weights.

In considering the value of the *denominator* of g it will be observed that it is derived by taking the value of h in each case in terms of t. With three weights, $h = 5t$; with four weights, $h = 7t$; and with five weights, $h = 9t$; so that in general, $h = (2n-1)t$. The denominator of the fraction generally, therefore, is $2n-1$.

The value of the coefficient is, consequently, $\frac{n^2}{2n-1}$, and the full effect at A of any number of equal weights equably disposed upon a beam is $\frac{n^2}{2n-1} L \frac{ht}{l}$.

201.—General Expression for Full Strain at Second Weight.—For the effect at the location B we have the expression $pL\frac{ht}{l}$; in which the same quantities occur as before, except in the case of the coefficient p.

This coefficient is composed of two classes of fractions. The first of these is based upon the relation between the distances EA and EB, and since EA is in all cases equal to $\frac{1}{3}$ of EB, therefore this part of the coefficient p will be equal to $\frac{1}{3}$.

In the second fraction of the coefficient, the numerator is, as in the case at A, equal to the sum of an arithmetical progression, but extending one less in the number of the terms, so that in place of n^2 we put $(n-1)^2$.

The denominator is found by taking $n-1$ for n, or $2(n-1)-1$, equal to $2n-3$, for $2n-1$. The value of

this fraction is therefore $\frac{(n-1)^2}{2n-3}$. To this, adding the first fraction, we have

$$p = \tfrac{1}{3} + \frac{(n-1)^2}{2n-3}$$

and for the full effect at B, of all the weights,

$$\left(\tfrac{1}{3} + \frac{(n-1)^2}{2n-3}\right) L\frac{ht}{l}$$

From the above, the value of the coefficient p is as follows:

with 2 weights, $p = \tfrac{1}{3} + \frac{(2-1)^2}{(2\times 2)-3} = \tfrac{1}{3} + \tfrac{1}{1} = \tfrac{4}{3}$

" 3 " $p = \tfrac{1}{3} + \frac{(3-1)^2}{(2\times 3)-3} = \tfrac{1}{3} + \tfrac{4}{3} = \tfrac{15}{9}$

" 4 " $p = \tfrac{1}{3} + \frac{(4-1)^2}{(2\times 4)-3} = \tfrac{1}{3} + \tfrac{9}{5} = \tfrac{32}{15}$

" 5 " $p = \tfrac{1}{3} + \frac{(5-1)^2}{(2\times 5)-3} = \tfrac{1}{3} + \tfrac{16}{7} = \tfrac{55}{21}$

" 6 " $p = \tfrac{1}{3} + \frac{(6-1)^2}{(2\times 6)-3} = \tfrac{1}{3} + \tfrac{25}{9} = \tfrac{84}{27}$

The numerators of these results are in the order of $2n$, $5n$, $8n$, $11n$ and $14n$; the numerals differing by 3. The denominators are the products of 1, 3, 5, 7 and 9, each by 3. We may continue therefore the values to any number of weights by following these laws, thus

for 7 weights, $p = \frac{17\times 7}{11\times 3} = \frac{119}{33}$

for 8 weights, $p = \frac{20\times 8}{13\times 3} = \frac{160}{39}$

or, in general, the effect at B for any number of weights may be had directly from the previous expression.

202.—General Expression for Full Strain at Any Weight.—For the sum of effects at C, it is seen that we have $kL\frac{ht}{l}$, and it can be shown that the coefficient k is the sum of two fractions—namely, $\frac{4}{5}$ and $\frac{(n-2)^2}{2n-5}$ or

$$k = \tfrac{4}{5} + \frac{(n-2)^2}{2n-5}$$

For the effect at D we have

$$u = \tfrac{9}{7} + \frac{(n-3)^2}{2n-7}$$

For the effect at E we have

$$q = \tfrac{16}{9} + \frac{(n-4)^2}{2n-9}.$$

or, putting them in sequence, we have

at	A	the	effect	$g = \frac{0}{1} + \frac{(n-0)^2}{2n-1}$
"	B	"	"	$p = \frac{1}{3} + \frac{(n-1)^2}{2n-3}$
"	C	"	"	$k = \frac{4}{5} + \frac{(n-2)^2}{2n-5}$
"	D	"	"	$u = \frac{9}{7} + \frac{(n-3)^2}{2n-7}$
"	E	"	"	$q = \frac{16}{9} + \frac{(n-4)^2}{2n-9}$
"	F	"	"	$v = \frac{25}{11} + \frac{(n-5)^2}{2n-11}$

and so for any number of weights upon one end of the beam.

An examination of this series shows that in the first of the two fractions the numerator is equal to the square of the number of weights preceding the one under consideration; for instance, at A, where there are no weights preceding, we have the numerator 0; at B there is one weight preceding, and hence the numerator is 1^2 equals 1; at C there are two weights preceding, hence the numerator equals 2^2 equals 4; at D there are three weights, hence the numerator equals 3^2 equals 9; etc. For the denominator of the first fraction we have, for the several cases in consecutive order, the values 1, 3, 5, 7, etc.; an arithmetical series of the odd numbers.

In the second fraction we have a numerator equal to the square of the difference between n and the number of weights preceding the one at which the strain is being measured; and a denominator of $2n$ minus the denominator of the first fraction.

Let r represent in any case the number of weights preceding the one at the location of which we wish to know the strain. Then we shall have, as the coefficient of the effect at that point,

$$x = \frac{r^2}{2r+1} + \frac{(n-r)^2}{2n-(2r+1)}$$

and for the full effect, or the destructive energy,

$$D = L\frac{ht}{l}\left(\frac{r^2}{2r+1} + \frac{(n-r)^2}{2n-(2r+1)}\right) \qquad (68.)$$

in which L represents one of the equal weights with which the beam is loaded; h the distance from the weight at which the strain in the beam is being measured to the right-hand end of the beam; t the distance from the same point to the left-hand end; $l = h + t$ the length of the beam between sup-

ports; n the number of equal weights equally disposed upon the beam, as in *Fig.* 41; and r the number of weights between the point where the strain is measured and the left-hand end of the beam, not including the one at the point where the strain is measured.

203.—Example.—What is the strain at the fifth weight from the left-hand end of a beam 22 feet long, loaded with 11 weights of 100 pounds each; the weights placed at equal distances from centres, and the distance from each end of the beam to the centre of the nearest weight being equal to half the distance between the centres of any two adjoining weights? Here the distance between centres of weights will be 2 feet, t will equal 9 feet, and h will equal 13 feet, $L = 100$, $n = 11$, and $r = 4$.

From these the strain at the fifth weight will be (*form. 68.*)

$$D = 100 \times \frac{13 \times 9}{22} \left(\frac{4^2}{8+1} + \frac{(11-4)^2}{22-(8+1)} \right) = 2950$$

QUESTIONS FOR PRACTICE.

204.—A beam 12 feet long is loaded at 4 feet from the left-hand end with 4000 pounds. What is the strain at that point?

205.—What are the strains, respectively, at 2, 4, and 6 feet from the right-hand end?

206.—A beam 14 feet long is loaded with two weights; one, A', weighing 3000 pounds, is located at 4 feet from the left-hand end; the other, B', weighing 5000 pounds, is at 6 feet from the right-hand end.

What strain is caused by these two weights at the point A?

What strain is caused at B?

207.—In the above beam what strain is caused by the two weights at a point 2 feet from the left-hand end?

What strain is caused at a point 2 feet from the right-hand end?

What strain is produced at the middle of the beam?

208.—A beam 20 feet long is loaded with three weights; one, A', of 3000 pounds, at 3 feet from the left-hand end; one, B', of 2000 pounds, at 11 feet from the same end; and the third weight, C', of 4000 pounds, at 4 feet from the right-hand end.

What is the full effect of the three weights at the location of each weight, at 2 feet from the left-hand end, at 2 feet from the right-hand end, at 6 feet from the same end, and at the middle of the beam?

209.—A beam 16 feet long is loaded with 20 weights of 100 pounds each, the weights being equally distributed.

What strain do these weights produce in the beam at the ninth weight from one end?

CHAPTER X.

STRAINS FROM UNIFORMLY DISTRIBUTED LOADS.

ART. **210.—Distinction Between a Series of Concentrated Weights and a Thoroughly Distributed Load.**—The distribution of the load upon a beam, as shown in *Figs.* 39, 40 and 41, is essentially that of a uniform distribution over the entire length of the beam. For if the beam be divided into as many parts as there are weights, by vertical lines located midway between each two weights, it is seen that the parts into which these lines divide the beam are all equal one with another, and the weight upon each part is located in a vertical line passing through the centre of gravity of that part. Hence this beam, taken with the loads upon it, is an apparently parallel case with a beam having an equally distributed load.

An application of formula (*68.*), however, will show that the case is that of a beam loaded with a series of *concentrated* weights, and not with a thoroughly distributed load, although it closely approximates the latter. We find that the results of computations made with this formula differ according to the number of weights upon the beam, but approach a certain limit as the number of weights is increased; a limit which is that of a beam with an equally distributed load.

211.—Demonstration.—For example, let us find by formula (*68.*) the effects at the middle of the beam under differing numbers of weights.

We may modify the formula to suit this case, for $L \times n = U$, when U equals the total weight upon the beam, or $L = \frac{U}{n}$, and $h = t = \frac{1}{2}l$.

By substituting these values, we have

$$D = \frac{U}{n} \times \frac{\frac{1}{2}l \times \frac{1}{2}l}{l} \left(\frac{r^2}{2r+1} + \frac{(n-r)^2}{2n-(2r+1)} \right) = x$$

$$x = \frac{Ul}{4n} \left(\frac{r^2}{2r+1} + \frac{(n-r)^2}{2n-(2r+1)} \right) \qquad (69.)$$

To apply this modified formula to the question :

First. Let there be five weights equally disposed, or $n = 5$; then $r = 2$, and we have

$$x = \frac{Ul}{4 \times 5} \left(\tfrac{4}{5} + \tfrac{9}{5} \right) = \tfrac{13}{25} \times U \frac{l}{4}$$

Second. Let there be nine weights or $n = 9$, then $r = 4$, and we have

$$x = \frac{Ul}{4 \times 9} \left(\tfrac{16}{9} + \tfrac{25}{9} \right) = \tfrac{41}{81} \times U \frac{l}{4}$$

Third. If $n = 25$, then $r = 12$, and

$$x = \frac{Ul}{4 \times 25} \left(\tfrac{144}{25} + \tfrac{169}{25} \right) = \tfrac{313}{625} \times U \frac{l}{4}$$

Fourth. If $n = 101$, then $r = 50$, and

$$x = \frac{Ul}{4 \times 101} \left(\tfrac{2500}{101} + \tfrac{2601}{101} \right) = \tfrac{5101}{10201} \times U \frac{l}{4}$$

Comparing the coefficients of these several results, we have

when	$n = 5$,	the coefficient	$= \frac{13}{25}$	$= \frac{1}{2} + \frac{1}{50}$
"	$n = 9$,	" "	$= \frac{41}{81}$	$= \frac{1}{2} + \frac{1}{162}$
"	$n = 25$,	" "	$= \frac{313}{625}$	$= \frac{1}{2} + \frac{1}{1250}$
"	$n = 101$,	" "	$= \frac{5101}{10201}$	$= \frac{1}{2} + \frac{1}{20402}$

The result in all cases is equal to a half, plus a fraction which decreases as n increases, or which has unity for its numerator, and a denominator equal to twice the square of n. The coefficient may be expressed then by $\frac{1}{2} + \frac{1}{2n^2}$

Now, when the number of weights is unlimited, or the load thoroughly and equally distributed over the whole length, then n is infinite, and the denominator of the last fraction becomes infinity. In this case, the fraction itself equals zero and consequently vanishes.

Hence the coefficient tends towards $\frac{1}{2}$, and with the loads subdivided to the last degree, and infinite in number, actually becomes $\frac{1}{2}$; for, with these conditions fulfilled the case is actually that of an equally distributed load, and then

$$x = \tfrac{1}{2}U\frac{l}{4} = \tfrac{1}{8}Ul. \qquad \text{(See } Art. \textbf{ 59}.)$$

This value of the coefficient may be concisely derived by the use of the calculus, as will now be shown.

212.—Demonstration by the Calculus.—To obtain a formula to represent the strain caused at any point by an equally distributed load, let $RPTS$, *Fig.* 42, represent graphically an equally distributed load, SR being equal to TP, and let it be required to find the ordinate EF, equal to the effect at any point E, caused by the whole load.

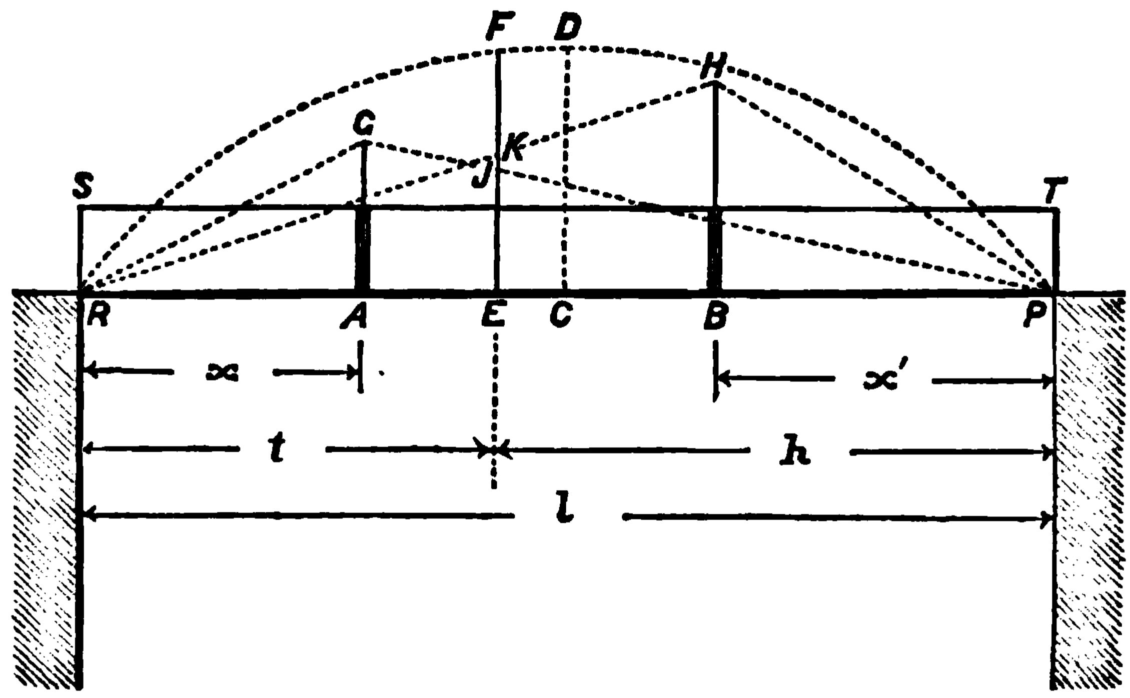

FIG. 42.

To do this we may proceed as follows: Let the ordinate AG represent by scale the strain caused at A by a small weight A', concentrated at A. Then will EJ represent (*Art.* **190**) the effect of A' at E. Again, let the ordinate BH represent by scale the strain at B caused by a small weight B', concentrated at B. Then will EK represent the effect of B' at E. The sum of these, $EJ+EK$, will equal the joint effect of the weights A' and B' at E. Or (*Art.* **190**)

$$D=\frac{A'hx}{l}+\frac{B'tx_{,}}{l}$$

Let the loads A' and B' be very small; equal to a small portion of the equally distributed load $SRPT$, and represented graphically by the thin vertical slices at A and B respectively, and let these slices be reduced to the smallest possible thickness. By the rules of the calculus we may represent the thickness of the slices, when infinitely reduced, by dx, the *differential* of x, or rate of increase. If e be put to represent the weight per lineal foot of the equally distributed load $SRPT$, then edx will represent the weight of the thin slice at A, or equal A'. So also $edx_{,}$ will represent the weight of the slice at B, or equal B'.

Substituting these values for A' and B' in the above expression, we obtain

$$D = \frac{ehxdx}{l} + \frac{etx_{,}dx_{,}}{l} = \frac{e}{l}\,(hxdx + tx_{,}dx_{,})$$

This is the effect at E of the two loads at A and B, but these loads are infinitesimally small, therefore the expression is to be considered merely as the sum of the *differential*, or rates of increase of the strains produced by the two parts into which the whole of the equally distributed load $RSTP$ is divided by the ordinate EF. The strain itself is to be had by the integral which is to be derived from the above differential of the strain. Therefore, by integration, we have (*Arts.* **462** and **463**)

$$\int \frac{e}{l}\,(hxdx + tx_{,}dx_{,}) = \frac{e}{l}\,(\tfrac{1}{2}hx^2 + \tfrac{1}{2}tx_{,}^2) = y$$

By integrating between $x = 0$ and $x = t$, also between $x_{,} = 0$ and $x_{,} = h$, or making the integral definite, we have

$$y = \frac{e}{l}\,(\tfrac{1}{2}ht^2 + \tfrac{1}{2}th^2)$$

$$y = \frac{et}{2l}(ht + h^2) = EF$$

but $\qquad h = l - t$

therefore $\qquad ht = (l - t)\,t$

and $\qquad h^2 = (l - t)^2$

therefore

$$ht + h^2 = (l-t)t + (l-t)^2 = lt - t^2 + l^2 - 2lt + t^2 = l^2 - lt$$

and the formula

$$y = \frac{et}{2l}(ht + h^2) \qquad \text{becomes}$$

$$y = \frac{et}{2l}(l^2 - lt)$$

$$y = \tfrac{1}{2}et\,(l - t) \qquad (70.)$$

This result gives the value of the ordinate y, drawn at any point, and is comparable with the formula for the parabola*, in which l equals the base, and the maximum ordinate, y, equals the height. Therefore, if the curve line *RFDP* be that of the parabola, it will limit all the ordinates, y, which may be drawn from the line *RP*.

In the above discussion e was put for the weight of one foot lineal of the load, therefore the whole load U equals el, or $e = \frac{U}{l}$. If in formula (*70.*) we substitute for e this value of it, we have

$$y = \tfrac{1}{2}U\frac{t}{l}(l - t)$$

$$y = \tfrac{1}{2}U\frac{ht}{l} \qquad (71.)$$

and when $h = t = \frac{1}{2}l$ we have, for the ordinate at its maximum or at the centre,

$$y = \tfrac{1}{2}U\frac{\frac{1}{2}l \times \frac{1}{2}l}{l} = \tfrac{1}{2} \times \frac{Ul}{4}$$

$$y = \tfrac{1}{8}Ul \qquad (72.)$$

---

* For here we have an ordinate to the curve from any point in the base, which is in proportion to the rectangle $[t \times (l - t)]$ of the two parts into which the base is divided by that point, a property of the parabola. (See Cape's Mathematics, 1850, Vol. II., p. 48.)

We thus see that the true value of the coefficient discussed in *Art.* **211** is equal to one half.

This result ($\frac{1}{2}Ul$) is the effect at the middle of the beam, and shows that an equally distributed load will need to be twice the weight of a concentrated load to produce like effects upon any given beam; a like result with that which was obtained in another way at *Art.* **59.**

213.—Distinction Shown by Scales of Strains.—By the calculus, the coefficient, as has just been shown, is equal to $\frac{1}{2}$, but those by formula (*69.*) exceed $\frac{1}{2}$ by a certain fraction (*Art.* **211**).

A comparison of the scales of strains in *Figs.* 41 and 42 will show that the line limiting the ordinates is not a parabola, but a *polygonal* line. In proportion to the increase in the number of the weights, and their consequent diminution in size and distance apart, this polygonal figure approximates the parabolic curve; and in like proportion do the corresponding coefficients approach the coefficient obtained by the calculus; until finally, when the number of the weights becomes infinite, or the load is absolutely an equably distributed one, then the coefficients are identical. The difference between the two expressions is that which is shown between the areas of the polygonal and parabolic figures.

214.—Effect at Any Point by an Equally Distributed Load.—One other lesson may be learned from this discussion.

It has been shown (*Arts.* **59** and **61**) that the effect at the *middle* of the beam, from an equably distributed weight, is equal to that which would be produced by just one half of the weight if concentrated there; and now we see (*Arts.* **211** and **212**) that this proportion holds good, not only at the *middle* of the beam, but also at *any point* in its length.

The expression (*71.*) just obtained,

$$y = \tfrac{1}{2} U \frac{ht}{l}$$

gives the effect produced by an equally distributed load at any point in the beam.

It was shown (*Art.* **56**) that the effect at any point of a load concentrated at that point, is equal to

$$W \frac{mn}{l} = W \frac{ht}{l}$$

Now when the effects in the two cases are equal, we have

$$\tfrac{1}{2} U \frac{ht}{l} = W \frac{ht}{l}$$

or,

$$\tfrac{1}{2} U = W$$

showing that when the effects at any point are equal, the concentrated load is equal to just half of the uniformly distributed load.

215.—Shape of Side of Beam for an Equably Distributed Load.—We have seen (*form. 71.*) that the effect at any point in a beam from an equably distributed load is

$$y = \tfrac{1}{2} U \frac{ht}{l}$$

and that the curve drawn through the ends of a series of ordinates obtained by this formula is a parabola (*Art.* **212**, foot note).

From this may easily be derived the form of the depth of a beam (the breadth being constant), which shall be equally strong throughout its length to bear safely an equably dis-

tributed load. The formula (*71.*) gives the strain at any point, and when put equal to the resistance (*Art.* **35**) is

$$\tfrac{1}{2}U\frac{ht}{l} = Sbd^2$$

Substituting for S its value $\frac{B}{4}$ we have for the safe weight (*Art.* **73**)

$$\frac{2Uaht}{l} = Bbd^2$$

from which $$d^2 = \frac{2Uaht}{Bbl}$$

This gives the square of the depth at any point, and when $h = t = \frac{1}{2}l$ we have

$$d^2 = \frac{Ual}{2Bb} \qquad (73.)$$

equals the square of the depth at the middle.

Now make *CD*, *Fig.* 43, equal by formula (*73.*) to d^2 equals $\frac{Ual}{2Bb}$, and through *D* draw the parabolic curve *RDFP*. Across the figure draw a series of ordinates, as *CD* and *EF*. Then any one of these ordinates is equal to d^2 or the square of the required depth of the beam at the location of that ordinate. To find *d*, the depth, at each of these points, we have but to make *CG* equal to the square root of *CD*, and *EH* equal to the square root of *EF*, and in like manner find corresponding points to *G* and *H* on each ordinate, and draw the curve line *RGHP* through these points; then this curve line will define the top edge of a beam (*RP* being the bottom edge), which shall be equally strong at all points to bear safely the equably distributed load.

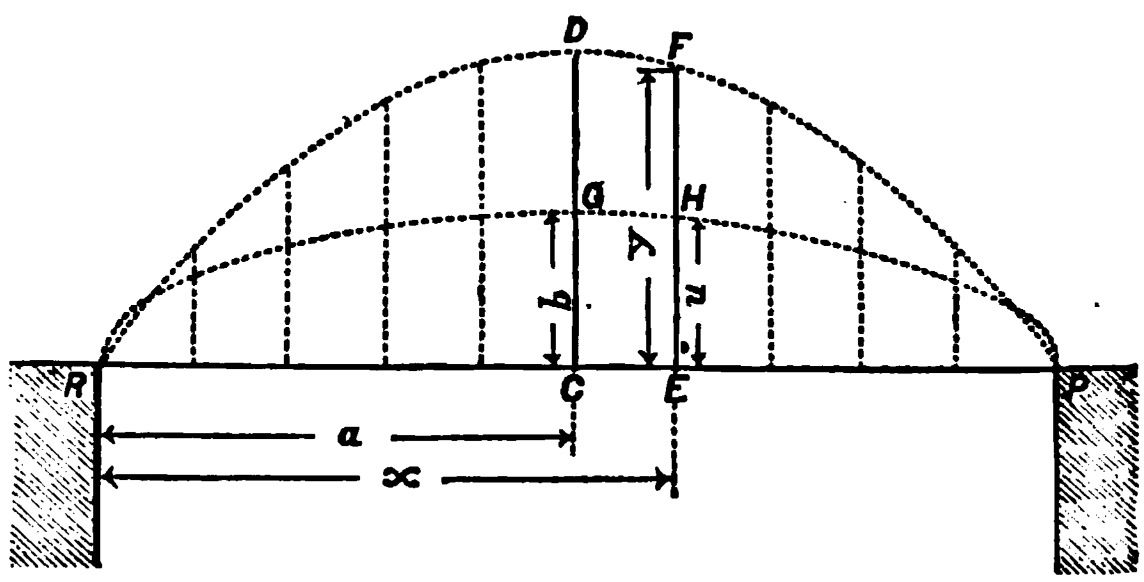

FIG. 43.

216.—The Form of Side of Beam a Semi-ellipse.—The form of the top edge of the beam as obtained in the last article is elliptical, as may be shown thus:

The equation to the ellipse, the co-ordinates taken as in *Fig.* 43, is*

$$u^2 = \frac{b^2}{a^2}(2ax - x^2)$$

in which x (= RE, *Fig.* 43) is the abscissa, u (= EH) is its ordinate, a (= $RC = \frac{1}{2}l$) is the semi-transverse diameter, and b (= $CG = \sqrt{CD}$) is the semi-conjugate diameter: therefore $b^2 = \overline{CG}^2 = CD$ and, by formula (*72.*), in which CD, the height of the parabola at the middle in *Figs.* 42 and 43, is represented by y, at its maximum we have $y = \frac{1}{8}Ul$. In the above value of u^2 substituting for a, and b, their values as here shown, we have

$$u^2 = \frac{U}{2l}(lx - x^2)$$

and since $lx - x^2 = x(l - x) = th$ of *Fig.* 42, therefore

$$u^2 = \tfrac{1}{2}U\frac{ht}{l}$$

By referring to formula (*71.*) it will be seen that this value of u^2 is identical with that given for y, the ordinate to the

* Cape's Mathematics, Vol. II., p. 21, putting u for y.

parabola, consequently $y = u^2$, and therefore the curve *RGHP* is elliptical.

To obtain the shape of the beam, instead of drawing a series of ordinates in a parabola, and taking the square root of each ordinate, we may at once draw the semi-ellipse *RGHP*.

Formula (*73.*) gives the value of d^2 at middle, therefore for d at middle make *CG*, *Fig.* 44, equal to

$$d = \sqrt{\frac{Ual}{2Bb}} \qquad (74.)$$

and through *RGP* draw a semi-ellipse, then *RGPCR* will be the shape of the beam.

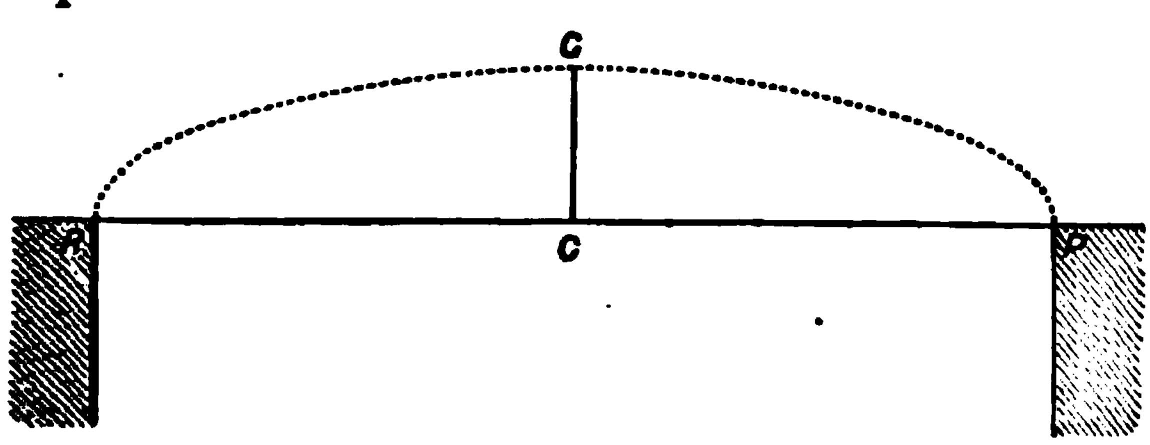

FIG. 44.

As an example :—With a beam of white pine 10 feet long, 5 inches broad, and loaded with 10,000 pounds equably distributed, and with a factor of safety $a = 4$, what should be the height at the middle?

Formula (*74.*) becomes

$$d = \sqrt{\frac{10000 \times 4 \times 10}{2 \times 500 \times 5}} = 8 \cdot 94$$

or the height of the beam is to be 9 inches, and the form of the side is to be that of a semi-ellipse, with 10 feet for its transverse diameter, and 9 inches for its semi-conjugate diameter.

QUESTIONS FOR PRACTICE.

217.—In a scale of strains for an equally distributed load, what curve forms the upper edge?

218.—In a beam, 10 feet long, having 1000 pounds equably distributed over its length, what are the strains at 2, 3, and 4 feet respectively, from one end?

219.—What should be the depth at the middle of this beam, if it be of white pine, if the breadth be made equal to $\frac{8}{10}$ of the depth, and if 4 be the value of the factor of safety?

220.—In order that the beam be of equal strength throughout its length, of what form should the upper edge be when the lower edge is straight, and the beam of parallel breadth throughout?

CHAPTER XI.

STRAINS IN LEVERS, GRAPHICALLY EXPRESSED.

ART. 221.—**Scale of Strains for Promiscuously Loaded Lever.**—In *Fig.* 45 we have a semi-beam loaded promiscuously with the concentrated weights *A*, *B*, *C* and *D*.

FIG. 45.

To construct a scale of strains for this case, make *EF*, by any convenient scale, equal to the product of the weight *A* into the distance *EK*; make *FG* equal to $B \times EU$; make *GH* equal to $C \times EV$; and *HJ* equal to $D \times ET$. From each weight erect a perpendicular, join *K* and *F*, *L* and *G*, *M* and *H*, and *N* and *J*; then any vertical ordinate, as *QP* or *RS*, drawn from the line *EK* to the line *JNMLK*, will, when measured by the same scale as that with which the points *F*, *G*, *H* and *J* were obtained, give, at the location of the ordinate, the effect produced by the four weights.

In the construction of this figure, each triangle of strains is made upon the principle shown in *Art.* **168**, and the several triangles are successively added. An ordinate crossing all these triangles must necessarily be equal to the sum of the strains at its location caused by all the weights.

The strain at any ordinate may also be found arithmetically, by taking the sum of the products of each weight into its horizontal distance to the ordinate, measured from the weight towards the wall; those weights which occur between the ordinate and the wall not being considered, as they add nothing to the strain at the ordinate.

222.—Strains and Sizes of Lever Uniformly Loaded.—When the weights are equably distributed over a semi-beam, the equation to the curve *CFA*, *Fig.* 46, limiting the ordi-

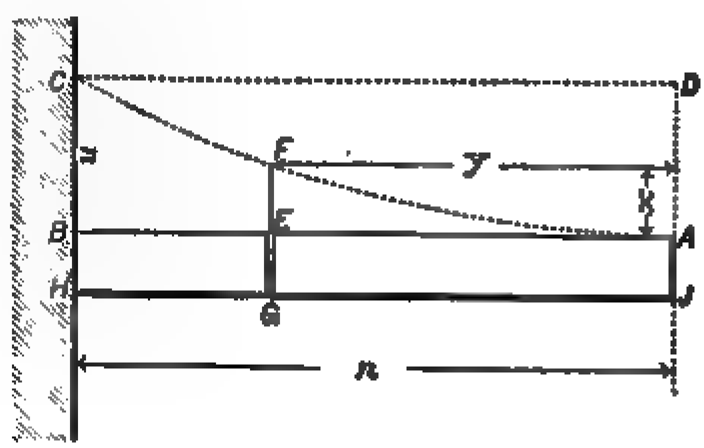

FIG. 46.

nates of strains, may be found by the use of the calculus, as in *Art.* **212**; for if *ABHJ* be taken to represent the equably distributed load, then in considering the effect at the wall of a very thin slice of this load, as *EG* (reducing it infinitely) we obtain the differential of the strain.

:t $AE = y$, then dy, its differential, may be taken as hickness of the thin slice of the load at EG, when :ed to its smallest possible limits. Putting e for the ɔt of a lineal foot of the load, then edy will equal the ɪt of the thin slice. The effect or moment of this slice,

at the wall, equals its weight into its distance from the wall, therefore we have for the differential of the moment

$$edy \times (n-y) = du \qquad \text{or,}$$
$$endy - eydy = du$$

The integral of this expression is (*Arts.* **462** and **463**)

$$\int (endy - eydy) = eny - \tfrac{1}{2}ey^2 = u$$

Applying this, or integrating between y equals zero and y equals n, we have

$$en^2 - \tfrac{1}{2}en^2 = \tfrac{1}{2}en^2 = BC = u$$

or for the strain at the wall, BC,

$$u = \tfrac{1}{2}en^2 \qquad (75.)$$

and for the strain at any point, E,

$$x = \tfrac{1}{2}ey^2 \qquad (76.)$$

From this latter, by transposing, we have

$$y^2 = \frac{2}{e}x$$

which is the equation to the parabola;* a proof that the curve CFA is that of a semi-parabola, in which A is the apex, and CD the base.

These considerations pertain to the scale for *strains.* A scale for *depths* may be had by proceeding as follows:

The value of e in formulas (*75.*) and (*76.*) is, from $U = en$ (in which U equals the whole load upon the semi-beam)

---

* For, putting $\frac{1}{e} = p$, then $y^2 = \frac{2}{e}x$ becomes $y^2 = 2px$, the equation to the parabola. See Cape's Mathematics, Vol. II., p. 47.

$e = \dfrac{U}{n}$. Substituting this value for e in formula (*75.*) we have

$$u = \tfrac{1}{2}\frac{U}{n}n^2 = \tfrac{1}{2}Un$$

Putting this equal to the resistance (*Art.* **35**) gives us

$$\tfrac{1}{2}Un = Sbd^2$$

and substituting for S its equivalent $\tfrac{1}{4}B$, and inserting the symbol for safety (*Art.* **73**), we have

$$4a\tfrac{1}{2}Un = Bbd^2 \qquad \text{or,}$$
$$2Uan = Bbd^2$$

[which agrees with formula (*20.*)] for the size of the semi-beam at the wall.

Again, subjecting formula (*76.*) to like changes, we have for the size of the semi-beam at any point

$$2U\frac{a}{n}y^2 = Bbd^2 \qquad (77.)$$

in which y is the distance of that point from the free end of the semi-beam.

223.—The Form of Side of Lever a Triangle.—If a semi-beam, subjected to an equally distributed load, be of rectangular section throughout, and of constant breadth, then, in order that it may be equally strong at all points of its length, the form of its side must be a triangle.

This may be shown as follows:

Formula (*77.*) gives by transposition

$$d^2 = \frac{2Ua}{Bbn}y^2 \qquad (78.)$$

in which the coefficient $\dfrac{2Ua}{Bbn}$, for the case above cited, is

composed of constant factors; hence d^2 will vary as y^2, and therefore d will be in proportion to y. From this, formula (78.) is shown to be the equation to a straight line, and in such form that when y equals zero, d also becomes zero. From this, the side elevation of the semi-beam must be a triangle, with the depth at the wall (for then y becomes equal to n) equal [from formula (78.) or (20.)] to

$$d = \sqrt{\frac{2Uan}{Bb}} \qquad (79.)$$

As an example, let it be required to define the depth of a semi-beam of white pine, 10 feet long and 5 inches broad, carrying 5000 pounds equably distributed along its length, and with a factor of safety, a, equal to 4.

Formula (79.) becomes

$$d = \sqrt{\frac{2 \times 5000 \times 4 \times 10}{500 \times 5}} = 12{\cdot}65$$

This is the depth at the wall, as at AC, *Fig.* 47, in which AB is the length of the semi-beam. By joining B and C we have ABC for the shape of the side of the required semi-beam.

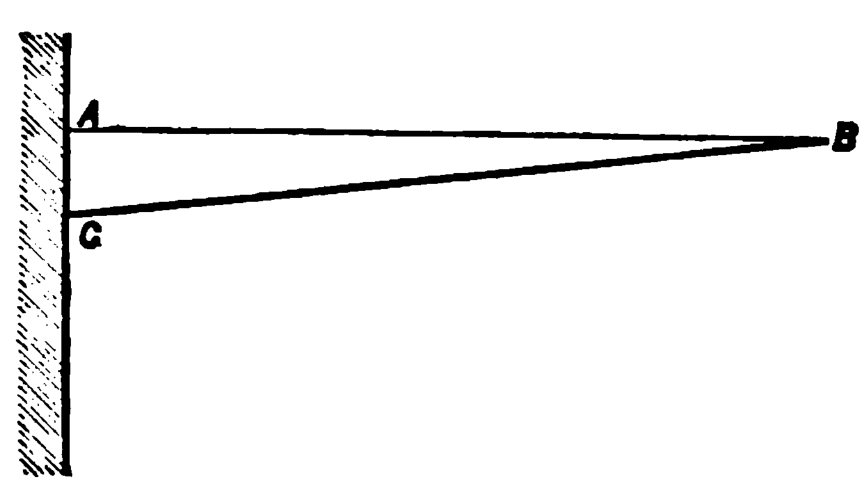

FIG. 47.

224.—Combinations of Conditions.—The forms of strain scales for loads under various simple conditions having been defined, we may now consider those arising from combinations of conditions.

225.—Strains and Dimensions for Compound Load.—Take the case of a semi-beam or lever, carrying an equably distributed load, and also a concentrated load at the free end.

Let the line AB, *Fig.* 48, represent the length of the

FIG. 48.

lever, R a weight suspended from its free end, and DC the face of the wall into which the lever is secured. In formula (*75.*) we have the strain at the wall, in which e equals the weight per lineal foot of the load, or $e = \frac{U}{n}$. Substituting this value in the formula, we have $u = \frac{1}{2}Un$ as the strain at the wall; therefore make $AD = \frac{1}{2}Un$, and by the same scale make $AC = R \times AB = Rn$. Join B and C, and describe a semi-parabola from B to D with the apex at B, and the base extended from D parallel with AB; then any vertical ordinate drawn from the curve DB to the straight line CB will measure the strain at the point of intersection with the line AB.

The scale here given is that for *strains;* the scale for *depths* will now be shown.

We have seen in *Art.* **223** that the form of the side of a lever required by a uniformly distributed load is that of a triangle, the vertical base of which is determined by formula (*79.*); and it is shown at *Art.* **178**, that the form, for a load concentrated at the end of a lever, is a semi-parabola, with its apex at the free end of the lever, and its base vertical at the fixed end and equal to

$$d = \sqrt{\frac{4Pan}{Bb}}$$

Therefore let AB, *Fig.* 49, be the length of the lever

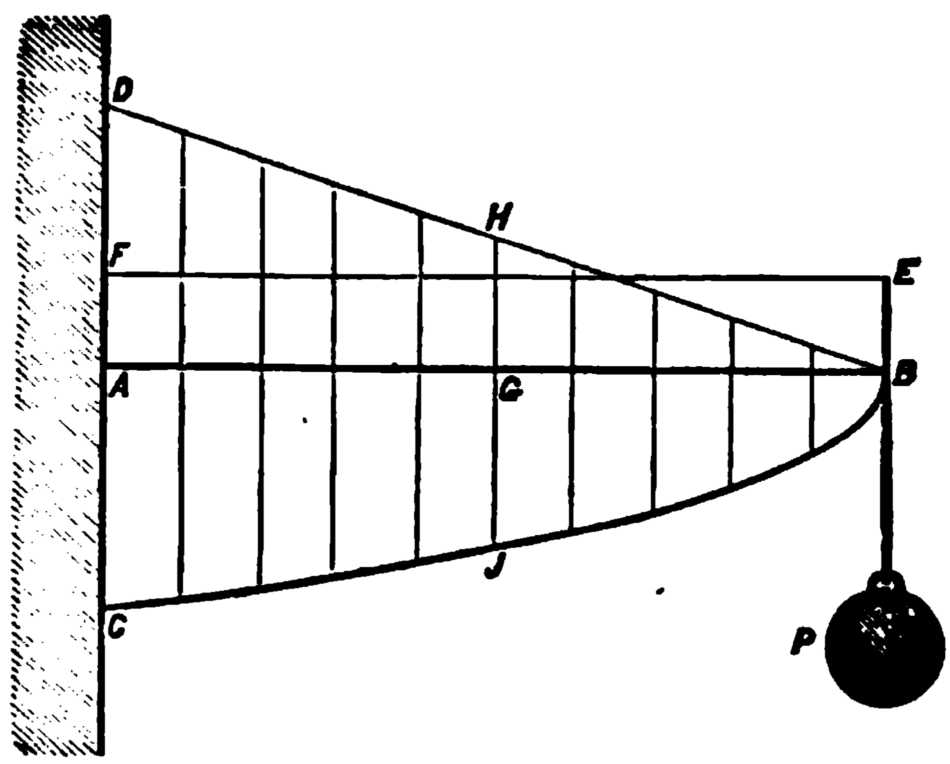

FIG. 49.

secured at A in the wall DC, and having suspended from its free end, B, the weight P, and also carrying an equably distributed load $ABEF$. Make, by formula (79.),

$$AD = \sqrt{\frac{2Uan}{Bb}}$$

and join B and D; then ABD is the scale for the depths required by the equally distributed load U. Make, as above,

$$AC = \sqrt{\frac{4Pan}{Bb}}$$

and upon AC as a base and AB for the height describe the semi-parabola ABC, which gives the scale for depths due to the concentrated load P.

Now, an ordinate drawn at any point, as G, vertically across the combined scales of depths, as H to J, measures, by scale, the required depth for the lever at the point G.

The length of any ordinate, as HJ, may be determined analytically thus. The portion of the ordinate representing the equably distributed load is, by formula (77.),

$$\sqrt{\frac{2Ua}{Bbn}y^2} = d$$

For the remaining part of the ordinate we have formula (*36.*) (in which x is equivalent to the y of this case),

$$\sqrt{\frac{4Pa}{Bb}y} = d''$$

Adding these we have for the full length of the ordinate HJ, or for the depth at the point G,

$$d = \sqrt{\frac{2Ua}{Bbn}y^2} + \sqrt{\frac{4Pa}{Bb}y} \qquad (80.)$$

in which U is the weight equably distributed over the length of the lever; P, the weight concentrated at the end of the lever; n, the length of the lever; y, the horizontal distance from the free end of the lever to the location of the ordinate at which the strain is being measured; a, the factor of safety; b, the breadth of the lever, and B the resistance to rupture as per Table XX.

226.—Scale of Strains for Compound Loads.—*Fig.* 50 represents the case of a semi-beam like the preceding, except that the concentrated load is located at some other point than the extreme end.

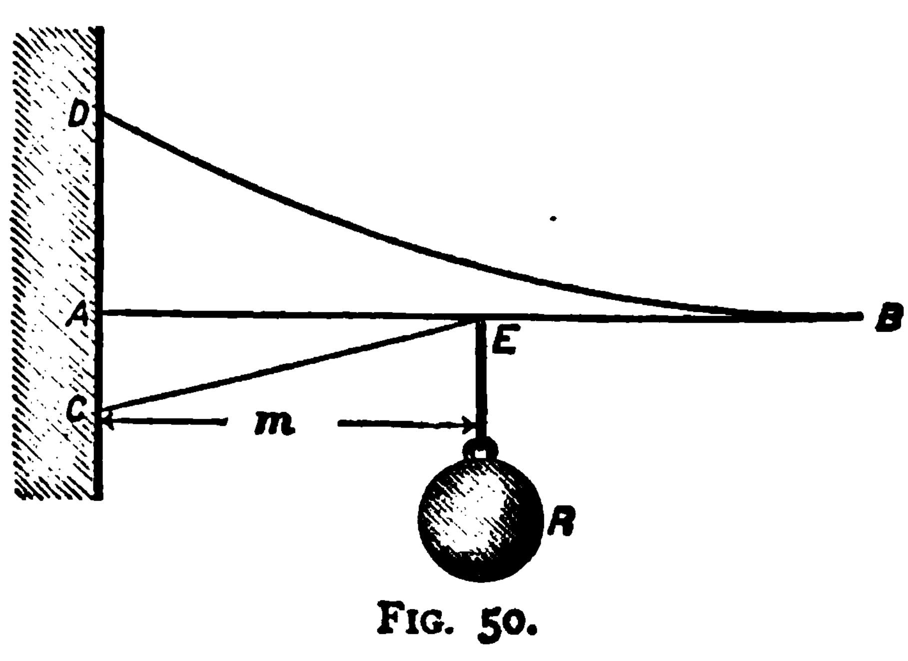

FIG. 50.

The curve DB is found as in *Fig.* 48, and the line CE in the same manner as there, except that, in finding AC, the distance m from the wall to the weight R is to be substituted for n, the length of the lever.

227.—Scale of Strains for Promiscuous Load.—A semi-beam, equably loaded, may also have to carry two or more concentrated loads. In this case, for the scale of strains we combine the methods required for the two kinds of loads, as in *Fig.* 51. Here AB represents the length of the semi-beam; the curve DB, for the equably distributed load, is

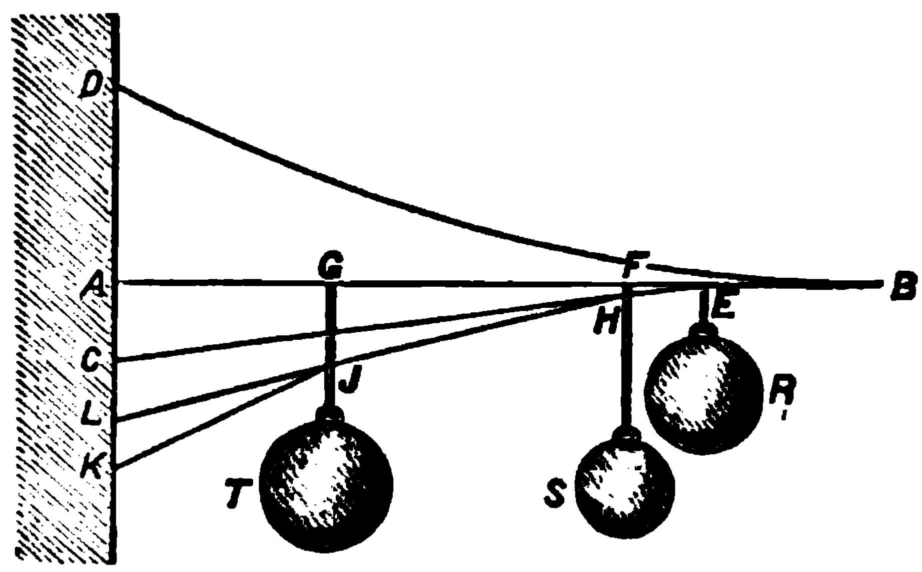

Fig. 51.

obtained as in *Art.* **222**; and the triangles for the concentrated weights are found as in *Art.* **221**.

A vertical ordinate drawn anywhere across the figure, and terminated by the curve DB and the line $KJHEB$, will measure the strain at the location of that ordinate. The depth of the beam at that point may be found by putting the strain as above found equal to the resistance; or,

$$D = Sbd^2$$

or (*Art.* **35**),

$$D = \tfrac{1}{4}Bbd^2$$

from which,

$$\sqrt{\frac{4Da}{Bb}} = d$$

in which D represents the destructive energy or the strain as shown by the length of the vertical ordinate obtained as above directed; a, the symbol for safety (*Art.* **73**); B equals the resistance to rupture as per Table XX., and b and d are the breadth and depth, respectively—the breadth being constant.

QUESTIONS FOR PRACTICE.

228.—In a semi-beam 6 feet long, carrying 500 pounds at 2 feet from the wall, and 300 pounds at 5 feet from the wall, what are the respective strains at 1, 2, 3, 4 and 5 feet from the free end?

What is the strain at the wall?

229.—In a scale of strains for a semi-beam equably loaded, what curve limits the upper edge?

230.—A semi-beam, 8 feet long, is equably loaded with 100 pounds per foot lineal.

What is the strain produced at 5 feet from the free end?

231.—Of what form is the side of the last-named semi-beam required to be, in order that the beam may be of equal strength at all points, the breadth being constant?

232.—In a semi-beam 7 feet long, carrying 1000 pounds at its free end, and 100 pounds per foot lineal, equably distributed, what are the respective strains at 3, 5 and 7 feet from the free end?

233.—In a semi-beam 10 feet long, carrying an equably distributed load of 1000 pounds, and concentrated loads of 800, 500 and 700 pounds, at the several distances of 3, 6 and 8 feet from the free end, what are the respective strains at 2, 4, 7 and 9 feet from the free end?

CHAPTER XII.

COMPOUND STRAINS IN BEAMS, GRAPHICALLY EXPRESSED.

ART. 234.—**Equably Distributed and Concentrated Loads on a Beam.**—We have now to consider the effect of compound weights upon whole beams.

Of this class we shall take first the case of an equably distributed weight, together with a concentrated one, as in *Fig.* 52.

In this figure the curve of strains $RFDP$ for the equably distributed load is a parabola, with its apex at D. The

FIG. 52.

height CD is, by formula (*72.*), to be made equal to $\frac{1}{8}Ul$; and HJ, by the same scale, and by *Art.* **192**, is to be made equal to $A'\frac{mn}{l}$. Join J with R and with P. Then any vertical ordinate FG drawn across the figure, and termi-

nated by the curve *RFDP* at top, and by the line *RJP* at bottom, will measure the strain, y, at E, the point of intersection of the ordinate with the line *RP*.

To obtain this strain analytically, we have, for the ordinate *EF*, formula (*71.*), which is (putting u for y)

$$u = \tfrac{1}{2}U\frac{ht}{l}$$

and, for the ordinate *EG*, formula (*44.*), which is (putting b' for y, A' for W and h for x)

$$b' = A'\frac{m}{l}h$$

Now, since $b'+u = EG+EF = y$, therefore

$$y = u+b' = \tfrac{1}{2}U\frac{ht}{l} + A'\frac{m}{l}h$$

$$y = \frac{h}{l}(\tfrac{1}{2}Ut + A'm) \qquad (81.)$$

equals the strain at any point between H and P.

To find the requisite depth of the beam at any point, the breadth being constant, we put the strain equal to the resistance, or (*Art.* **35**)

$$y = Sbd^2 = \tfrac{1}{4}Bd^2$$

or, for the safe weight,

$$4ay = Bbd^2$$ from which

$$d = \sqrt{\frac{4ah(\tfrac{1}{2}Ut + A'm)}{Bbl}} \qquad (82.)$$

235.—Greatest Strain Graphically Represented.—To find the longest ordinate, and consequently the greatest strain, arising from the compound loads of *Fig.* 52, draw the tangent *KL* parallel with *JP*; then an ordinate *FG* drawn from

the point of contact, F, will be greater than any other which may be drawn across the figure.

236.—Location of Greatest Strain Analytically Defined. —The *point* of contact between a curve and its tangent is not easily found by mere inspection, but analytically its exact position may be defined.

Fig. 52.

To do this, let (*Fig.* 52) $a' = HJ$, $b' = EG$, $u = EF$, $h = EP$ and $h + t = l = RP$.

We now have, from the similar triangles HJP and EGP,

$$n : a' :: h : b' = \frac{a'h}{n}$$

From formula (*70.*), in which $y = u = \frac{1}{2}et(l-t)$, we have

$$u = \tfrac{1}{2}eh(l-h) = \tfrac{1}{2}ehl - \tfrac{1}{2}eh^2 \qquad \text{therefore}$$

$$\frac{a'h}{n} + \tfrac{1}{2}ehl - \tfrac{1}{2}eh^2 = b' + u = FG = y$$

$$y = h\left(\frac{a'}{n} + \tfrac{1}{2}el\right) - \tfrac{1}{2}eh^2 \qquad (83.)$$

This is the value of an ordinate drawn at any point between H and P. But it is required to find where this

ordinate will be at its maximum. This may be done by the calculus. Obtain the differential of formula (*83.*), and placing it equal to zero, derive its integral; from which the value of h will be obtained. This represents the distance from P to the ordinate y, when at its maximum, and therefore determines the point E, the location of the ordinate, as required.

237.—Location of Greatest Strain Differentially Defined. —*First.* For the value of h we are to find the differential of formula (*83.*) and put it equal to zero; thus:

$$dy = \left(\frac{a'}{n} + \tfrac{1}{2}el\right)dh - \tfrac{1}{2}e \times 2hdh = 0$$

$$\left(\frac{a'}{n} + \tfrac{1}{2}el\right)dh = ehdh$$

$$\frac{a'}{n} + \tfrac{1}{2}el = eh$$

$$\frac{\frac{a'}{n} + \tfrac{1}{2}el}{e} = \frac{a'}{en} + \tfrac{1}{2}l = h$$

Now, since $el = U$, therefore $e = \frac{U}{l}$, and

$$h = \frac{a'}{\frac{U}{l}n} + \tfrac{1}{2}l = \tfrac{1}{2}l + \frac{a'l}{Un}$$

Again, $\qquad a' = HY = A'\frac{mn}{l} \qquad$ therefore

$$h = \tfrac{1}{2}l + \frac{A'\frac{mn}{l}l}{Un}$$

$$h = \tfrac{1}{2}l + \frac{A'm}{U} \qquad (84.)$$

or the distance of the ordinate from the remote end of the beam is equal to half the length of the beam, plus a fraction which has for its numerator the product of the concentrated weight into its distance from the nearest bearing, and for its denominator the weight which is equably distributed along the beam.

This formula of the value of h is limited in its application to those cases in which n exceeds h in value. When, on the contrary, h exceeds n, then the longest ordinate is at the location of the concentrated weight, and n is to be substituted for h. The reason for this may be seen by an inspection of the figure.

238.—Greatest Strain Analytically Defined.—*Second.* To find the length of the ordinate y, we have, by formula (*83.*),

$$y = \frac{a'h}{n} + \tfrac{1}{2}elh - \tfrac{1}{2}eh^2$$

and by substituting for l its value, $h + t$,

$$y = \frac{a'h}{n} + \tfrac{1}{2}eh\,(h+t) - \tfrac{1}{2}eh^2$$

$$y = \frac{a'h}{n} + \tfrac{1}{2}(eh^2 + eht) - \tfrac{1}{2}eh^2$$

$$y = \frac{a'h}{n} + \tfrac{1}{2}eht$$

Now, $a' = A'\frac{mn}{l}$, and $e = \frac{U}{l}$, therefore

$$y = \frac{A'\frac{mn}{l}h}{n} + \tfrac{1}{2}U\frac{ht}{l}$$

$$y = \frac{h}{l}(A'm + \tfrac{1}{2}Ut)$$

which gives the greatest strain resulting from both the concentrated and distributed loads.

This formula is identical with formula (*81.*), obtained by another process.

239.—Example.—As an example, let it be required to find the location and length of the longest ordinate of strains produced by a load of 4000 pounds, concentrated at three feet from one end of a beam 16 feet long, together with a load of 3000 pounds, equably distributed over its length.

First. The location of the ordinate, or the value of h. This, from formula (*84.*), is

$$8 + \frac{4000 \times 3}{3000} = 8 + 4 = 12 = h$$

or the longest ordinate is situated within one foot of the location of the concentrated weight.

Second. The amount of strain at this ordinate. This, by the above formula, is

$$y = \frac{12}{16}(\overline{4000 \times 3} + \overline{\tfrac{1}{2} \times 3000 \times 4}) = 13500$$

or the greatest resulting strain at any one point of the combined weights equals 13,500 pounds.

240.—Dimensions of Beam for Distributed and Concentrated Loads.—The amount of strain just found is the actual moment of the loads. Putting this equal to the resistance (*Art.* **35**), we have, for the safe weight,

$$a\frac{h}{l}(A'm + \tfrac{1}{2}Ut) = Sbd^2 = \tfrac{1}{4}Bbd^2 \qquad \text{or}$$

$$4a\frac{h}{l}(A'm + \tfrac{1}{2}Ut) = Bbd^2 \qquad (85.)$$

which is a rule for obtaining the dimensions requisite for resisting effectually the greatest strain arising from the combined action of a *concentrated* and an *equably distributed* load; and in which A' equals the concentrated load, and U the equably distributed load, both in pounds; l is the length of the beam between bearings; m the distance from the concentrated weight to the nearer end of the beam; h the distance from the location of the greatest strain to the more distant end of the beam; and t equals $l - h$. l, m, h and t are all to be taken in feet, and the value of h is to be had from formula (*84.*); care being exercised that when h exceeds n in value, then n is to be used in place of h, and m in place of t. In the latter case formula (*85.*) becomes

$$4a\,\frac{n}{l}(A'm + \tfrac{1}{2}Um) = Bbd^2$$

$$4a\,\frac{mn}{l}(A' + \tfrac{1}{2}U) = Bbd^2 \qquad (86.)$$

241.—Comparison of Formulas, Here and in *Art.* 150.— Formula (*29.*), given in *Art.* **150**, for a carriage beam with one header, is for a case similar to that of the last article, but is not strictly accurate. Instead of the two strains being taken at the same point, E (the location of the longest ordinate), as in *Fig.* 52, they are taken, the one for the concentrated load, at the location of this load, and the other, that for the equably distributed load, at the middle of the beam; or, the maximum strain for each load.

Taken in this manner the result is in excess of the truth, as $HJ + CD$ is greater than FG. The error is upon the safe side, the strains being estimated greater than they really are. In most cases this error would not be large, and the only objection to it would be that it requires a little more material in the beam. Formula (*29.*) may therefore be employed

in ordinary cases where a low priced material, such as wood for example, is used for the beams; but where a more costly material is involved, economy would dictate that the strain be not over-estimated, and that it be correctly obtained by the use of formula (*85.*) in *Art.* **240.** (See also caution in *Art.* **88.**)

242.—Location of Greatest Strain Differentially Defined.—In *Fig.* 53 we have a scale of strains, $RABPF$, by which is found the effect arising at any point in the length of the beam from *two* concentrated loads, together with an equably distributed load.

The curve RFP is a parabola (foot note, *Art.* **212**) found as in *Fig.* 52, and the moment of the two concentrated loads equals AH at H and BJ at J, and is found as in *Art.* **194** and *Fig.* 37. FG is the ordinate for strains occurring between H and J, and is defined thus:

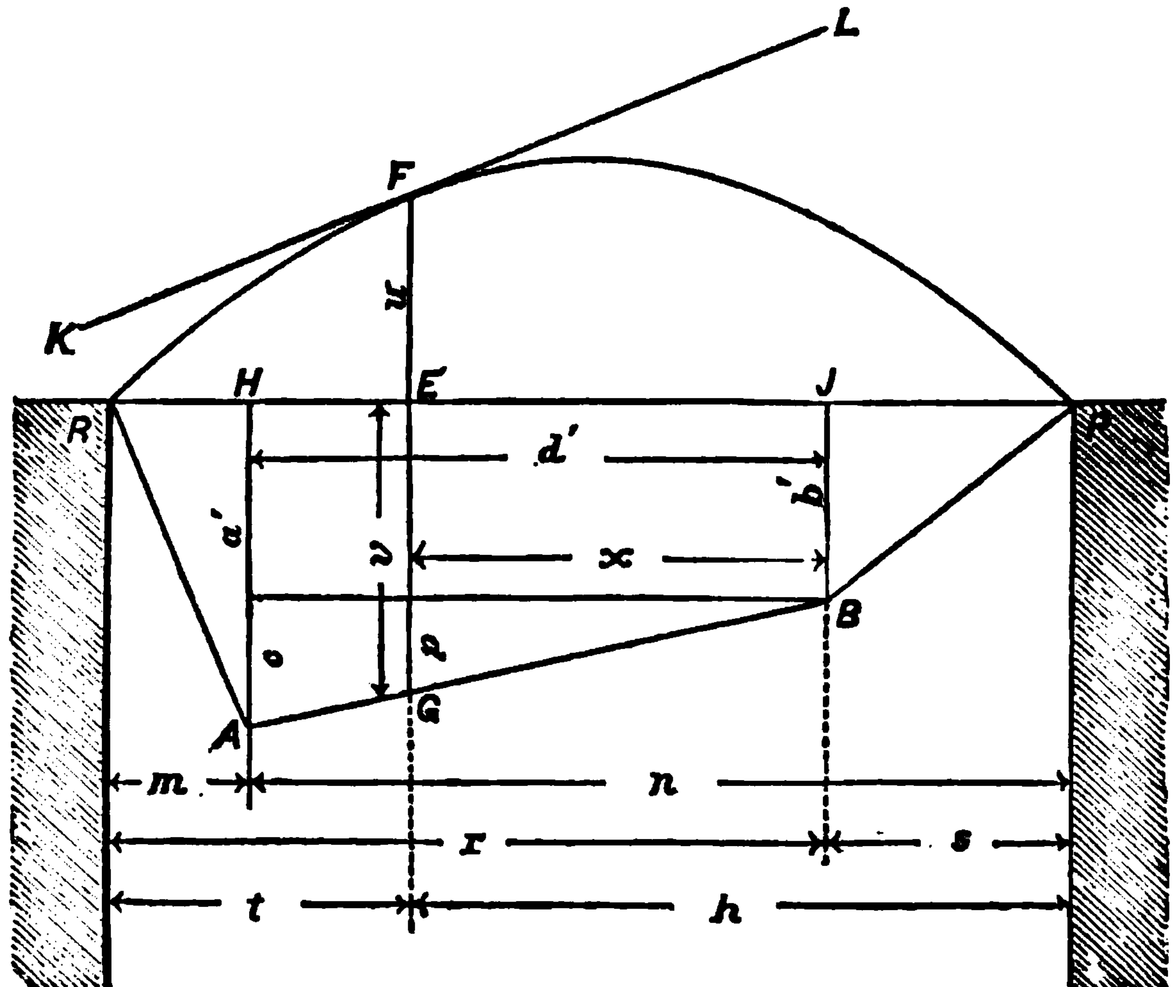

FIG. 53.

Let $HJ = d'$, $EJ = x$, $EG = v = b'+p$, $EF = u$, $AH = a'$, $BJ = b'$ and $a'-b' = c$. Then, from similar triangles,

$$d' : c :: x : p = \frac{c}{d'}x$$

and, since $x = h - s$, $v = b' + p$ and $c = a' - b'$, therefore

$$p = \frac{c}{d'}(h-s)$$

and
$$v = b' + \frac{a'-b'}{d'}(h-s)$$

Formula (*70.*),

$$y = \tfrac{1}{2}et\,(l-t)$$

gives [putting ht for $t(l-t)$ and u for y]

$$u = \tfrac{1}{2}eht$$

and since $y = u + v$, consequently

$$y = \tfrac{1}{2}eht + b' + \frac{a'-b'}{d'}(h-s) \qquad (87.)$$

This is the value of the ordinate for the strain at any point between H and J.

To obtain the *longest* ordinate which can be drawn here, proceed as in *Arts.* **235** to **237**, and as follows:

First reduce formula (*87.*) thus,

$$\tfrac{1}{2}eht = \tfrac{1}{2}eh\,(l-h) = \tfrac{1}{2}ehl - \tfrac{1}{2}eh^2$$

$$\frac{a'-b'}{d'}(h-s) = \frac{a'-b'}{d'}h - \frac{a'-b'}{d'}s$$

then
$$v + u = y = \tfrac{1}{2}ehl - \tfrac{1}{2}eh^2 + b' + \frac{a'-b'}{d'}h - \frac{a'-b'}{d'}s$$

In this expression, rejecting the quantities unaffected by the variable h, we have, for the differential of y,

$$dy = \left(\tfrac{1}{2}el + \frac{a'-b'}{d'}\right)dh - ehdh = 0$$

or,
$$\left(\tfrac{1}{2}el + \frac{a'-b'}{d'}\right)dh = ehdh$$

or, its integral gives

$$h = \tfrac{1}{2}l + \frac{a'-b'}{d'e} \qquad (88.)$$

243.—Greatest Strain and Dimensions.—The above gives the value of h. To obtain the value of y at its maximum take formula (*87.*). In this, for the value of a' we have AH, equal to the joint effect at H of the two concentrated loads; or, putting a' for the D of formula (*51.*),

$$a' = \frac{m}{l}(A'n + B's)$$

and for the value of b' (*form. 52.*)

$$b' = \frac{s}{l}(B'r + A'm)$$

The value of e (from $el = U$) is equal to $\frac{U}{l}$. By substituting this value for e we have

$$y = U\frac{ht}{2l} + b' + \frac{a'-b'}{d'}(h-s) \qquad (89.)$$

This equals the strain from the compound weights of *Fig.* 53, and is the same as (*87.*), for $\frac{U}{2l} = \tfrac{1}{2}e$.

Either formula will give the strain at any required point between H and J (*Fig.* 53) by putting h equal to the dis-

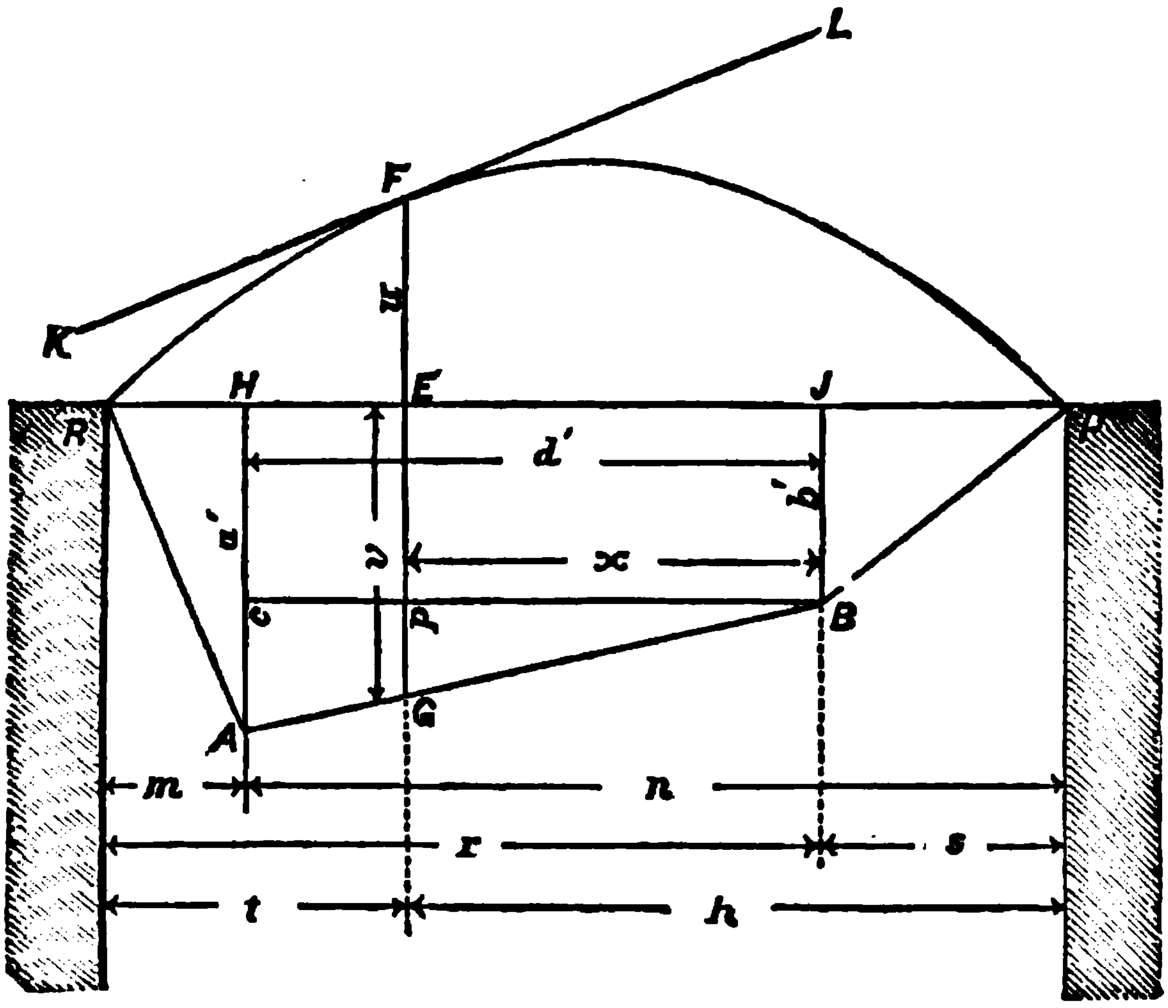

FIG. 53.

tance between that point and P; but when the *greatest* strain is required, h must be obtained from its value in formula (*88.*). To obtain the dimensions in this case, we put the strain equal to the resistance, and have, with a as the factor for safe weight (*Arts.* **35** and **73**)

$$a\left[U\frac{ht}{2l}+b'+\frac{a'-b'}{d'}(h-s)\right] = Sbd^2 = \tfrac{1}{4}Bbd^2$$

$$4a\left[U\frac{ht}{2l}+b'+\frac{a'-b'}{d'}(h-s)\right] = Bbd^2 \qquad (90.)$$

and from this formula may be found the dimensions required for resisting effectually the greatest strain in the beam, the value of h being derived from formula (*88.*).

244.—Assigning the Symbols.—It is important to observe here that of the two moments a' and b', a' designates the larger of the two, while m and n represent the distances from a' to the two ends of the beam, m being the distance to that support which may be reached without passing the

other weight. Again r and s are to be regarded as the distances from b' to the two ends of the beam; h and s dating from the same end of the beam as n; and as n is the greatest possible value of h, it is to be substituted for it when by the formula for h its value is found equal to or greater than n.

In order to ascertain which of the two moments a' and b' is the greater, a trial must be had by the use of the expressions in the last article designating their respective values. When the two concentrated weights are equal, then the nearer weight to the middle of the beam will produce the greater moment, and may at once be designated as a'.

245.—Example—Strain and Size at a Given Point.—As an example, let a beam, 10 feet long, be required to carry an equably distributed load of 100 pounds per foot lineal, a concentrated load of 2000 pounds at a point two feet from the left-hand end, and a second concentrated load of 800 pounds located at 3 feet from the right-hand end. What will be the resulting strain at 4 feet from the right-hand end?

Formula (*87.*) is

$$y = \tfrac{1}{2}eht + b' + \frac{a'-b'}{a'}(h-s)$$

equals the required strain.

In designating m and s we find (*Art.* **243**) for the larger weight

$$\frac{2}{10}(\overline{2000 \times 8} + \overline{800 \times 3}) = 3680$$

naller

$$\frac{3}{10}(\overline{800 \times 7} + \overline{2000 \times 2}) = 2880$$

t. **153**) $m = 2$ and $s = 3$.

We now have $e = 100$, $h = 4$, $l = 10$, $t = l - h = 6$, $m = 2$, $n = 8$, $s = 3$, $r = 7$ and $d' = 5$.

$$\tfrac{1}{2}eht \text{ becomes } \tfrac{1}{2} \times 100 \times 4 \times 6 = 1200$$

With $A' = 2000$ and $B' = 800$, $a' = \frac{m}{l}(A'n + B's) =$ (as above) 3680, and $b' = \frac{s}{l}(B'r + A'm) =$ (as above) 2880 and $a' - b' = 3680 - 2880 = 800$.

We therefore have, as a resulting value of y in formula (*87.*),

$$1200 + 2880 + \frac{800}{5}(4 - 3) = 4240 = y$$

This equals the effect at 4 feet from the right-hand end produced by the three weights.

To find the dimensions of the beam at this point, make the strain just found equal to the resistance [see *Art.* **243** at formula (*90.*)], and we have

$$4a \times 4240 = Bbd^2$$

and, if $a = 4$ and $B = 500$ (see Table XX.), we have

$$bd^2 = \frac{4 \times 4 \times 4240}{500} = 135 \cdot 68$$

Let $b = 3$, then we have $d = 6 \cdot 73$; or, the beam at 4 feet from the right-hand end should be $3 \times 6 \cdot 73$ inches in cross-section.

246.—Example—Greatest Strain.—Again, let it be required to show the *greatest* strain produced at any one point by the three weights of the last article.

The first dimension required here is that of h. For this we have, as per formula (*88.*),

$$h = \tfrac{1}{2}l + \frac{a'-b'}{d'e}$$

from which $$h = 5 + \frac{800}{5 \times 100} = 6 \cdot 6$$

This result being less than n, since n equals 8, is therefore the correct value of h, and from it we obtain (from $t+h=l$) $t = 3 \cdot 4$. Formula (*89.*) now gives

$$y = 1000 \frac{6 \cdot 6 \times 3 \cdot 4}{2 \times 10} + 2880 + \frac{800}{5}(6 \cdot 6 - 3) = 4578$$

which is the required greatest strain.

247.—Example—Dimensions.—What sized beam of equal cross-section throughout would be required to carry safely the loads upon the beam of the last article, when $B = 500$ and $a = 4$?

The greatest strain at any point was found to be 4578 pounds, therefore

$$4a \times 4578 = Bbd^2$$

$$bd^2 = \frac{4 \times 4 \times 4578}{500} = 146 \cdot 5$$

and with b taken equal to 3, then $d = 6 \cdot 99$. The beam must be 3×7 inches.

248.—Dimensions for Greatest Strain when h Equals n.—When, in formula (*90.*), $h = n$, or is greater than n, then $t = m$, $h - s = d'$, and

$$b' + \frac{a'-b'}{d'}(h-s) = b' + \left(\frac{a'-b'}{d'} \times d'\right) = b' + a' - b' = a'$$

also,

$$U\frac{ht}{2l} = \tfrac{1}{2}U\frac{mn}{l}$$

and the formula becomes

$$4a\left(\tfrac{1}{2}U\frac{mn}{l}+a'\right)=Bbd^2$$

or, supplying the value of a' (*Art.* **243**),

$$4a\left[\tfrac{1}{2}U\frac{mn}{l}+\frac{m}{l}(A'n+B's)\right]=Bbd^2 \qquad (91.)$$

which is a rule for a beam carrying two concentrated loads and a uniformly distributed load, when $h=n$ as above stated.

249.—Dimensions for Greatest Strain when h is Greater than n.—As an example under this rule, what are the breadth and depth of a Georgia pine beam 20 feet long, carrying 2000 pounds uniformly distributed over its whole length, 10,000 pounds at 7 feet from the left-hand end, and 8000 pounds at 5 feet from the right-hand end; the factor of safety being 4?

Here $a=4$, $U=2000$, $l=20$, $B=850$, $m=7$ and $s=5$ (since $7\times 10{,}000=70{,}000$ exceeds $5\times 8000=40{,}000$), $n=13$, $r=15$ and $d'=8$. The value of h is to be tested, to know whether it is equal to or greater than n.

By formula (*88.*), and *Art.* **243**,

$$h=\tfrac{1}{2}l+\frac{a'-b'}{d'e}=\tfrac{1}{2}l+\frac{a'-b'}{d'\dfrac{U}{l}}$$

$$a'=\frac{m}{l}(A'n+B's)=\frac{7}{20}(\overline{10000\times13}+\overline{8000\times5})=59500$$

$$b'=\frac{s}{l}(B'r+A'm)=\frac{5}{20}(\overline{8000\times15}+\overline{10000\times7})=47500$$

$$a'-b' = 59500 - 47500 = 12000$$

$$h = 10 + \frac{12000}{8 \times \frac{2000}{20}} = 25$$

This gives a value to h greater than that of n and shows (*Art.* **244**) that n must be substituted for h, and that the problem is a proper one for solving by formula (*91.*); therefore

$$bd^2 = \frac{4 \times 4 \left[1000 \frac{7 \times 13}{20} + \frac{7}{20} (\overline{10000 \times 13} + \overline{8000 \times 5}) \right]}{850} = 1205 \cdot 65$$

If the breadth b be taken at 8 inches, then $d = 12 \cdot 28$; that is, the beam should be $8 \times 12\frac{1}{4}$ inches.

250.—Rule for Carriage Beams with Two Headers and Two Sets of Tail Beams.—By proper modifications, formula (*90.*) may be adapted to the requirements of a carriage beam with two headers, as in *Fig.* 25. These modifications are as follows: By *Art.* **150** we have

$$U = \tfrac{1}{2} cfl$$

hence

$$U \frac{ht}{2l} = \tfrac{1}{4} cfht$$

also, from *Arts.* **153** and **243**,

$$a' = \frac{m}{l} (A'n + B's)$$

and, from *Art.* **155**,

$$A' = \tfrac{1}{4} fgm \quad \text{and} \quad B' = \tfrac{1}{4} fgs$$

therefore

$$a' = \frac{m}{l}(\tfrac{1}{4}fgmn + \tfrac{1}{4}fgs^2)$$

$$a' = fg\frac{m}{4l}(mn+s^2)$$

Similarly we find

$$b' = fg\frac{s}{4l}(rs+m^2)$$

To obtain the maximum strain, h is to be determined by formula (*88.*), in which for e we have

$$e = \frac{U}{l} = \frac{cfl}{2l} = \tfrac{1}{2}cf$$

and therefore

$$h = \tfrac{1}{2}l + \frac{a'-b'}{\tfrac{1}{2}cd'f}$$

In these deductions, f equals the weight per superficial foot of the floor, c the distance apart from centres at which the beams in the floor are placed, and g the length of the header. (For cautions in distinguishing between m and s, and between a' and b', see *Art.* **244.**) By formula (*90.*) and the modifications proposed, we therefore have

$$4a\left[\tfrac{1}{2}cfht + b' + \frac{a'-b'}{d'}(h-s)\right] = Bbd^2 \qquad (92.)$$

and as auxiliary thereto we have, as above,

$$a' = fg\frac{m}{4l}(mn+s^2)$$

$$b' = fg\frac{s}{4l}(rs+m^2) \qquad \text{and}$$

$$h = \tfrac{1}{2}l + \frac{a'-b'}{\tfrac{1}{2}cd'f}$$

and thus we have in formula (*92.*) a rule for a carriage beam carrying two headers and two sets of tail beams. (See caution in *Art.* **88**).

251.—Example.—To show the application of this rule, let it be required to find the breadth of a white pine carriage beam, 20 feet long and 10 inches deep; the beam to carry two headers 10 feet long, one located 9 feet from the left-hand end, and the other 6 feet from the right-hand end. The floor beams are to be placed 15 inches from centres, and the floor is to carry 100 pounds per superficial foot, with the factor of safety $a = 4$.

Here the header at the left-hand end is the nearer of the two to the middle of the carriage beam, and therefore (*Art.* **244**) $m = 9$.

From formula (*92.*) we have, for the value of b,

$$b = \frac{4a}{Bd^2}\left[\tfrac{1}{4}cfht + b' + \frac{a'-b'}{d'}(h-s)\right] \qquad (93.)$$

in which $a = 4$, $B = 500$, $d^2 = 10^2$, $f = 100$, $c = 1\tfrac{1}{4}$, $g = 10$, $l = 20$, $m = 9$, $n = 11$, $r = 14$, $s = 6$ and $d' = 5$.

From the auxiliary formulas of *Art.* **250**,

$$a' = 100 \times 10 \times \frac{9}{4 \times 20}\left(\overline{9 \times 11} + 6^2\right) = 15187{\cdot}5$$

$$b' = 100 \times 10 \times \frac{6}{4 \times 20}\left(\overline{14 \times 6} + 9^2\right) = 12375$$

$$a'-b' = 15187{\cdot}5 - 12375 = 2812{\cdot}5$$

$$h = 10 + \frac{2812{\cdot}5}{\frac{1}{2} \times 1\frac{1}{4} \times 5 \times 100} = 19$$

Here n, since it is but 11, is less in value than h, and must be used in its place; we therefore have recourse to formula (*91.*), *Art.* **248**. By this formula the value of b is

$$b = \frac{4a}{Bd^2}\left[\tfrac{1}{2}U\frac{mn}{l} + \frac{m}{l}(A'n+B's)\right]$$

This is a general rule. To make it conform to the requirements for a carriage beam, we have for U the equally distributed load $\frac{1}{2}cfl$ (*Art.* **150**).

$A' = \frac{1}{4}fgm$ (*Art.* **250**), and $B' = \frac{1}{4}fgs$. Hence

$$b = \frac{4a}{Bd^2}\left[\tfrac{1}{4}cfmn + \frac{m}{l}(\tfrac{1}{4}fgmn + \tfrac{1}{4}fgs^2)\right]$$

$$b = \frac{4a}{Bd^2}\left[\tfrac{1}{4}cfmn + \frac{fgm}{4l}(mn + s^2)\right]$$

$$b = \frac{afm}{Bd^2}\left[cn + \frac{g}{l}(mn + s^2)\right] \qquad (94.)$$

$$b = \frac{4\times100\times9}{500\times100}\left[\overline{1\tfrac{1}{4}\times11} + \frac{10}{20}(\overline{9\times11} + 36)\right] = 5\cdot85$$

or, the carriage beam should be $5\frac{7}{8}$ or, say 6 inches broad.

In this computation, no allowance is made for the weakening effect of mortising, it being understood that no mortises are to be made; the headers being hung in bridle irons (*Art.* **147**). (See *Art.* **88**).

252.—Carriage Beam with Three Headers.—It sometimes occurs in the plan of a floor that two openings, the one a stairway at the wall, the other an opening for light at or near the middle of the floor, are opposite each other, as in *Fig.* 54.

In this arrangement the carriage beam has three headers to carry, besides its load as an ordinary floor beam.

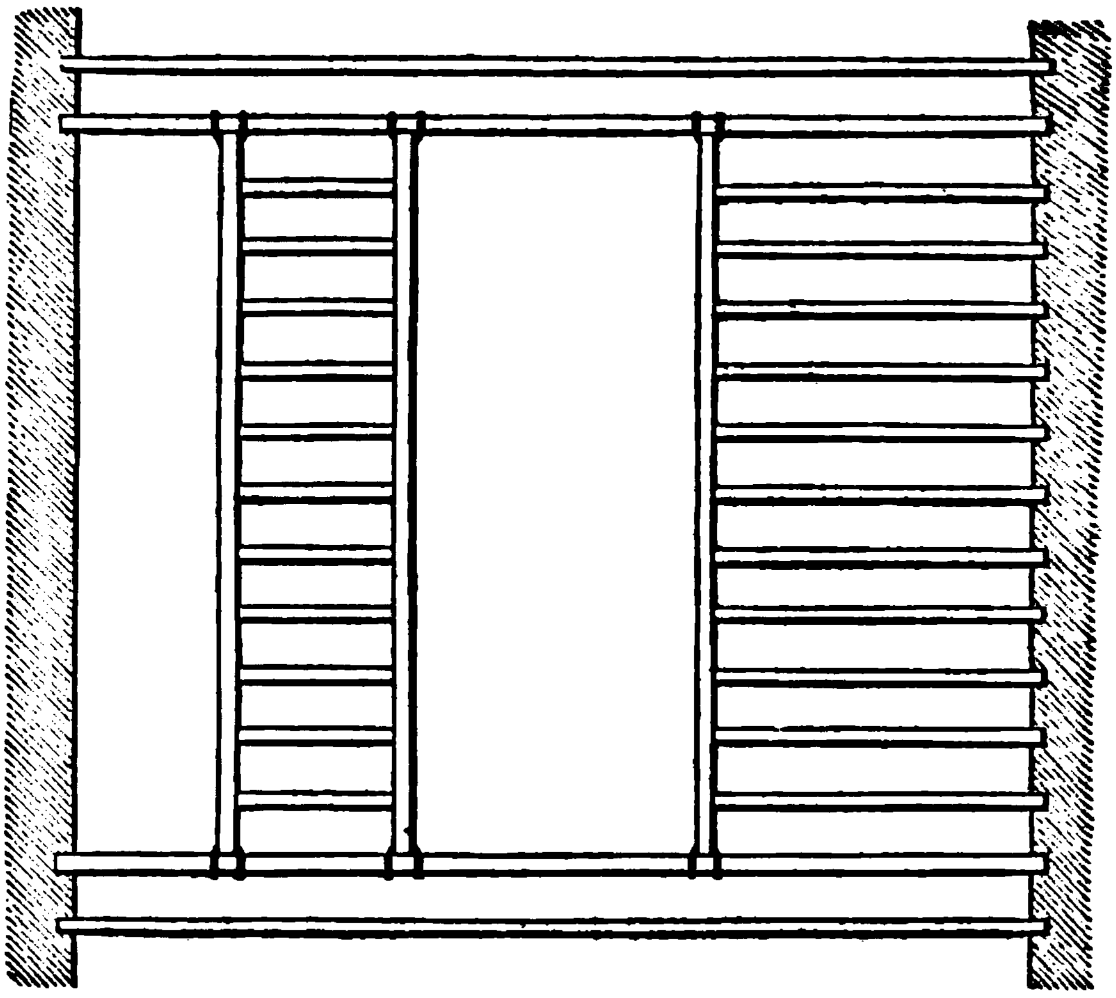

FIG. 54.

Cases of this kind may be divided into two classes: one that in which the header causing the greatest strain occurs between the other two; the other, that in which it occurs next to one of the walls. We will first consider the latter case.

253.—Three Headers—Strains of the First Class.—When the well hole for light occurs at the middle of the distance between the walls, its two headers will be equally near the centre of the length of the carriage beam; and, were their loads alike, the headers would produce equal strains upon the carriage beam; but the loads are not alike, for the tail beams carried by one header, those which reach to the wall, are longer than those carried by the other.

Hence the header carrying the tail beams, one end of which rest on the wall, has the heavier load; and, as it has the same leverage as the header on the other side of the well hole, it will therefore have the greater moment, and will produce the greater strain upon the carriage beam.

The stair header will add to the strains upon the carriage beam at the points of location of the other two headers, and this addition will be greater at the middle header than at the farther one, but still not so much greater as to cause the total strains at the one to preponderate over those at the other.

254.—Graphical Representation.—Let *Fig.* 55, constructed similarly with *Fig.* 53, represent the strains in a carriage beam supporting three headers, one of the outside ones, as at *A*, producing the greatest strain. In this figure the curve *DKE* is a parabola (*Art.* **212**) and is the curve of strains for the uniformly distributed load upon the carriage beam,

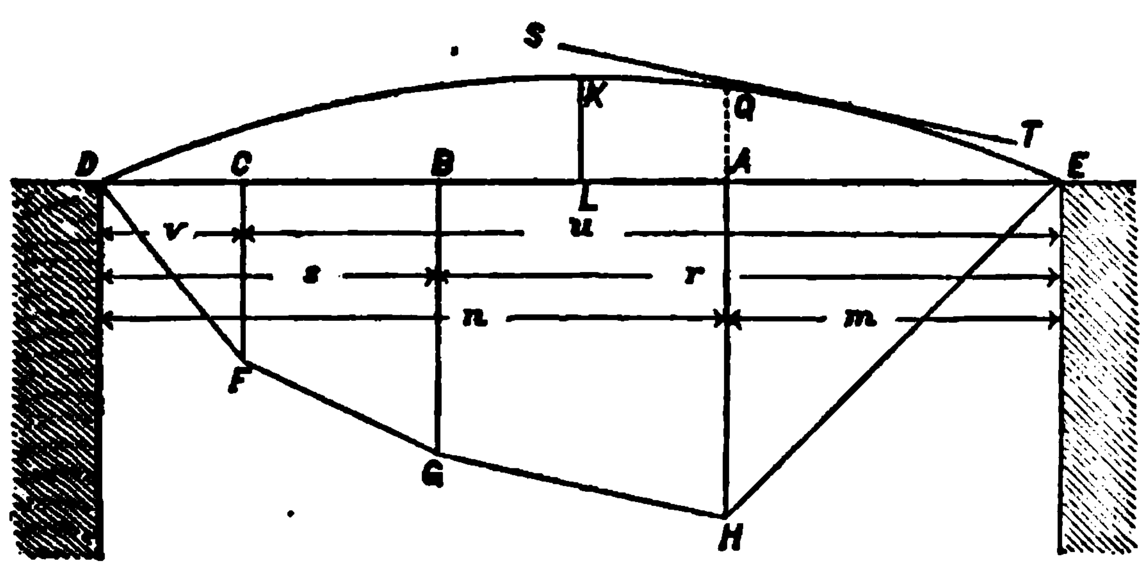

FIG. 55.

of which *KL* represents the strain at the middle of the beam; and *CF*, *BG* and *AH*, vertical lines, by the same scale, represent the strains caused by the three headers at the points *C*, *B* and *A*, respectively. Any ordinate drawn, parallel to *AH*, across this figure, and terminated by the boundary line *DFGHEKD*, will measure the strain in the carriage beam at its location. Hence that point at which an ordinate thus drawn proves to be longest of any which may be drawn, is the point where the strain upon the carriage beam is the greatest, and the length of this ordinate measures the amount of this strain.

Draw the tangent ST parallel to GH. If its point of contact with the curve occurs between Q and E, then HQ will be the longest possible ordinate; but, if it occur between K and Q, then HQ will not be the longest. When AH and BG are equal, the point of contact will be at K. In the case under consideration (the well hole in the middle of the floor) the tangent will usually touch between Q and E, giving HQ as the longest ordinate.

255.—Greatest Strain.—With the loads A, B and C in position as in *Fig.* 55, the longest ordinate may be found

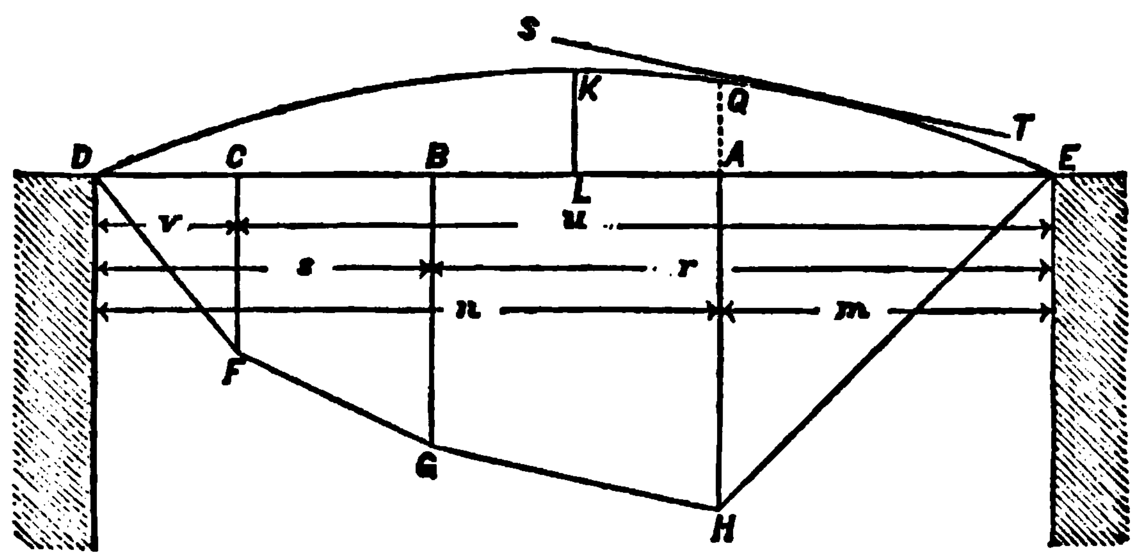

FIG. 55.

by formula (*87.*), where

$$y = \tfrac{1}{2}eht + b' + \frac{a'-b'}{d'}(h-s)$$

and in which $m+n = r+s = h+t = l$ (for the position of these letters see *Art.* **244**), $\frac{1}{2}eht$ represents the strain from the uniformly distributed load, and $b' + \frac{a'-b'}{d'}(h-s)$ stands for the length of an ordinate drawn from GH to BA at the distance h from D towards A, and represents at the location of the ordinate the strain from the three concentrated loads. In all cases, except where b' is very nearly or quite equal to a', h will exceed n, and, in

general, for all problems of the class of which we are treating, it may be assumed, without material error, that h *will* always exceed n. Then m and n take the place of t and h in formula (*87.*), and it becomes (*Art.* **248**)

$$y = \tfrac{1}{2}emn + a'$$

or,

$$y = \tfrac{1}{2}U\frac{mn}{l} + a'$$

The value of a' is (*form. 58.*)

$$a' = \frac{m}{l}(A'n + B's + C'v)$$

hence

$$y = \tfrac{1}{2}U\frac{mn}{l} + \frac{m}{l}(A'n + B's + C'v) \qquad (95.)$$

In this formula y equals the greatest strain in the beam.

256.—General Rule for Equably Distributed and Three Concentrated Loads.—Putting the strain y of last article equal to the resistance (*Art.* **35**) gives us

$$\tfrac{1}{2}U\frac{mn}{l} + \frac{m}{l}(A'n + B's + C'v) = Sbd^2$$

and with $B = 4S$ and a as the coefficient of safety,

$$4a\frac{m}{l}(\tfrac{1}{2}Un + A'n + B's + C'v) = Bbd^2 \qquad (96.)$$

which is a general rule for beams carrying a uniformly distributed load and three concentrated loads similarly placed with those in *Fig.* 55. In this rule, U is the uniformly distributed load, and A', B' and C' the three loads concentrated at A, B and C in the figure.

257.—Example.—As an example, we will ascertain the required breadth of a Georgia pine beam of average quality,

20 feet long and 14 inches deep, with a load of 2000 pounds equally distributed over its length, a concentrated load of 4000 pounds at 3 feet from the left-hand end, a like load at 7 feet from the same end, and one of 7000 pounds at 7 feet from the right-hand end. Take as the factor of safety $a = 4$. Then $l = 20$, $m = 7$, $n = 13$, $s = 7$, $r = 13$, $v = 3$, $u = 17$, $d = 14$, $U = 2000$, $A' = 7000$, $B' = 4000 = C'$ and $B = 850$, and from formula (*96.*)

$$b = \frac{4 \times 4 \times 7}{850 \times 14^2 \times 20}\left(\tfrac{1}{2} \times 2000 \times 13 + 7000 \times 13 + 4000 \times 7 + 4000 \times 3\right) = 4{\cdot}84$$

or the breadth should be $4\frac{7}{8}$ inches.

258.—Rule for Carriage Beams with Three Headers and Two Sets of Tail Beams.—To modify formula (*96.*) so as to make it applicable to a carriage beam, we have for U, the uniformly distributed load, (*Art.* **150**) $U = \frac{1}{2} cfl$; for the load at A, caused by the header carrying the tail beams, one end of which rests upon the wall, $A' = \frac{1}{2} fgm$; for the load at B, $B' = \frac{1}{2} fg(s-v)$; and for the load at C the same, $C' = \frac{1}{2} fg(s-v)$. Formula (*96.*) now becomes

$$b = \frac{4am}{Bd^2l}\left[\tfrac{1}{2}cfln + \tfrac{1}{2}fgmn + \tfrac{1}{2}fg(s-v)s + \tfrac{1}{2}fg(s-v)v\right]$$

$$b = \frac{amf}{Bd^2l}\left[cnl + gmn + g(s-v)(s+v)\right]$$

$$b = \frac{amf}{Bd^2l}\left[cnl + g(mn + s^2 - v^2)\right] \qquad (97.)$$

which is a rule for carriage beams carrying three headers and two sets of tail beams, located, as in *Fig.* 55, with A, the heaviest strained header in an outside position relative to the other two headers.

259.—Example.—Under the above rule, what should be the breadth of a spruce carriage beam 20 feet long and 12

inches deep, carrying three headers 15 feet long, located as in *Fig.* 54. The well-hole for light, in the middle of the width of the floor, is 6 feet wide, and the stairway opening, at one of the walls, 3 feet wide. The beams of the floor are placed 15 inches from centres, and are to carry 90 pounds per superficial foot, with 4 as the factor of safety.

Here $l = 20$, $m = s = \frac{l-6}{2} = 7$, $n = 13$, $v = 3$, $g = 15$, $d = 12$, $c = 1\frac{1}{4}$, $f = 90$, $a = 4$ and $B = 550$.

By formula (97.)

$$b = \frac{4 \times 7 \times 90}{550 \times 12^2 \times 20}[1\tfrac{1}{4} \times 13 \times 20 + 15\,(7 \times 13 + 7^2 - 3^2)] = 3{\cdot}64$$

or the breadth should be $3\frac{5}{8}$ inches.

260.—Three Headers—Strains of the Second Class.— We will now consider the other class named in *Art.* **252**, that in which the header causing the greatest strain occurs between the other two.

The conditions of this class of cases are represented in *Fig.* 56, in which $AH = a'$, $BG = b'$ and $CF = c'$, representing by scale the combined concentrated strains at A, B

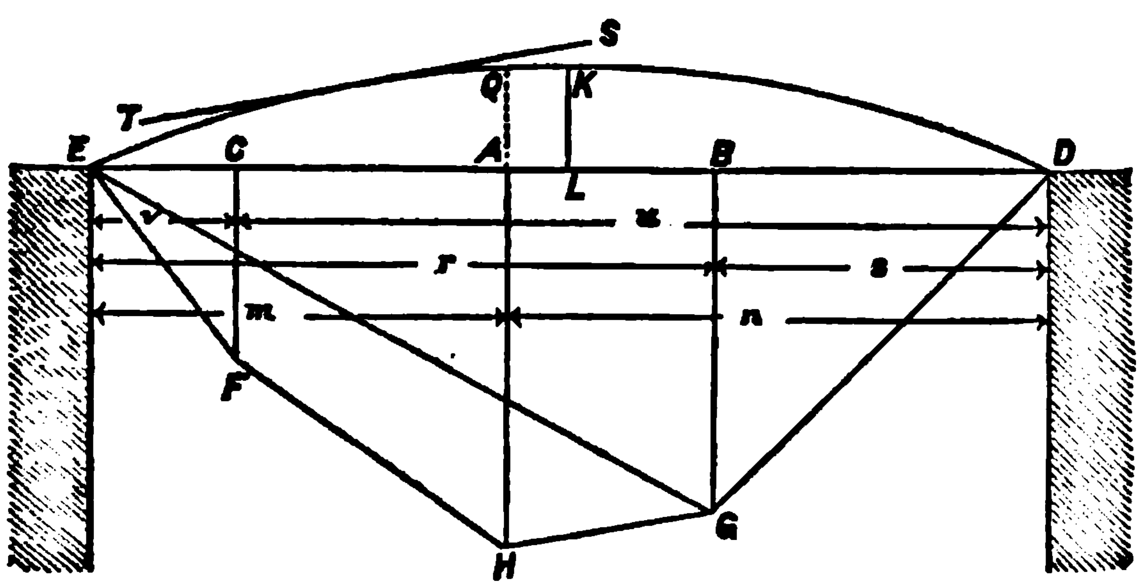

FIG. 56.

and C respectively, and KL is the strain at the middle due to the uniformly distributed load. The parabolic curve

(*Art.* **212**) EKD and the line $DGHFE$ form the boundaries of the scale of strains, as in *Art.* **254**.

For the proper assignment of the symbols m, n, r, s, etc. see *Art.* **244**, taking the two *larger* of the strains of *Fig.* 56 for the two given in that article.

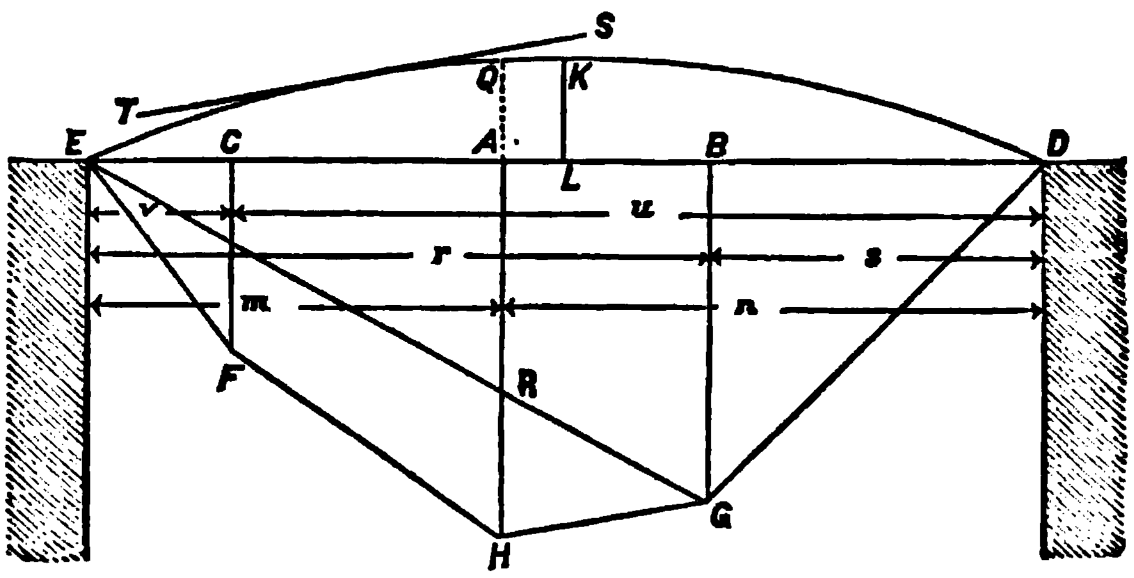

FIG. 56.

The longest vertical ordinate across the scale of strains will ordinarily be at QH; the exceptions being when the strain at B is nearly or quite equal to that at A. In the latter case, however, the diminution at QH will be so small that that ordinate may be assumed, without material error, to be the greatest. Taking it as the greatest, formula (*87*.) becomes, as in *Art.* **255**,

$$y = \tfrac{1}{2} U \frac{mn}{l} + a'$$

261.—Greatest Strain.—The manner of obtaining the value of a', the strain produced by the three concentrated loads, will now be shown.

The strain at A, produced by the load A', is (*Art.* **56**) $A'\frac{mn}{l}$. The strain at B, produced by B', is $B'\frac{rs}{l}$. The effect at A of the strain at B may be had by the proportions shown in the triangles BGE and ARE; for the effect is proportional to the horizontal distance from E (see *Art.* **192**); therefore,

$$r : m :: B'\frac{rs}{l} : B'\frac{rsm}{rl} = B'\frac{ms}{l}$$

equals the effect of the weight B'. at the point A.

Also, for the effect at A of the weight at C, the effect of C' at C being $C'\frac{uv}{l}$, we have

$$u : n :: \frac{C'uv}{l} : \frac{C'nuv}{lu} = \frac{C'nv}{l}$$

equals the effect at A of the weight at C.

The joint effect at A of the three weights is therefore

$$a' = A'\frac{mn}{l} + B'\frac{ms}{l} + C'\frac{nv}{l}$$

or,

$$a' = \frac{m}{l}(A'n + B's) + C'\frac{nv}{l}$$

Adding this to the effect of the uniformly distributed load, $\frac{1}{2}U\frac{mn}{l}$, gives

$$y = \frac{1}{2}U\frac{mn}{l} + \frac{m}{l}(A'n + B's) + C'\frac{nv}{l} \qquad \text{or}$$

$$y = \frac{m}{l}(\tfrac{1}{2}Un + A'n + B's) + C'\frac{nv}{l} \qquad (98.)$$

This represents the greatest strain arising from the uniformly distributed load and the three weights disposed as in *Fig.* 56; A' at the middle being the greatest strain and B' the next greatest.

262.—General Rule for Equally Distributed and Three Concentrated Loads.—Putting the strain [*form.* (*98.*)] in equilibrium with the resistance (*Art.* **35**) we have

$$\frac{m}{l}(\tfrac{1}{2}Un + A'n + B's) + C'\frac{nv}{l} = Sbd^2 = \tfrac{1}{6}Bbd^2$$

and with the symbol for safety added,

$$b = \frac{4a}{Bd^2l}[m(\tfrac{1}{2}Un + A'n + B's) + C'nv] \qquad (99.)$$

which is a rule for beams loaded with an equally distributed load and with three loads relatively disposed as in *Fig.* 56; A' being the greatest strain, and B' the next greatest, and A' being at the middle.

263.—Example.—As an example under this rule: What should be the breadth of a Georgia pine beam of average quality, 20 feet long and 12 inches deep, carrying 4000 pounds uniformly distributed, 6000 pounds at 4 feet from the left-hand end, 6000 pounds at 9 feet from the same end, and 7000 pounds at 6 feet from the right hand end; with the factor of safety $a = 4$?

Assigning the symbols to the loads and spaces as in *Fig.* 56, we have

$a = 4$, $B = 850$, $d = 12$, $l = 20$, $m = 9$, $n = 11$, $r = 14$, $s = 6$, $v = 4$, $U = 4000$, $A' = 6000$, $B' = 7000$ and $C' = 6000$.

Substituting these values in formula (*99.*) gives

$$b = \frac{4 \times 4}{850 \times 12^2 \times 20}[9(\overline{\tfrac{1}{2} \times 4000 \times 11} + \overline{6000 \times 11} + \overline{7000 \times 6}) + (6000 \times 11 \times 4)] = 9 \cdot 37$$

or the breadth should be $9\frac{3}{8}$ inches.

264.—Assigning the Symbols.—In working a problem of the kind just given, it is of prime importance to have the symbols denoting the weights and distances properly located. In doing this, the first point to settle is as to which of the two classes (*Fig.* 55 or 56) the case in hand belongs.

Make a sketch, such as *Fig.* 55 or 56, according to the probable position of the largest strain, letter the weights and

distances as there shown, and then compute the three strains by the following formulas.

For *Fig* 55 the strains will be as follows (*Art.* 195):

At A, the strain $a' = \frac{m}{l}(A'n + B's + C'v)$ (*100.*)

" B, " $b' = \frac{s}{l}(A'm + B'r) + C'\frac{rv}{l}$ (*101.*)

" C, " $c' = \frac{v}{l}(A'm + B'r + C'u)$ (*102.*)

In the diagram, AH is to be made, by any convenient scale, equal to a', BG to b', and CF to c', as found by these three formulas, and KL, the height of the parabola, is, by the same scale, to be made equal to $\frac{1}{2}U\frac{mn}{l}$. U is the load equably diffused over the beam; A', B' and C' are the loads concentrated at A, B and C respectively, and l is the span, or length of the beam between bearings.

For *Fig.* 56 the strains will be as follows:

At A, the strain $a' = \frac{m}{l}(A'n + B's) + C'\frac{nv}{l}$ (*103.*)

" B, " $b' = \frac{s}{l}(A'm + B'r + C'v)$ (*104.*)

" C, " $c' = \frac{v}{l}(A'n + B's + C'u)$ (*105.*)

In the case of a carriage beam the loads A', B' and C' in the formulas (*100.*) to (*105.*) are those from the headers; and equal $\frac{1}{2}fgm$, etc. In this, f and g are constant, as to the three loads in any given case, and m represents the length of one set of tail beams; consequently the loads A' B' and C' will vary as the length of the tail beams.

Hence, in the preliminary work required to ascertain to which of the two classes any given case belongs, it will suffice to use simply the length of the tail beams, instead of the full weights A', B' and C'.

For example: Take the case given in *Art.* **259**, where $l = 20$, $m = 7$, $s = 7$, $n = 13$, $r = 13$ and $v = 3$, the letters being assigned as required by *Fig.* 55. Here the tail beams carried by the header at A are 7 feet long, and those carried by the two other headers are 4 feet; therefore $A = 7$ and $B = C = 4$, and by formulas (*100.*) and (*101.*)

$$a' = \frac{7}{20}(\overline{7\times13} + \overline{4\times7} + \overline{4\times3}) = 45\cdot85$$

$$b' = \frac{7}{20}(\overline{7\times7} + \overline{4\times13}) + \frac{4\times13\times3}{20} = 43\cdot15$$

The result here obtained, a' being larger than b', shows that the case has been rightly assigned to the first class, that of *Fig.* 55.

265.—Reassigning the Symbols.—The result of a computation of the strains may show that the arrangement of the symbols was erroneous; instead of the greatest strain being in the middle it may be found at one side, or *vice versa.* Then the lettering of the loads and spaces must be changed, to agree with the proper diagram and formulas, before computing the dimensions of the beam; using formula (*96.*) or (*97.*) for the class shown in *Fig.* 55, and formula (*99.*) for the class shown in *Fig.* 56.

266.—Example.—As an illustration of the above, take a case presumably belonging to the class first treated (*Fig.* 55), where the greatest strain is an outside one. Let $l = 20$; and let the greatest load, 1750 pounds, be designated by A', with its distances $m = 7$ and $n = 13$; the second load, 1250 pounds, be designated by B', with its distances $r = 12$ and $s = 8$; and the third load, 1250 pounds, be called C', and its distances $v = 3$ and $u = 17$. To find

the united effect at each station, we have, according to formulas (*100.*) and (*101.*),

$$a' = \frac{7}{20}\left(\overline{1750 \times 13} + \overline{1250 \times 8} + \overline{1250 \times 3}\right) = 12775$$

$$b' = \frac{8}{20}\left(\overline{1750 \times 7} + \overline{1250 \times 12}\right) + \frac{1250 \times 12 \times 3}{20} = 13150$$

Here b' exceeds a' and shows that a mistake has been made as to the class to which the case belongs. We must change the symbols and arrange them for the second class (*Fig.* 56).

The middle weight is to be called A'; the weight before called A', at 7 feet from one of the walls, is now to be B'; and the third weight C'. With these changes made, we have $A' = 1250$, $B' = 1750$, $C' = 1250$, $l = 20$, $m = 8$, $n = 12$, $s = 7$, $r = 13$ and $v = 3$; and, from formulas (*103.*) and (*104.*),

$$a' = \frac{8}{20}\left(\overline{1250 \times 12} + \overline{1750 \times 7}\right) + \frac{1250 \times 12 \times 3}{20} = 13150$$

$$b' = \frac{7}{20}\left(\overline{1250 \times 8} + \overline{1750 \times 13} + \overline{1250 \times 3}\right) = 12775$$

The result is now satisfactory, and shows that the problem belongs to the second class, the one in which the greatest strain occurs at the middle, and this notwithstanding the fact that the greatest of the three *weights* is at the outside. It will be seen that the results of the two trials are the same, but reversed, that which was at first taken for a' being now taken for b'.

267.—Rule for Carriage Beam with Three Headers and Two Sets of Tail Beams.—Formula (*99.*) may be transformed so as to make it specially applicable to carriage beams.

If, in *Fig.* 56, we suppose the spaces EC and AB to be openings in the floor, then one set of tail beams will extend

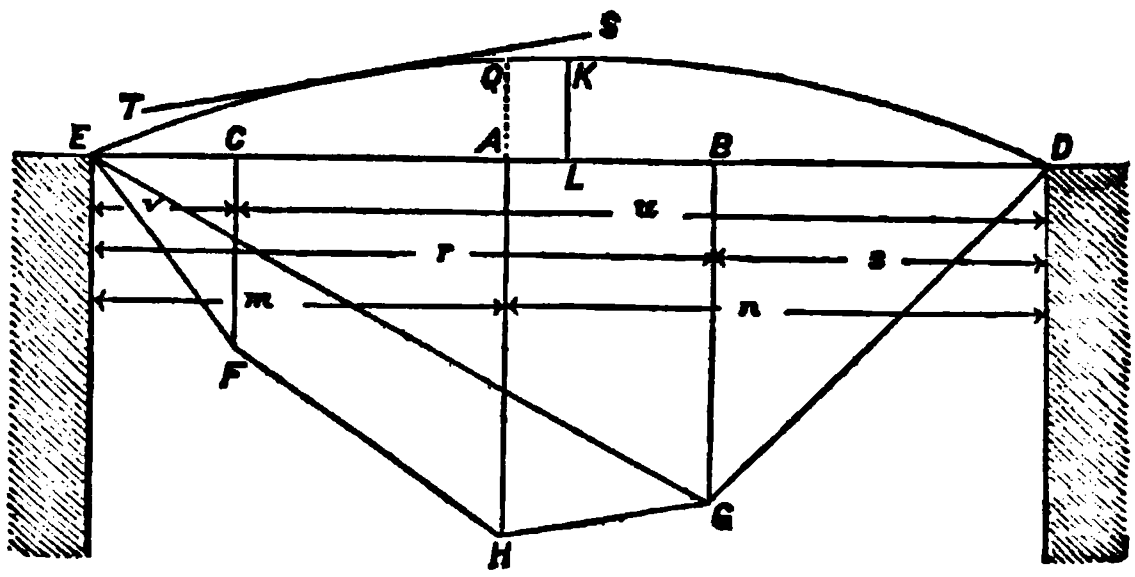

FIG. 56.

from C to A, and another from B to D, giving three headers, one each at A, B and C. The load on the header A will equal that upon C, and will equal one quarter of the load upon the space occupied by the tail beams AC, or $\frac{1}{4}fg(m-v)$. Similarly the load at B will be $\frac{1}{4}fgs$. Of the several factors composing formula (*99.*) we now have

$$A'n = \tfrac{1}{4}fgn\,(m-v)$$
$$B's = \tfrac{1}{4}fgs^2$$
$$C'nv = \tfrac{1}{4}fgnv\,(m-v)$$

and since

$$U = \tfrac{1}{2}cfl$$
$$\tfrac{1}{2}Un = \tfrac{1}{4}cfln$$

and the formula itself becomes

$$b = \frac{4a}{Bd^2l}[m(\tfrac{1}{4}cfln + \tfrac{1}{4}fgn\,\overline{m-v} + \tfrac{1}{4}fgs^2) + \tfrac{1}{4}fgnv(m-v)]$$

$$b = \frac{4a}{Bd^2l}[\tfrac{1}{4}fm\,(cnl + gn\,\overline{m-v} + gs^2) + \tfrac{1}{4}fgnv\,(m-v)]$$

$$b = \frac{4af}{4Bd^2l}[cnlm + gmn(m-v) + gms^2 + gnv(m-v)]$$

$$b = \frac{af}{Bd^2l}[cnlm + gn(m-v)(m+v) + gms^2]$$

$$b = \frac{af}{Bd^2l}[m(cnl + gs^2) + gn(m^2 - v^2)] \qquad (106).$$

which is a rule for carriage beams carrying three headers and two sets of tail beams relatively placed as in *Fig.* 56, the header producing the greatest strain being between the other two.

268.—Example.—What should be the width of a carriage beam 20 feet long, 12 inches deep, of Georgia pine of average quality, carrying three headers 14 feet long; the headers placed so as to afford a stair opening 4 feet wide at one wall, and a light well 5 feet wide, 6 feet from the other wall? The floor beams are 15 inches from centres and carry 200 pounds per foot superficial, with the factor of safety $a = 4$.

In this case we have $B = 850$, $f = 200$, $a = 4$, $c = 1\frac{1}{4}$, $d = 12$, $l = 20$, $v = 4$, $m = 9$, $n = 11$, $s = 6$, $r = 14$ and $g = 14$, and by formula (*106.*)

$$b = \frac{4 \times 200}{850 \times 12^2 \times 20} [9(\overline{1\tfrac{1}{4} \times 11 \times 20} + \overline{14 \times 6^2}) + 14 \times 11(\overline{9^2 - 4^2})] = 5 \cdot 56$$

or the breadth should be, say 6 inches.

QUESTIONS FOR PRACTICE.

269.—In a beam 20 feet long, carrying an equably distributed load of 2000 pounds, and, at 4 feet from one end, a concentrated load of 5000 pounds, what is the greatest strain produced, and where is it located?

270.—In a floor composed of beams 12 inches deep, and set 15 inches from centres, there is a Georgia pine carriage beam 22 feet long, carrying two headers with an opening between them. The headers are 14 feet long, and are placed at 5 and 12 feet respectively from the left-hand wall. The floor is required to carry 200 pounds per superficial foot, with the factor of safety $a = 4$.

What must be the breadth of the carriage beam?

CHAPTER XIII.

DEFLECTING ENERGY.

ART. 271.—**Previously Given Rules are for Rupture.**—In the discussion of the subject of transverse strains, the rules adduced thus far have all been based upon the resistance of the material to *rupture*, or the power of the material to resist the *destructive* effect produced by the load which the beam is required to carry.

272.—**Beam not only to Be Safe, but to Appear Safe.**—It is requisite in good construction that a loaded beam be not only *safe*, but that it also *appear* safe; or, that the amount of *deflection* shall not appear to be excessive. In determining the pressure a beam may receive without injury, real or apparent, it is requisite to investigate the power of a beam to resist *bending*, rather than *breaking*—that is, to ascertain the Laws of Deflection.

273.—**All Materials Possess Elasticity.**—Any load, however small, will bend a beam. If the load be not excessive, the beam will, upon the removal of the load, recover its straightness.

The power of the beam by which it returns to its original shape upon the removal of its load, is due to the *elasticity* of the material. All materials possess elasticity, though some, as lead and clay, have but little, while others, as india-rubber and whalebone, have a large measure of it.

274.—Limits of Elasticity Defined.—When a beam is bent, some of its fibres are extended and some compressed, as was shown at *Art.* **22**; and when the pressure by which the bending was effected is removed, the fibres resume their original length. Should the pressure, however, have been excessive, then the resumption will not be complete, but the extended fibres will remain a trifle longer than they were before the pressure, and the compressed fibres a trifle shorter. When this occurs, the elasticity is said to be injured; or, the pressure has exceeded the limits of elasticity.

When the fibres are thus injured, they are not only incapable of recovering their original length, but (the pressure being renewed and continued) they are not able to maintain even their present length, and therefore the deflection must gradually increase, and the fibres continue to alter in length, until finally *rupture* will ensue.

275.—A Knowledge of the Limits of Elasticity Requisite.—To secure durability, it is evident that a beam subject to transverse strain should not be loaded beyond its limit of elasticity. Hence the desirability of ascertaining this limit.

276.—Extension Directly as the Force.—Let the effect of force in producing extension be first considered. Suspend a weight of one pound, by a strip of india-rubber one foot long, and measure the increase in the length of the rubber. Then, double the weight, and it will be found that the increase in length will be double. If the extension caused by one pound be one inch, then that caused by two pounds will be two inches. Three pounds will increase the length by three inches; or, whatever weight be suspended, it will be found that the extensions will be directly in proportion to the forces producing them, provided always that the force

applied shall not be so great as to destroy the elasticity of the material; shall not so injure it as to prevent it from recovering its original length upon the removal of the force.

277.—Extension Directly as the Length.—The above shows the relation between the weight and the extension. The relation will now be shown between the extension and the length of the piece extended. At 5 inches from the upper end of a strip of rubber attach a one-pound weight. This will produce an extension of, say a quarter of an inch. Detach the weight and re-attach it at double the length, or at 10 inches from the upper end. It will now be found that the 10 inches has become $10\frac{1}{2}$ inches; the elongation being a half inch, or double what it was before. Again, remove the weight and attach it at 15 inches from the upper end, and the strip will be extended to $15\frac{3}{4}$ inches; an elongation of three quarters of an inch, or three times the amount of the first trial. From this we conclude that, under the same amount of pressure, the extensions will vary directly as the lengths of the pieces extended.

278.—Amount of Deflection.—When the projecting beam *ABCD*, *Fig.* 57, is deflected by a weight, *P*, suspended from the free end, it bends the beam, not only at the point *A*, at the wall, but also at every point of its length from *A* to *B*, so that the line *AB* becomes a convex curve, as shown.

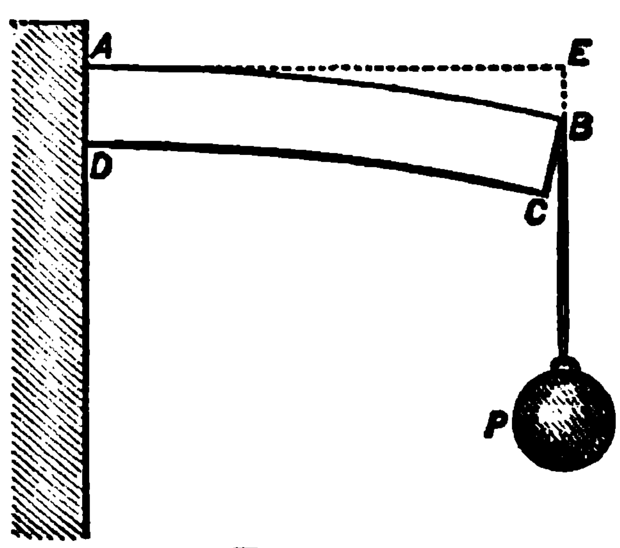

FIG. 57.

The exact shape of this *elastic curve* is defined by writers upon that subject. A full discussion

of the laws of deflection would include the development of this curve. The purpose of this work, however, will be attained without carrying the discussion so far. All that will here be attempted will be to show the *amount* of deflection; or, in the present example, the distance, EB, which the point B is depressed from its original position.

279.—The First Step.—In bending a beam, the fibres at the concave side are shortened and those at the convex side are lengthened. The first step, therefore, in finding the amount of deflection, will be to ascertain the manner of this change in length of fibre, and the method by which the amount of alteration may be measured.

280.—Deflection to be Obtained from the Extension.—It is manifest that the elongation of the fibres in the upper edge of the beam AC, *Fig.* 57, must occur not only at A, but at every point in the length of the line AB. The fibres at every point suffer an exceedingly small elongation, and if we can determine the sum of this large number of small elongations, we shall have the amount of extension of the line AB. This may be done in a simple manner, for we may, without serious error in the result to be obtained, consider them all as though they were collected and concentrated at one place in the line, instead of considering each one at the point where it occurs.

To effect this, let the line AB be drawn straight, as in *Fig.* 58, and the line FG be drawn at right angles to FK, the *neutral* line—the line which divides between those fibres which are extended and those which are compressed,

and therefore a line in which the fibres are not altered in length. The line AG may be taken as the sum of the numerous small extensions which have occurred in the fibres at the line AB of *Fig.* 57.

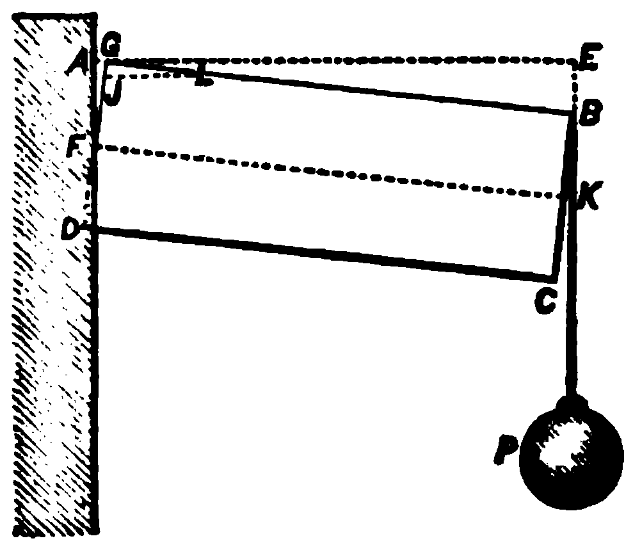

FIG. 58.

In order to show the relation between the extension and the deflection, we will investigate the proportion between AG, the measure of the one, and EB, the measure of the other.

281.—Deflection Directly as the Extension.—Make GJ, *Fig.* 58, equal to AG, and draw JL parallel with AE. The two triangles AGF and JGL are both right-angled triangles, and if AGF be revolved ninety degrees upon G as a centre, then the line AG will coincide with the line GJ, the line GF with the line GL, and AF with JL; and we have the triangle JGL, equal in all respects to the triangle AGF.

The triangle GJL is homologous with the triangle EBA, for the right line AB cuts the two parallel lines AE and JL, making the angles GLJ and EAB equal; the angles at E and G are by construction right angles, and hence the remaining angles at J and B must be equal, and the two triangles, having all their respective angles equal, must have their respective sides in proportion, or be homologous. Now, since the triangle JGL is identical with the triangle AGF, we have the two triangles AGF and BEA with their corresponding sides in proportion, or

$$GF : AE :: AG : EB$$

and as AG measures the extension and EB the deflection,

it results that the extension is in direct proportion to the deflection.

282.—Deflection Directly as the Force, and as the Length.—By the experiment of *Art.* **276**, it was shown that the extensions are in proportion to the forces producing them, and since, as just shown, they are also in proportion to the deflection, therefore the deflections are in direct proportion to the forces producing them.

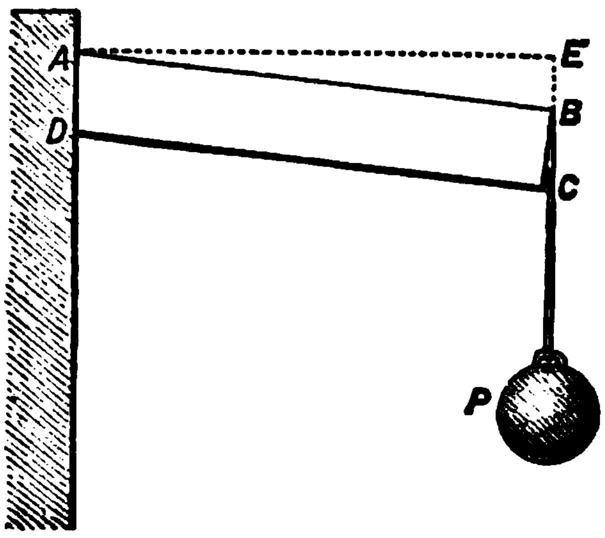

FIG. 59.

In the case of a semi-beam projecting from a wall, as *AC*, *Fig.* 59, the force producing the deflection *EB*, is the product of the weight *P*, into the arm of leverage *AE*, at the end of which the weight acts; or, the force producing the deflection is in proportion to the weight and the length.

This is shown in *Fig.* 60. Here let it be required that the weight *P* remain constant in amount and location, while the length of the semi-beam be increased. We shall then

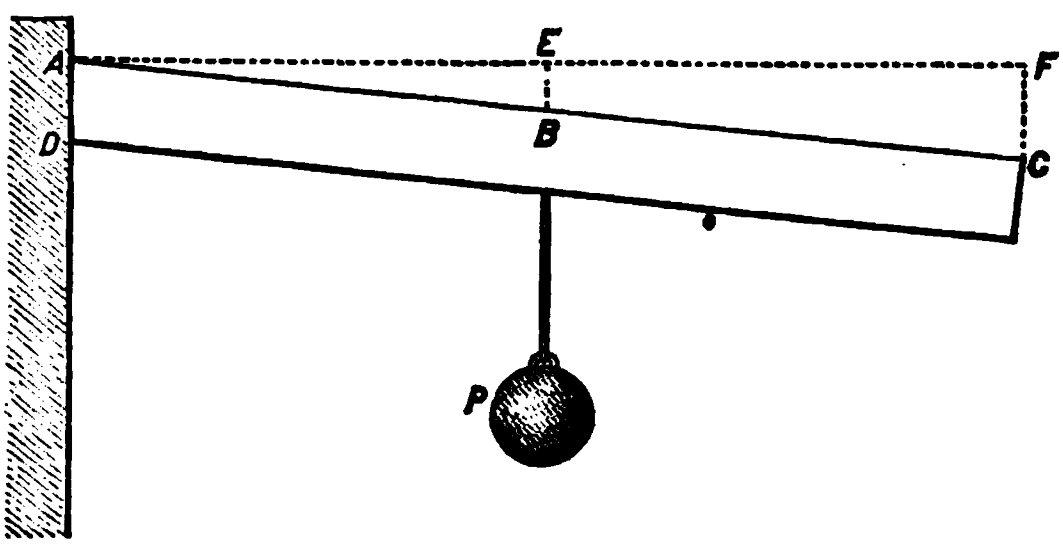

FIG. 60.

have at *E*, in *Fig.* 60, the same deflection as at *E* in *Fig.* 59, because the force producing the deflection $(P \times AE)$ is the same in each figure. But at *F*, the end of the increased

length, the deflection is greater, owing to an increase in the size of the triangle *AEB*, from *AEB* to *AFC*. The increase at *F* over that at *E* is in proportion to the increase of *AF* over *AE*, because *EB* and *FC*, the lines measuring the deflections, are similar sides of the two homologous triangles *AEB* and *AFC*; and *AE* and *AF*, the lines measuring the lengths, are also similar sides of these triangles. For example, if *AF* equal twice *AE*, then we will have *FC* equal to twice *EB*; or, in whatever proportion *AF* is to *AE*, we shall have the like proportion between *FC* and *EB*. In every case, the deflections will be in direct proportion to the lengths.

283.—Deflection Directly as the Length.—Again: If the weight be moved from *E* to *F*, *Fig.* 61, the end of the above increased length, then the force with which it acts is increased, and the deflection *FC*, caused by the weight when

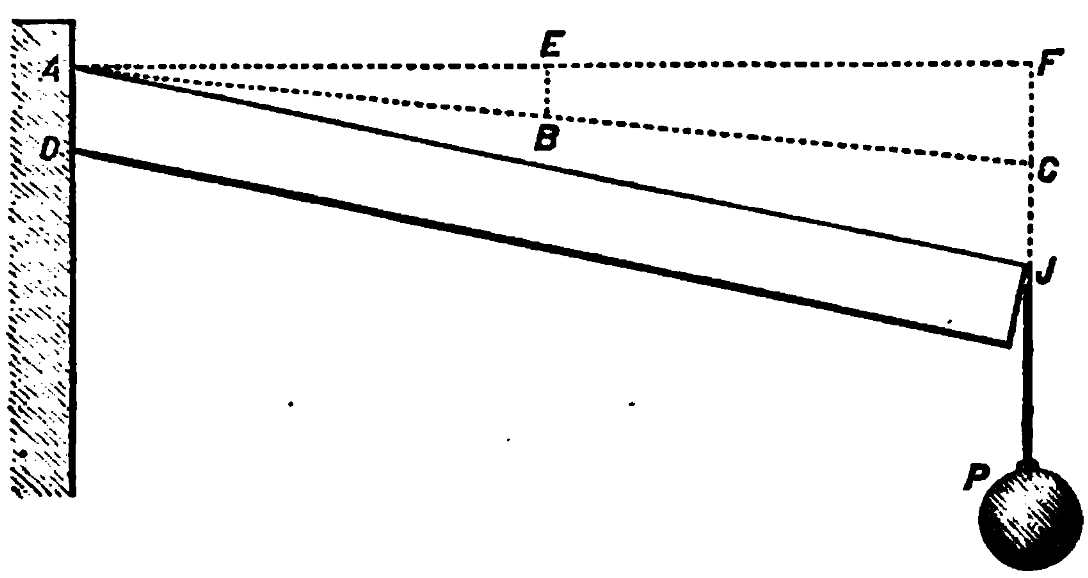

FIG. 61.

located at *E*, now becomes *FJ*. If *AF* equals twice *AE*, then the force producing deflection is doubled, because the leverage at which the weight acts is doubled; and since the deflections are in proportion to the forces producing them, *FJ* is double *FC*; and in whatever proportion the arm of leverage be increased, it will be found that the deflections at the two locations will be in proportion to the dis-

tances of the weights from the wall *AD*, or in proportion to the lengths.

284.—Deflection Directly as the Length.—Once more: When the weight was located at *E*, the length of fibres suffering extension was from *A* to *E*, but now this length is increased to *AF*.

This increase in length of fibres will increase the extension (*Art.* **277**), and consequently the deflection (*Art.* **281**). If *AF*, *Fig.* 62, be double the length of *AE*, then, owing to the extension of double the length of fibres, the deflection *FJ*, *Fig.* 61, will be doubled, or increased to *FK*, *Fig.* 62;

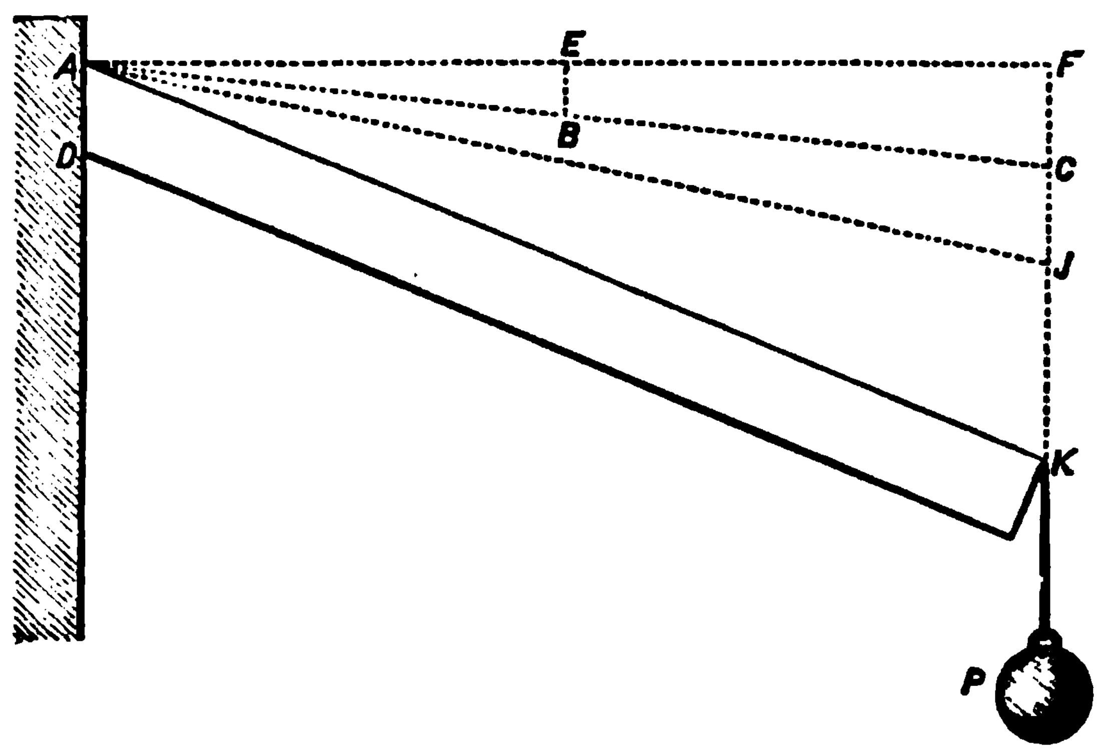

Fig. 62.

and in whatever proportion the beam be lengthened, the deflection will increase in like proportion, or the deflections will be in proportion to the lengths.

285.—Total Deflection Directly as the Cube of the Length.—Summing up the results as found in the above several steps in the increase of deflection, we find, by a comparison of *Figs.* 59 and 62, that, owing to an increase of the beam to twice its original length, we have an increase in deflection to eight times its original amount. If $EB = 1$,

then $FC = 2$, $FJ = 2FC = 4$, and $FK = 2FJ = 8EB$. With lengths of beam in proportion as 1 to 2, the deflections are as 1 to 8, or as the cubes of the lengths.

This is true not only when the length is doubled, but also for any increase of length, for a reference to the discussion will show that the deflection was found to be in proportion to the length on three several considerations: first (*Art.* **282**), on account of an increase in the size of the triangle containing the line measuring the deflection; second (*Art.* **283**), on account of the additional energy given to the weight by the increase of the leverage with which it acted; and, third (*Art.* **284**), on account of the extension of an additional length of fibres. The deflection and the length being necessarily of the same denomination, and the deflection being taken in inches, we therefore take the length, N, in inches, and we have the deflection in proportion to NNN or to N^3.

286.—Deflecting Energy Directly as the Weight and Cube of the Length.—From *Art.* **276** the extensions are in proportion to the weights, and since, from *Art.* **281**, the deflections are as the extensions, therefore we have the deflections in proportion to the weights. Combining this with the result in the last article, we have, for the sum of the effects, the deflection in proportion to the weight and the cube of the length; or,

$$\delta : PN^3$$

QUESTIONS FOR PRACTICE.

287.—The rules given in former chapters for beams exposed to cross strains were based upon the power of resistance to rupture.

Upon what power of the material may other rules be based?

288.—To what degree may beams be deflected without injury?

289.—What relation exists between extensions and the forces producing them?

290.—What relation exists between extensions and deflections?

291.—What relation, in a beam, is there between the deflections, the weights and the lengths?

CHAPTER XIV.

RESISTANCE TO FLEXURE.

ART. **292.—Resistance to Rupture, Directly as the Square of the Depth.**—Having considered, in the last chapter, the power exerted by a weight in *bending* a beam, attention will now be given to the *resistance* of the beam.

It was shown in the third chapter, that the resistance to *rupture* is in proportion to the *square* of the depth of the beam. It will now be shown that the resistance to *bending* is in proportion to the *cube* of the depth.

293.—Resistance to Extension Graphically Shown.—For the greater convenience in measuring the extension of the fibres at the top of a bent lever (*Fig.* 57), it was proposed in *Art.* **280** to consider this extension as occurring at one point; at the wall. In an investigation of the resistance to bending, the whole extension may still be considered as being concentrated at that point.

Let the triangle AGF, *Fig.* 63, represent the triangle AGF of *Fig.* 58, in which AF is the face of the wall, and AG, at the top edge of the lever, is the measure of the extension of the fibres there; while at F, the location of the neutral line, the fibres are not extended in any degree.

It is evident that the fibres suffer extension in proportion to their distance from F towards G, so that the lines BC, DE, etc., severally measure the extensions at their respective locations. Within the limits of elasticity, the *resistance*

of a fibre to extension is measured by its reaction when released from tension. Thus, the line BC measures the *extension* of the fibres at that location, and when the load is removed from the lever these fibres contract and resume their original length. Hence, BC also measures the *resistance* to extension. The resistance of the lever to bending, therefore, is in proportion to the sum of the extensions. The extensions of that portion of the lever occurring between the lines AG and BC

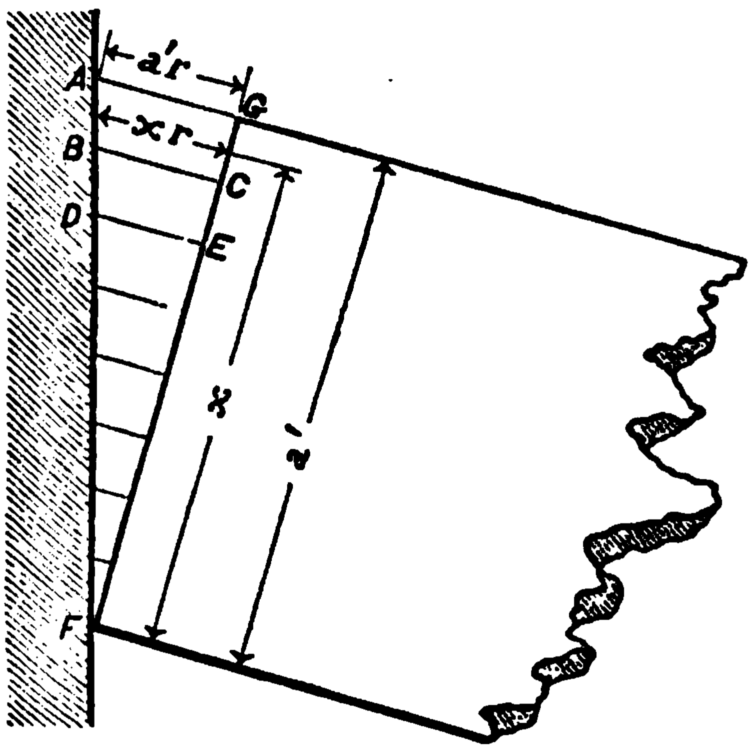

FIG. 63.

is measured by the sum of the lengths of all the fibres within the space $ABCG$. The average length of these fibres will be that of the one at the middle, and the number of fibres is measured by CG, the width they occupy. The sum, therefore, of the lengths of all the fibres will be equal to the area of the figure $ABCG$.

Again, the sum of the lengths of all the fibres between the lines BC and DE is equal to the area of the figure $BDEC$; so in each of the other figures into which the triangle AGF is divided a similar result is found. From this we conclude that the sum of the lengths of all the fibres exposed to tension is equal to the area of the whole triangle AGF; and, therefore, that the *resistance* of the lever is in proportion to the area of this triangle.

294.—Resistance to Extension in Proportion to the Number of Fibres and their Distance from Neutral Line.— In the measure of the extensions, we have the reaction or power of resistance; but there is still another fact connected

with the act of bending which needs consideration. The power of a fibre to resist deflection will be in proportion to its distance from F, the location of the neutral line; or, to the leverage with which it acts, as was shown in *Figs.* 8 and 9. Thus at AG a fibre will resist more than one at DE, while farther down, each fibre resists less until at F, where there is no leverage, the power to resist entirely disappears. It may, therefore, be concluded that the power of each fibre to resist is in proportion to its distance from F; and adding this power of resistance to that before named, we have, as the total resistance, the sum of the products of the lengths of the several fibres into their respective distances from F.

295.—Illustration.—As an illustration of the above, we may find an approximate result thus:

Let the line FG, *Fig.* 63, be divided into any number of equal parts, and through these points of division draw the lines BC, DE, etc., parallel with AG. These lines will divide the triangle into the thin slices $ABCG$, $BDEC$, etc. Now, the resistance of the top slice, $ABCG$, will be approximately equal to its area into its distance from F; or, if CG, the thickness of the slice, be represented by t, and the average length of fibres in the slice, $\frac{1}{2}(AG+BC)$, by $b_{\prime\prime}$ then the area of the slice will equal $b_{\prime}t$; and, if $a_{\prime}$ be put for FG, the average distance of the slice from F will be $a_{\prime}-\frac{1}{2}t$; and therefore the resistance of the top slice will be

$$R = b_{\prime}t(a_{\prime}-\tfrac{1}{2}t)$$

In like manner, if $c_{\prime}$ be put for the average length of the fibres of the second slice, we shall have, to represent its resistance,

$$R_{\prime} = c_{\prime}t(a_{\prime}-\tfrac{3}{2}t)$$

For the third we shall have

$$R_{,,} = d_{,}t(a_{,}-\tfrac{5}{2}t)$$

Thus, obtaining the resistance of *all* the slices and adding the results, we have the total resistance.

296.—Summing up the Resistances of the Fibres.—To make a general statement, let x be put for the distance from F to the middle of the thickness of any one of the slices into which the triangle is divided, and let r, a constant, be the length of an ordinate, as DE, located at the distance unity from F. Then we have by similar triangles the proportion

$$1 : r :: x : xr$$

and therefore xr will equal the breadth of the slice at any point distant x from F, or putting x equal to the distance from F to the middle of the slice, then xr will be equal to the average length of the fibres of the slice. The resistance then of one of the slices, say the top slice, will be $x \times xr \times t = x^2rt$. For the top slice, $x = a_{,}-\frac{1}{2}t$, therefore

$$x^2rt = (a_{,}-\tfrac{1}{2}t)^2rt = R$$

Again; for the second slice, $x = a_{,}-\frac{3}{2}t$ therefore

$$(a_{,}-\tfrac{3}{2}t)^2rt = R_{,}$$

For the third slice we have

$$(a_{,}-\tfrac{5}{2}t)^2rt = R_{,,}$$

In like manner we obtain the resistance of each successive slice, each result being the same as the preceding one, excepting the fractional coefficient of t, which differs as shown, the numerator increasing by the constant number 2. When

n represents the total number of slices, then the last result or the resistance of the last slice will be

$$[a_{,}-\tfrac{1}{2}(2n-1)t]^2 rt = R_n$$

and the sum of all the resistances, or

$$R_{,} + R_{,,} + R_{,,,} + \text{etc.} + R_n = M$$

will equal the total resistance of all the fibres, thus:

$$M = (a_{,}-\tfrac{1}{2}t)^2 rt + (a_{,}-\tfrac{3}{2}t)^2 rt + \text{etc.} + (a_{,}-\tfrac{1}{2}\overline{2n-1}\,t)^2 rt$$

$$M = rt\ [(a_{,}-\tfrac{1}{2}t)^2 + (a_{,}-\tfrac{3}{2}t)^2 + \text{etc.} + (a_{,}-\tfrac{1}{2}\overline{2n-1}\,t)^2]$$

Now the number of slices multiplied by the thickness of each will equal FG, or $nt = a_{,}$, from which $t = \frac{a_{,}}{n}$, and, by substituting this value,

$$a_{,}-\tfrac{1}{2}t = a_{,}-\tfrac{1}{2}\frac{a'}{n} = a_{,}\left(1-\frac{1}{2n}\right) = a_{,}\frac{2n-1}{2n}$$

and $$(a_{,}-\tfrac{1}{2}t)^2 = a_{,}^2\frac{(2n-1)^2}{4n^2}$$ therefore

$$M = rt\left[a_{,}^2\frac{(2n-1)^2}{4n^2} + a_{,}^2\frac{(2n-3)^2}{4n^2} + \text{etc.} + a_{,}^2\frac{(2n-\overline{2n-1})^2}{4n^2}\right]$$

$$M = \frac{rta_{,}^2}{4n^2}[(2n-1)^2 + (2n-3)^2 + \text{etc.} + (2n-\overline{2n-1})^2]$$

Now,
$$\begin{aligned}(2n-1)^2 &= 4n^2 - \overline{1 \times 4n} + 1\\ (2n-3)^2 &= 4n^2 - \overline{3 \times 4n} + 9\\ (2n-5)^2 &= 4n^2 - \overline{5 \times 4n} + 25\end{aligned}$$

To get the sum of these, we have, first, for the sum of the first terms, $n \times 4n^2 = 4n^3$.

The coefficients of the second terms, namely, 1, 3, 5, etc., equal in amount the sum of an arithmetical series composed of these odd numbers; or, n^2 (*Art.* **200**), and hence the sum of these several second terms is $n^2 \times 4n = 4n^3$. The first and second terms summing up alike cancel each other, and we have but the third terms remaining. The sum of these is that of the squares of the odd numbers 1, 3, 5, etc., and our last formula becomes

$$M = \frac{a_{,}^{3}rt}{4n^{2}}\left[1^2 + 3^2 + 5^2 + \text{etc.} + (2n-1)^2\right]$$

Now, $t = \frac{a_{,}}{n}$ and $\frac{a_{,}^{3}rt}{4n^{2}} = \frac{a_{,}^{3}r}{4n^{3}}$, therefore

$$M = a_{,}^{3}r\left(\frac{1^2 + 3^2 + 5^2 + \text{etc.} + (2n-1)^2}{4n^3}\right) \qquad (107.)$$

297.—True Value to which these Results Approximate. —As an example to test this formula, let $n = 3$, then

$$M = a_{,}^{3}r\left(\frac{1+9+25}{4 \times 27}\right) = \tfrac{35}{108}a_{,}^{3}r$$

Again, let $n = 4$, then

$$M = a_{,}^{3}r\left(\frac{1+9+25+49}{4 \times 4^3}\right) = \tfrac{84}{256}a_{,}^{3}r$$

and if $n = 5$, then

$$M = a_{,}^{3}r\left(\frac{1+9+25+49+(\overline{2 \times 5-1})^2}{4 \times 5^3}\right) = \tfrac{165}{500}a_{,}^{3}r$$

If $n = 10$, then

$$M = \tfrac{1330}{4000}a_{,}^{3}r$$

If $n = 20$, then

$$M = \frac{10660}{32000} a_1^2 r$$

Reducing these five fractions to their least common denominator, 43,200, we have

When	$n = 3$,	the numerator		$= 14{,}000$
"	$n = 4$,	"	"	$= 14{,}175$
"	$n = 5$,	"	"	$= 14{,}256$
"	$n = 10$,	"	"	$= 14{,}364$
"	$n = 20$,	"	"	$= 14{,}391$

It will be noticed that these numerators increase as n increases, but not so rapidly. As n becomes larger, the increase in the numerator is more gradual, but still remains an increase, for however large n becomes, the numerator will still increase, until n becomes infinite, when its limit is reached.

This limit is equal in this particular case to 14,400, or one third of 43,200, the denominator; or, in general, the value of the fraction tends towards $\frac{1}{3}$, and

$$M = \frac{1}{3} a_1^2 r$$

298. — True Value Defined by the Calculus. — This definite result is reached more easily and directly by means of the calculus.

Taking the notation of *Art.* **296**, we have, for the resistance of one of the slices, the expression

$$R = x^2 rt$$

This gives the resistance for a slice at any distance, x, from F, and if the thickness of the slice be reduced to the smallest conceivable dimension, then t, its thickness, may

be taken for the differential of x, or dx, and we have as the differential of the resistance

$$dR = x^2 r dx$$

from which, by integration, is obtained (*Art.* **463**)

$$R = \tfrac{1}{3} x^3 r$$

and when the result is made definite by taking the integral between limits, or between $x = 0$ and $x = a_{,}$, we have

$$R = \tfrac{1}{3} a_{,}^3 r \qquad (108.)$$

299.—Sum of the Two Resistances, to Extension and to Compression.—The foregoing discussion has been confined to the resistance offered by that portion of the lever the fibres of which suffer extension.

A similar result may be obtained from a consideration of the resistance offered by the remaining fibres to compression.

If c be put to represent the depth of that part of the beam in which the fibres are compressed, then it will be found that the resistance to compression will, from (*108.*), be equal to

$$R_{,} = \tfrac{1}{3} c^3 r \qquad (109.)$$

and the total resistance offered by the lever will be

$$R + R_{,} = \tfrac{1}{3} a_{,}^3 r + \tfrac{1}{3} c^3 r = \tfrac{1}{3} r (a_{,}^3 + c^3)$$

It may be shown also, by a farther investigation, that in levers suffering small deflections, or when not deflected beyond the limits of elasticity, $a_{,} = c$, or the neutral line is at the middle of the depth. In the latter case, we have $a_{,} = c = \tfrac{1}{2} d$, and therefore

$$R + R_{,} = \tfrac{1}{3} r [(\tfrac{1}{2} d)^3 + (\tfrac{1}{2} d)^3] = \tfrac{2}{3} r \tfrac{1}{8} d^3 = \tfrac{1}{12} r d^3$$

300.—Formula for Deflection in Levers.—The above is the result in a lever one inch broad. A lever two inches broad would bear twice as much; one three inches broad would bear three times as much; or, generally, the resistance will be in proportion to the breadth. We have then for a lever of any breadth

$$R + R_{,} = \tfrac{1}{12} rbd^{2} \qquad (110.)$$

This expression gives the resistance to the deflecting energy, which is (*Art.* **286**) equal to PN^{2}. This power, PN^{2}, however, not only overcomes the resistance, $\frac{1}{12}rbd^{2}$, but in the act also accomplishes the deflection; moves the lever through a certain distance. Representing this distance by δ, we have, as the full measure of the work accomplished, $\delta \times \frac{1}{12}rbd^{2}$. When the power and the work are equal, we have

$$PN^{2} = \tfrac{1}{12}\delta rbd^{2} \qquad \text{from which,}$$

$$\delta = \frac{PN^{2}}{\frac{1}{12}rbd^{2}} \qquad (111.)$$

301.—Formula for Deflection in Beams.—The expression (*111.*) is for a semi-beam or lever. When a full beam, supported at each end, is deflected by W, a weight located at the middle, we have to consider that for P we must take $\frac{1}{2}W$, and for N take $\frac{1}{2}L$ (see *Art.* **35**). These alterations will produce

$$\delta = \frac{\frac{1}{2}W(\frac{1}{2}L)^{2}}{\frac{1}{12}rbd^{2}} \qquad \text{or}$$

$$\delta = \frac{WL^{2}}{\frac{2}{3}rbd^{2}} \qquad (112.)$$

for the deflection of a beam supported at both ends and loaded in the middle.

302.—Value of F, the Symbol for Resistance to Flexure. —In formula (*112.*) the dimensions are all in inches. As it is more convenient that the length be taken in feet, let l represent the length in feet, then

$$\frac{L}{12} = l, \quad L = 12l \text{ and } L^3 = \overline{12l}^3 = 1728l^3$$

By substitution in formula (*112.*) we have

$$\delta = \frac{1728\,Wl^3}{\frac{4}{3}rbd^3} = \frac{1296\,Wl^3}{rbd^3} \qquad \text{or}$$

$$\frac{r}{1296} = \frac{Wl^3}{bd^3\delta}$$

The symbol r is a measure of the extension, differing in different materials, but constant, or nearly so, in each. Putting for $\frac{r}{1296}$ the letter F we have

$$F = \frac{Wl^3}{bd^3\delta} \qquad (113.)$$

The respective values of F for several materials have been obtained by experiment, and may be found in Table XX. Its value in each case is that for a beam supported at each end, and with the load in pounds applied at the middle of l, the distance in feet between the bearings; while b, d and δ are in inches; δ being the deflection within the limits of elasticity.

303.—Comparison of F with E, the Modulus of Elasticity.—The common expression for flexure of beams when laid on two supports and loaded at the middle is [Tate's Strength of Materials, London, 1850, p. 24, formula (*49.*)]

$$\delta = \frac{WL^3}{48EI} \qquad (114.)$$

in which E represents what is termed the *Modulus* or *Coefficient of Elasticity*, which is (same work, p. 3), "that force, which is necessary to elongate a uniform bar, one square inch section, to double its length (supposing such a thing possible) or to compress it to one half its length"; and I represents the *Moment of Inertia* (*Arts.* **457** to **463**) of the cross-section of the beam.

In this expression the dimensions are all in inches. To change L to feet we have $\frac{L}{12} = l$ equals the length in feet,

or
$$L^3 = \overline{12l}^3 = 1728l^3.$$

Substituting this value in (*114.*) we obtain

$$\delta = \frac{1728\,Wl^3}{48EI} = \frac{36\,Wl^3}{EI}$$

or
$$\frac{E}{36} = \frac{Wl^3}{I\delta}$$

In formula (*113.*) we have

$$F = \frac{Wl^3}{bd^3\delta}$$

Multiplying this by 12 gives

$$12F = \frac{Wl^3}{\frac{1}{12}bd^3\delta}$$

and since $\frac{1}{12}bd^3 = I$ (see *Art.* **463**)

$$12F = \frac{Wl^3}{I\delta} \qquad (115.)$$

Comparing this with above value of $\frac{E}{36}$ we have

$$\frac{E}{36} = 12F \quad \text{or} \quad E = 432F$$

304.—Relative Value of F and E.—In Table XX. the value of F for wrought iron is, from experiments on rolled iron beams, 62,000. Then

$$432F = E = 432 \times 62000 = 26784000,$$

equals the modulus of elasticity for the wrought-iron of which these beams were made. They were of American metal. Tredgold found the value for English iron to be $E = 24{,}900{,}000$; and Hodgkinson from 19,000,000 to 28,000,000.

An average of the results in seven cases gives 25,300,000 as the modulus of elasticity for English wrought-iron.

305.—Comparison of F with E common, and with the E of Barlow.—Barlow, in his "Materials and Construction," p. 93, foot-note (Ed. of 1851), uses the expression $\frac{L^3W}{bd^3\delta} = E$, instead of $\frac{L^3W}{3bd^3\delta}$, for a lever loaded at one end; and on p. 94, $\frac{L^3W}{16bd^3\delta} = E$. The dimensions are all in inches. Changing the length to feet, we have

$$E = \frac{1728l^3W}{16bd^3\delta} = \frac{108l^3W}{bd^3\delta}$$

or

$$\frac{E}{108} = \frac{Wl^3}{bd^3\delta}$$

Comparing this with (*113.*), which is

$$F = \frac{Wl^3}{bd^3\delta}$$

we have $\frac{E}{108} = F$, or $108F = E$. We found before (*Art.* 303) that $E = 432F$, and since $4 \times 108 = 432$, therefore

the E of Barlow equals one quarter of the E in common use, and his values of E are equal to 108 times the values of F as given in this book.

For example; on p. 147, in an experiment on New England fir, he gives, by an error in computation, $E = 547800$, but which, corrected, equals 373026. Dividing this by 108 as above, gives

$$\frac{373026}{108} = F = 3454$$

By reference to Table XX. we find that for spruce, the wood most probably intended for New England fir,

$$F = 3500$$

Again; taking Barlow's four experiments on oak, p. 146, and correcting the arithmetical errors, we have $E = 361758$, 482344, 291227 and 242860. This gives an average of 344547, and dividing it by 108 as above, we have

$$F = 3190$$

By reference to Table XX. we find that by my experiments

$$F = 3100$$

306.—Example under the Rule for Flexure.—To make a practical application of the rule in formula (*113.*), let it be required to find the depth of a white pine beam 10 feet long between bearings and 4 inches broad; and which, with a load of 2000 pounds at the middle of its length, shall be deflected 0·3 of an inch.

We obtain from (*113.*)

$$d^2 = \frac{Wl^2}{Fb\delta}$$

or, in this case,

$$d^2 = \frac{2000 \times 10^3}{2900 \times 4 \times 0 \cdot 3}$$

$$d^2 = \frac{2000000}{3480} = 574 \cdot 7$$

$$d = 8 \cdot 31$$

or the depth should be $8 \cdot 31$, say $8\frac{1}{3}$ inches.

QUESTIONS FOR PRACTICE.

307.—How may the resistance of a fibre to extension be measured when the elasticity remains uninjured?

308.—In a beam exposed to transverse strain, what is the resistance to extension in proportion to?

309.—When the bending energy and the resistance of a beam are in equilibrium, what is the expression for this relation?

310.—Given a white pine beam 20 feet long, 6 inches broad and 12 inches deep, and loaded with 1000 pounds at the middle. What will be the deflection, the value of F being 2900?

CHAPTER XV.

RESISTANCE TO FLEXURE—LIMIT OF ELASTICITY.

ART. 311.—**Rules for Rupture and for Flexure Compared.**—The rules for determining the *strength* of materials differ from those denoting their *stiffness*. The former are more simple; all their symbols being unaffected except one, and this only to the second power, or square; in the latter, two of the symbols are involved to the third power or cube.

Many, in determining the dimensions of timbers exposed to transverse strains, are induced, by the greater simplicity of the rules for strength, to use them in preference to those for stiffness, even when the latter only should be used.

A beam apportioned by the rules for strength will not bend so as to strain the fibres beyond their elastic limit, and will therefore be safe; but in many cases the beam will bend more than a due regard for appearance will justify.

When timbers, therefore, as those in the ceiling or floor of a room, might deflect so much as to be readily perceptible, and unpleasant to the eye, they should have their dimensions fixed by the rules for stiffness only.

312.—**The Value of *a*, the Symbol for Safe Weight.**—In order that the symbol *a* in the rules for strength, denoting the number of times the safe weight is contained in the breaking weight, may be of the proper value to preserve the fibres of the timber from being strained beyond the elastic

limit, a few considerations will now be presented showing the manner in which this value is ascertained.

In *Fig.* 64 let $ABCD$ represent a lever with one end, AD, imbedded in a wall, AD being the face of the wall, and carrying at the other end, BC, a weight P; the weight deflecting the lever from the line AE to the extent EB. The line FH is the neutral line, and FG is drawn at right angles to FH.

Fig. 64.

As in *Figs.* 58 and 63, so here the triangle AFG shows the elongation of the fibres in the upper half of the beam, and AG the elongation to the limits of elasticity of the fibres at the upper edge AB. The triangle AFG is in proportion to the triangle ABE, as shown in *Art.* **281**. If $AB = N$ (this being a semi-beam), and e equals the extension per unit of N, then $AG = eN$.

We have by similar triangles

$$AF : AB :: AG : EB$$

Then if $AD = d$ and $EB = \delta$

$$\tfrac{1}{2}d : N :: eN : \delta = \frac{eN^2}{\tfrac{1}{2}d}$$

$$\delta = \frac{2eN^2}{d}$$

The dimensions here are all in inches. To change N in inches to n in feet, we have

$$\frac{N}{12} = n, \quad N = 12n \quad \text{and} \quad N^2 = 144n^2$$

from which

$$\delta = \frac{288en^2}{d}.$$

and from this we obtain

$$e = \frac{d\delta}{288n^2}$$

in which δ is the deflection when at the limit of elasticity, and in which e, d and δ are in inches, and n in feet. This is for a semi-beam, and it will be perceived that the deflection EB, in *Fig.* 64, caused by the weight P, is precisely the same as would be produced in a full beam by double this weight placed at D, the beam being in a reversed position.

When, therefore, l equals the length of the full beam in feet, n will equal $\frac{1}{2}l$. Substituting this value of n in the above expressions, we have

$$\delta = \frac{288e(\frac{1}{2}l)^2}{d}$$

$$\delta = \frac{72el^2}{d} \qquad (116.)$$

and for the value of e,

$$e = \frac{d\delta}{72l^2} \qquad (117.)$$

In *Art.* **302** we have, for the stiffness of materials, formula (*113.*),

$$F = \frac{Wl^3}{bd^3\delta}$$

For δ substitute $\frac{72el^2}{d}$, its value as just found, and, in order to distinguish the weight used to produce flexure from that used to produce rupture, let us for the moment indicate the former by G, and the latter by W. Then,

from the above,

$$Gl^2 = Fbd^2 \frac{72el^2}{d}$$

$$Gl = 72Fbd^2e$$

The relation between F, the measure of the elasticity of materials, and B, the resistance to rupture, may be put thus:

$$B : F :: 1 : m = \frac{F}{B}; \text{ or, } F = Bm$$

Substituting this value for F in the above, we have

$$Gl = 72Bmbd^2e$$

$$\frac{Gl}{72em} = Bbd^2$$

Now the formula for strength, $B = \frac{Wl}{bd^2}$, [*form.* (*10.*) in *Art.* **36**] gives $Wl = Bbd^2$; a comparison of this value of Bbd^2 with that above shown gives

$$\frac{Gl}{72em} = Wl$$

Since G is the deflecting weight which bends the lever to the limit of elasticity, it is therefore the ultimate weight which may be trusted safely upon the beam, and as a is a symbol put to denote the number of times G is contained in W, the breaking weight, therefore

$$G : W :: 1 : a = \frac{W}{G} \text{ and } Ga = W$$

Substituting this value for W in the above, we have

$$\frac{Gl}{72em} = Gal$$

$$\frac{1}{72em} = a$$

As above found, $m = \frac{F}{B}$, therefore

$$a = \frac{1}{72e\frac{F}{B}}$$

$$a = \frac{B}{72Fe} \qquad (118.)$$

From this expression the values of a for various materials have been computed, and the results are to be found in Table XX.

313.—Rate of Deflection per Foot Length of Beam.— The value of a as just found is based upon the elasticity of the material, and is measured by this elasticity at its limit.

This limit is that to which bending is allowable in beams apportioned for strength. In beams required to sustain their loads without bending so much as to be perceptible or offensive to the eye, the bending is generally far within the elastic limit. The deflection in these beams is rated in proportion to the length of the beam; or, when r in inches equals the rate of deflection per foot in length of the beam, then $rl = \delta$. The deflection by formula (*116.*) is

$$\delta = \frac{72el^2}{d}$$

therefore

$$rl = \frac{72el^2}{d}$$

$$r = \frac{72el}{d} \qquad (119.)$$

This gives r at its greatest possible value, and shows that it should never exceed 72 times the ratio between the length and depth, multiplied by e; e being the measure of extension as recorded in Table XX. The ratio between

the length and depth is to be taken with l in feet and d in inches.

The value of r as required in beams of the usual proportions and deflection, will not be as great as that here shown to be allowable. In cases where the rate of deflection, r, is as great as 0·05 of an inch per foot, and the length of the beam is short in comparison with the depth $\left(\text{say } \frac{l}{d} \text{ is as small as } \frac{5}{7}\right)$, then there will be danger of r exceeding the limit fixed by this rule. When the fraction $\frac{l}{d}$ is less than $\frac{5}{7}$ then the rate r should be tested to know whether it has exceeded the proper limit. It is seldom, however, that a beam 7 inches high is used shorter than 5 feet, or one 14 inches high shorter than 10 feet. Generally the number of feet in the length exceeds the number of inches in the depth.

314.—Rate of Deflection in Floors.—The rate of deflection allowable so as not to be unsightly is a matter of judgment. Tredgold, in his rules for floor beams, fixed it at $\frac{1}{40}$ of an inch per foot of the length, or 0·025. This is thought by some to be rather small, especially since in floors the limit of the rate is seldom reached; in fact never, except when the floor is loaded to its fullest capacity, a circumstance which occurs but seldom, and then only for a limited period. For this reason, it is proper to fix the rate at say $\frac{1}{33}$, or 0·03 of an inch per foot. With this as the rate for a full load, the usual rate of deflection under ordinary loads will probably not exceed 0·01 or 0·015. In the rules, the symbol r is left undetermined, so that the rate may be fixed as judgment or circumstances may dictate in each special case.

QUESTIONS FOR PRACTICE.

315.—What is the distinction between the rules for *strength* and those for *stiffness*?

316.—What expression shows δ, the deflection at the elastic limit?

317.—What expression gives the measure of extension at the elastic limit?

318.—What expression shows the ultimate value of a, the factor of safety?

319.—What expression gives the ultimate value of r, the rate of deflection?

CHAPTER XVI.

RESISTANCE TO FLEXURE—RULES.

ART. **320.—Deflection of a Beam, with Example.**—The formula (*113.*) for the deflection of beams supported at each end and loaded at the middle, is

$$F = \frac{Wl^3}{bd^3\delta}$$

from which,

$$\delta = \frac{Wl^3}{Fbd^3} \qquad (120.)$$

This is the deflection of any beam placed and loaded as above. For example: What is the deflection of a white pine beam of 4×9 inches, set edgewise upon bearings 16 feet apart, and loaded with 5000 pounds at the middle; the value of F being 2900, the average of experiments, the results of which are recorded in Table XX.?

The deflection in this case will be

$$\delta = \frac{5000 \times 16^3}{2900 \times 4 \times 9^3} = \frac{50 \times 1024}{29 \times 729} = 2 \cdot 4218$$

This is a large deflection, much beyond what would be proper in a good floor, for at 0·03 inch per foot of the length of the beam, the rate of deflection adopted (*Art.* **314**), we should have

$$\delta = 16 \times 0 \cdot 03 = 0 \cdot 48$$

or, say half an inch, whereas the 5000 pounds upon this

beam produces five times this amount. Although so greatly in excess of what a respect for appearance will allow, it is still, however, within the limits of elasticity, as will be seen by the use of formula (*116.*), in which we have

$$\delta = \frac{72el^2}{d}$$

Obtaining from Table XX. the average value of e, equal 0·0014, we have

$$\delta = \frac{72 \times 0\cdot0014 \times 16^2}{9} = 8 \times 0\cdot0014 \times 256 = 2\cdot8672$$

as the greatest deflection allowable.

321.—Precautions as to Values of Constants F and e.— The above is the ultimate deflection within the limits of elasticity, and is 0·4454 in excess of the 2·4218 produced by the 5000 pounds. In general, it would be undesirable to load a beam so heavily as this, or to deflect it to a point so near the limit of elasticity, and, unless the timber be of fair quality, would hardly be safe.

Some pine timber would be deflected by this weight much more than is here shown—in fact, beyond the limits of elasticity. In the above computation, F was taken at 2900, the average value, and the measure of elasticity, e, was taken at 0·0014, also the average value; whereas, had these constants been taken at their lowest value, such as pertain to the poorer qualities of white pine, and in which $F = 2000$ and $e = 0\cdot001016$, the limits of elasticity would have been found at a trifle over 2 inches, while the deflection would have reached $3\frac{1}{2}$ inches.

322.—Values of Constants F and e to be Derived from Actual Experiment in Certain Cases.—For any important work, the capacity of the timber selected for use should be tested by actual experiment. This may be done by submitting several pieces to the test of known weights placed at the centre, by increasing the weights by equal increments, and by noting the corresponding deflections. From these deflections, the specific values of F and e for that timber may be ascertained; and with these values the timber may be loaded with certainty as to the result. In the absence of a knowledge of the elastic power of the particular material to be used, a sufficiently wide margin should be allowed, in order that the timber may not be loaded beyond what the poorer kinds would be able to carry safely.

323.—Deflection of a Lever.—The rule for deflection, as discussed in these last articles, is appropriate for a beam supported at both ends and loaded in the middle. A rule will now be developed for a semi-beam or lever; a timber fixed at one end in a wall, and with a weight suspended from the other. The deflection in this case is precisely the same as that produced by twice the weight, laid at the middle of a whole beam, double the length of the lever, and supported at each end.

Let the weight at the end of the lever be represented by P, and the length of the lever by n, then W of formula (*120*), which is $\delta = \frac{Wl^3}{Fbd^3}$, will equal $2P$, and l will equal $2n$, and we have, by substituting these values for W and l

$$\delta = \frac{2P \times \overline{2n}^3}{Fbd^3}$$

$$\delta = \frac{16Pn^3}{Fbd^3} \qquad (121.)$$

324.—Example.—The deflection above found is that produced in a lever by a weight suspended from its free end.

As an example: What would be the deflection caused by a weight of 1500 pounds suspended from the free end of a lever of Georgia pine, of average quality, 3×6 inches square and 5 feet long?

Here we have $P = 1500$, $n = 5$, $F = 5900$, $b = 3$ and $d = 6$; and therefore

$$\delta = \frac{16 \times 1500 \times 5^3}{5900 \times 3 \times 6^3} = \frac{80 \times 125}{59 \times 216} = 0 \cdot 7847$$

325.—Test by Rule for Elastic Limit in a Lever.—To test the above, to ascertain as to whether the deflection is within the limits of elasticity, take $l = 2n = 10$, and by formula (*116.*) we get

$$\delta = \frac{72el^2}{d} = \frac{72 \times 0 \cdot 00109 \times 10^2}{6} = 12 \times 0 \cdot 109 = 1 \cdot 308$$

This is satisfactory, as it shows that the lever has a deflection (0·7847) of not much more than half that within the elastic limit (1·308), and therefore a safe one.

326.—Load Producing a Given Deflection in a Beam.—By inversions of formulas (*120.*) and (*121.*), we may have rules for ascertaining the weight which any beam or lever will carry with a given deflection.

First; for a beam, we take formula (*120.*)

$$\delta = \frac{Wl^3}{Fbd^3}$$

and have

$$W = \frac{Fbd^3\delta}{l^3} \qquad (122.)$$

327.—Example.—For an example: What weight upon the middle of a beam of spruce, of average quality, 5 inches broad, 10 inches high, and 20 feet long between the bearings, will produce a deflection of 0·03 inch per foot, or 0·6 inch in all?

Here we have $F = 3500$, $b = 5$, $d = 10$, $\delta = 0{\cdot}6$ and $l = 20$; therefore

$$W = \frac{3500 \times 5 \times 10^3 \times 0{\cdot}6}{20^3} = \frac{10500}{8} = 1312{\cdot}5$$

328.—Load at the Limit of Elasticity in a Beam.—Again: What weight could be carried upon this beam if the deflection were permitted to extend to the limit of elasticity?

Formula (*116.*) gives us

$$\delta = \frac{72el^2}{d}$$

and from Table XX. we have the average value of e for spruce equal to 0·00098, and therefore

$$\delta = \frac{72 \times 0{\cdot}00098 \times 20^2}{10} = 72 \times 0{\cdot}00098 \times 40 = 2{\cdot}8224$$

Substituting this new deflection in the former statement, we have

$$W = \frac{3500 \times 5 \times 10^3 \times 2{\cdot}8224}{20^3} = \frac{49392}{8} = 6174$$

This 6174 pounds for good timber would be a safe load, but if there be doubts as to the quality, the load should be made less according to the lower values of F and e.

329.—Load Producing a Given Deflection in a Lever—Example.—*Second;* for a lever, we take formula (*121.*)

$$\delta = \frac{16Pn^3}{Fbd^3}$$

and find by inversion

$$P = \frac{Fbd^3\delta}{16n^3} \qquad (123.)$$

An application of this rule may be shown in the answer to the question: What weight may be sustained at the end of a hemlock lever, 6 inches broad and 9 inches high, firmly imbedded in a wall, and projecting 8 feet from its face? The hemlock is of good quality, and the deflection is limited to 1 inch.

Here we have $F = 2800$, $b = 6$, $d = 9$, $\delta = 1$, and $n = 8$; therefore

$$P = \frac{2800 \times 6 \times 9^3 \times 1}{16 \times 8^3} = 1495$$

that is, 1495 pounds at the end of the lever would deflect it one inch.

330.—Deflection in a Lever at the Limit of Elasticity.—What deflection in this lever would mark the limit of elasticity?

Formula (*116.*) is

$$\delta = \frac{72el^2}{d}$$

Taking l at twice n we have $l = 16$, $d = 9$, and $e = 0{\cdot}00095$; and as a result

$$\delta = \frac{72 \times 0{\cdot}00095 \times 16^2}{9} = 8 \times 0{\cdot}00095 \times 256 = 1{\cdot}9456$$

331.—Load on Lever at the Limit of Elasticity.—What weight would deflect this lever to the limit of elasticity?

For this we have

$$P = \frac{2800 \times 6 \times 9^2 \times 1 \cdot 9456}{16 \times 8^2} = 2909$$

This is nearly double the weight required to deflect it one inch, as before found; and the deflection is also nearly double. The weight and the deflection are directly in proportion. If 1500 pounds deflect a beam one inch, 3000 pounds will deflect it two inches.

332.—Values of W, l, b, d and δ in a Beam.—By a proper inversion of the formulas for beams, any one of the dimensions may be obtained, provided the other dimensions and the weight are known.

Thus we have (*form. 122.*)

$$W = \frac{Fbd^3\delta}{l^3}$$

and from this find

the length, $$l = \sqrt[3]{\frac{Fbd^3\delta}{W}} \qquad (124.)$$

the breadth, $$b = \frac{Wl^3}{Fd^3\delta} \qquad (125.)$$

and the depth, $$d = \sqrt[3]{\frac{Wl^3}{Fb\delta}} \qquad (126.)$$

and, as in formula (*120.*),

the deflection, $$\delta = \frac{Wl^3}{Fbd^3}$$

333.—Example—Value of l in a Beam.—Take an example under formula (*124.*). What should be the length of a beam of locust of average quality, 4 inches broad and 8 inches high, to carry 5000 pounds at the middle, with a deflection of one inch?

In formula (*124.*) $F = 5050$, $b = 4$, $d = 8$, $\delta = 1$ and $W = 5000$; hence

$$l = \sqrt[3]{\frac{5050 \times 4 \times 8^3 \times 1}{5000}} = 12 \cdot 74$$

or the answer is $12\frac{3}{4}$ feet.

334.—Example—Value of b in a Beam.—As an example under formula (*125.*), let it be required to know the proper breadth of an oak beam of average quality. The depth is 6 inches and the length 10 feet. The load to be carried is 500 pounds placed at the middle, and the deflection allowed is $0 \cdot 3$ inch.

In this case, $W = 500$, $l = 10$, $F = 3100$, $d = 6$ and $\delta = 0 \cdot 3$; and by substitution

$$b = \frac{500 \times 10^3}{3100 \times 6^3 \times 0 \cdot 3} = \frac{50000}{20088} = 2 \cdot 489$$

or $2\frac{1}{2}$ inches for the breadth.

335.—Example—Value of d in a Beam.—As an example under formula (*126.*), find the depth of a beam of maple of average quality, which is 5 inches broad and 20 feet long, and which is to carry 3000 pounds at the middle, with one inch deflection.

Here we have $F = 5150$, $W = 3000$, $l = 20$, $b = 5$ and $\delta = 1$; and hence

$$d = \sqrt[3]{\frac{3000 \times 20^3}{5150 \times 5 \times 1}} = 9 \cdot 768$$

or a depth of $9\frac{3}{4}$ inches.

336.—Values of P, n, b, d and δ in a Lever.—The rules for the quantities in a semi-beam or lever are derived from formula (*121.*), which is

$$\delta = \frac{16Pn^3}{Fbd^3}$$

and are as follows:

The load, $$P = \frac{Fbd^3\delta}{16n^3} \qquad (126.)$$

The length, $$n = \sqrt[3]{\frac{Fbd^3\delta}{16P}} \qquad (127.)$$

The breadth, $$b = \frac{16Pn^3}{Fd^3\delta} \qquad (128.)$$

The depth, $$d = \sqrt[3]{\frac{16Pn^3}{Fb\delta}} \qquad (129.)$$

337.—Example—Value of n in a Lever.—As an example under (*127.*): What length is required in a semi-beam or lever of ash of average quality, 3×7 inches cross-section, and carrying 200 pounds at the free end, with a deflection of half an inch?

In this example, $P = 200$, $F = 4000$, $b = 3$, $d = 7$ and $\delta = 0 \cdot 5$; and we have

$$n = \sqrt[3]{\frac{4000 \times 3 \times 7^3 \times 0 \cdot 5}{16 \times 200}} = \sqrt[3]{\frac{10290}{16}} = 8 \cdot 63$$

or the length is to be 8 feet $7\frac{1}{2}$ inches.

338.—Example—Value of b in a Lever.—Under formula (*128.*): What is the proper breadth for a lever of hickory of average quality, 3 inches deep, projecting 4 feet from

the wall in which it is fixed, carrying a load of 200 pounds at the free end, and having a deflection of one inch?

In the formula, $F = 3850$, $d = 3$, $\delta = 1$, $P = 200$ and $n = 4$. Substituting these values, we have

$$b = \frac{16 \times 200 \times 4^3}{3850 \times 3^3 \times 1} = 1 \cdot 97$$

The breadth must be 2 inches.

339.—Example—Value of *d* in a Lever.—What must be the depth of a bar of cherry of average quality, $1\frac{1}{2}$ inches broad, projecting 3 feet from the wall in which it is imbedded, and carrying at its end a load of 100 pounds, with a deflection of $\frac{3}{4}$ of an inch?

Here $P = 100$, $n = 3$, $F = 2850$, $b = 1 \cdot 5$ and $\delta = 0 \cdot 75$; and formula (*129.*) becomes

$$d = \sqrt[3]{\frac{16 \times 100 \times 3^3}{2850 \times 1 \cdot 5 \times 0 \cdot 75}} = 2 \cdot 38$$

The depth required is $2\frac{3}{8}$ inches.

340.—Deflection—Uniformly Distributed Load on a Beam.—The cases hitherto considered in this chapter have all had the load concentrated either at the middle of a beam or at the end of a lever. When the weight is distributed equably over the length of the beam or lever, the deflection is less than when the same weight is so concentrated.

In comparing the values of the deflecting energies producing equal deflections in the two cases, we have [formula (*511.*), p. 477, of "Mechanics of Engineering and Architecture," by Prof. Moseley, Am. ed. by Prof. Mahan, 1856,

and changing the symbols to agree with ours], for a beam loaded at the middle,

$$\delta = \frac{WL^3}{48EI}$$

and [formula (*530.*), p. 484, same work], for a beam uniformly loaded,

$$\delta = \tfrac{5}{8} \times \frac{UL^3}{48EI}$$

Comparing these two equal values of δ, we have

$$\frac{WL^3}{48EI} = \tfrac{5}{8}\frac{UL^3}{48EI} \qquad \text{or,}$$

$$W = \tfrac{5}{8}U$$

or, with equal deflections, the weight at the middle of the beam is equal to $\frac{5}{8}$ of the uniformly distributed load. Thus, 100 pounds uniformly distributed over the length of a beam will deflect it to the same extent that $62\frac{1}{2}$ pounds would were it concentrated at the middle of the length.

Then, since U represents a uniformly distributed load, $\frac{5}{8}U$ will equal the W of formula (*120.*), which formula is

$$\delta = \frac{Wl^3}{Fbd^3}$$

Substituting the value of W, as above, and transposing, we have

$$\tfrac{5}{8}Ul^3 = Fbd^3\delta \qquad (130.)$$

for the relation of the elements in the deflection of a beam by a uniformly distributed load.

341.—Values of U, l, b, d and δ in a Beam.—By inversions of formula (*130.*) we have the following rules—namely:

The weight, $$U = \frac{Fbd^3\delta}{\frac{5}{8}l^3} \qquad (131.)$$

The length, $$l = \sqrt[3]{\frac{Fbd^3\delta}{\frac{5}{8}U}} \qquad (132.)$$

The breadth, $$b = \frac{\frac{5}{8}Ul^3}{Fd^3\delta} \qquad (133.)$$

The depth, $$d = \sqrt[3]{\frac{\frac{5}{8}Ul^3}{Fb\delta}} \qquad (134.)$$

The deflection, $$\delta = \frac{\frac{5}{8}Ul^3}{Fbd^3} \qquad (135.)$$

342.—Example—Value of U, the Weight, in a Beam.—In a spruce beam of average quality, 20 feet long between bearings, 4 inches broad and 12 inches deep: What weight uniformly distributed over the beam will deflect it 2 inches?

In this example, $F = 3500$, $b = 4$, $d = 12$, $\delta = 2$ and $l = 20$; and by formula (*131.*)

$$U = \frac{3500 \times 4 \times 12^3 \times 2}{\frac{5}{8} \times 20^3} = 9676 \cdot 8$$

or the weight required is 9677 pounds.

343.—Example—Value of l, the Length, in a Beam.—In a 3×10 white pine beam of average quality: What is the proper length to carry 6000 pounds uniformly distributed, with a deflection of 2 inches?

Here $F = 2900$, $b = 3$, $d = 10$, $\delta = 2$ and $U = 6000$; and by the substitution of these in formula (*132.*)

$$l = \sqrt[3]{\frac{2900 \times 3 \times 10^3 \times 2}{\frac{5}{8} \times 6000}} = 16 \cdot 68$$

or the required length is 16 feet 8 inches.

344.—Example—Value of *b*, the Breadth, in a Beam.— Given a beam of average quality of Georgia pine, 20 feet long and 10 inches deep. If this beam carry a uniformly distributed load of 8000 pounds, with a deflection of $1\frac{3}{4}$ inches, what must be the breadth?

We have, as values of the known elements, $U = 8000$, $l = 20$, $F = 5900$, $d = 10$ and $\delta = 1 \cdot 75$; and formula (*133.*) gives us

$$b = \frac{5 \times 8000 \times 20^3}{8 \times 5900 \times 10^3 \times 1 \cdot 75} = 3 \cdot 874$$

The breadth must be $3\frac{7}{8}$ inches.

345.—Example—Value of *d*, the Depth, in a Beam.— A girder of average oak, 8 inches broad, and 10 feet long between bearings, is required to carry 10,000 pounds uniformly distributed over its length, with a deflection not to exceed $\frac{3}{10}$ of an inch. What must be its depth?

The elements of this case are $U = 10000$, $l = 10$, $F = 3100$, $b = 8$ and $\delta = 0 \cdot 3$. Applying formula (*134.*) we find

$$d = \sqrt[3]{\frac{5 \times 10000 \times 10^3}{8 \times 3100 \times 8 \times 0 \cdot 3}} = 9 \cdot 436$$

or we must make the depth $9\frac{1}{2}$ inches.

346.—Example—Value of δ, the Deflection, in a Beam.—We have a 3×6 inch beam of hemlock of average quality, 10 feet long. What amount of deflection would be produced by 3000 pounds uniformly distributed over its length? $U = 3000$, $l = 10$, $F = 2800$, $b = 3$ and $d = 6$; and the formula applicable, (*135.*), becomes

$$\delta = \frac{5 \times 3000 \times 10^3}{8 \times 2800 \times 3 \times 6^3} = 1 \cdot 0334$$

or a resulting deflection of 1 inch.

347.—Deflection—Uniformly Distributed Load on a Lever.—For a load at the free end of a lever [Moseley's Mechanics (cited in *Art.* **340**), formula (*509.*), p. 476, changing the symbols] we have

$$\delta = \frac{PN^3}{3EI}$$

and [page 482, same work, formula (*525.*)] for a lever with a uniformly distributed load, we have

$$\delta = \frac{UN^3}{8EI}$$

Comparing these equal values of δ we have

$$\frac{PN^3}{3EI} = \frac{UN^3}{8EI} \qquad \text{or,}$$

$$\frac{P}{3} = \frac{U}{8} \qquad \text{or,}$$

$$P = \tfrac{3}{8}U$$

or, the deflection by a uniformly distributed load is equal to that which would be produced by $\frac{3}{8}$ of that load if suspended from the end of the lever.

348.—Values of U, n, b, d and δ in a Lever.—In formula (*123.*), which is $P = \frac{Fbd^2\delta}{16n^3}$, we have the relations existing between the elements involved in the case of a lever under strain.

If the weight uniformly distributed over the length of the lever be represented by U, then $P = \frac{3}{8}U$ and formula (*123.*) becomes

$$\tfrac{3}{8}U = \frac{Fbd^2\delta}{16n^3}$$

and from this we have the following:

The weight, $$U = \frac{Fbd^2\delta}{6n^3} \qquad (136.)$$

The length, $$n = \sqrt[3]{\frac{Fbd^2\delta}{6U}} \qquad (137.)$$

The breadth, $$b = \frac{6Un^3}{Fd^2\delta} \qquad (138.)$$

The depth, $$d = \sqrt[2]{\frac{6Un^3}{Fb\delta}} \qquad (139.)$$

The deflection, $$\delta = \frac{6Un^3}{Fbd^2} \qquad (140.)$$

349.—Example—Value of U, the Weight, in a Lever.—In a Georgia pine lever of average quality, 6 inches broad and 10 inches deep, and projecting 10 feet from the wall in which it is imbedded: What weight uniformly distributed over the lever will deflect it 2 inches?

In this example, $F = 5900$, $b = 6$, $d = 10$, $\delta = 2$ and $n = 10$; and by formula (*136.*),

$$U = \frac{5900 \times 6 \times 10^2 \times 2}{6 \times 10^3} = 11800$$

or the uniformly distributed weight required is 11,800 pounds.

Three eighths of this weight, or 4425 pounds, concentrated at the free end of the lever, will deflect it the same amount, *viz.*: 2 inches.

350.—Example—Value of n, the Length, in a Lever.— In a lever of the same description as in the last article, except as to length and load: What is the proper length to carry 8000 pounds uniformly distributed, with a deflection of 2 inches?

Here we have $F = 5900$, $b = 6$, $d = 10$, $\delta = 2$ and $U = 8000$; and by the substitution of these in formula (*137.*)

$$n = \sqrt[3]{\frac{5900 \times 6 \times 10^3 \times 2}{6 \times 8000}} = \sqrt[3]{1475} = 11 \cdot 383$$

or the required length is 11 feet $4\frac{1}{2}$ inches.

351.—Example—Value of b, the Breadth, in a Lever.— Given a lever of like description as in *Art.* **349**, except as to breadth and load. If this lever carry a uniformly distributed load of 6000 pounds, what must be the breadth?

We have, as values of the known elements, $U = 6000$, $n = 10$, $F = 5900$, $d = 10$ and $\delta = 2$; and formula (*138.*) gives us

$$b = \frac{6 \times 6000 \times 10^3}{5900 \times 10^3 \times 2} = 3 \cdot 051$$

The breadth must be 3 inches.

352.—Example—Value of d, the Depth, in a Lever.— A lever of like description as in *Art.* **349**, except as to depth and load, is required to carry 10,000 pounds uniformly distributed over its length: What must be its depth?

The elements of this case are $U = 10000$, $n = 10$, $F = 5900$, $b = 6$ and $\delta = 2$. We apply formula (*139.*) and find

$$d = \sqrt[3]{\frac{6 \times 10000 \times 10^3}{5900 \times 6 \times 2}} = 9 \cdot 463$$

or we must make the depth $9\frac{1}{2}$ inches.

353.—Example—Value of δ, the Deflection, in a Lever.— We have a lever of like description as that in *Art.* **349**, except as to load and deflection: What amount of deflection would be produced by 5000 pounds uniformly distributed over its length?

$U = 5000$, $n = 10$, $F = 5900$, $b = 6$ and $d = 10$; and the formula applicable, (*140.*), becomes

$$\delta = \frac{6 \times 5000 \times 10^3}{5900 \times 6 \times 10^3} = 0 \cdot 8475$$

or a resulting deflection of $\frac{7}{8}$ of an inch.

QUESTIONS FOR PRACTICE.

354.—Given a *beam loaded at middle:* What are the rules by which to find the weight, length, breadth, depth and deflection?

355.—Given a *lever loaded at the free end:* What are the rules by which to find the weight, length, breadth, depth and deflection?

356.—In a *beam with the load uniformly distributed:* What are the rules by which to obtain the weight, length, breadth, depth and deflection?

357.—In a *lever with the load uniformly distributed:* What are the rules by which to obtain the weight, length, breadth, depth and deflection?

CHAPTER XVII.

RESISTANCE TO FLEXURE—FLOOR BEAMS.

ART. **358.—Stiffness a Requisite in Floor Beams.**—The rules given in Chap. VI. for the dimensions of floor beams are based upon the ascertained resistance of the material to rupture, and are useful in all cases in which the question of absolute strength is alone to be considered. For warehouses and those buildings in which strength is principally required, the rules referred to are safe and proper; but for buildings of good character, in which the apartments are finished with plastering, the floor timbers are required to possess stiffness as well as strength; for it is desirable that the deflection of the beams shall not be readily noticed, nor be injurious to the plastering.

359.—General Rule for Floor Beams.—The relations of the several elements in the question of stiffness, in beams uniformly loaded throughout their entire length, are found in formula (*130.*),

$$\tfrac{5}{8}Ul^3 = Fbd^3\delta$$

The load upon the floor beam is here represented by U, and its value is $U = cfl$ (see *Art.* **92**); in which c is the distance apart between the centres of the floor beams, f is the number of pounds weight upon each square foot of the floor, and l is the length of the beam; c and l both

being in feet. If for U we substitute this value, and for δ put rl (see *Arts.* **313** and **314**), we have

$$\tfrac{5}{8}cfl^3 = Fbd^3r \qquad (141.)$$

360.—The Rule Modified.—For the floors of dwellings and assembly rooms, f, the load per foot, may be taken (see *Art.* **115**) at 70 pounds for the loading and 20 pounds for the weight of the materials, or 90 pounds in all; and r, the rate of deflection per foot of the length, at 0·03 (see *Art.* **314**). Formula (*141.*) thus modified becomes

$$90 \times \tfrac{5}{8}cl^3 = 0{\cdot}03Fbd^3$$

$$\frac{90 \times 5}{8 \times 0{\cdot}03}cl^3 = Fbd^3$$

$$1875cl^3 = Fbd^3$$

$$cl^3 = \frac{F}{1875}bd^3 \qquad (142.)$$

This coefficient, $\frac{F}{1875}$, taking F at its average value for six of the woods in common use, reduces to

$\frac{5900}{1875} = 3{\cdot}15$ for Georgia pine,
$\frac{5050}{1875} = 2{\cdot}69$ " locust,
$\frac{3100}{1875} = 1{\cdot}65$ " oak,
$\frac{3500}{1875} = 1{\cdot}87$ " spruce,
$\frac{2900}{1875} = 1{\cdot}55$ " white pine,
$\frac{2800}{1875} = 1{\cdot}49$ " hemlock.

361.—Rule for Dwellings and Assembly Rooms.—For the coefficient in (*142.*), $\frac{F}{1875}$, putting the symbol i, we

have this simple rule for problems involving the dimensions of floor beams in dwellings and assembly rooms, namely,

$$cl^3 = ibd^3 \qquad (143.)$$

and we have the value of i for average qualities of six of the more common woods, as taken in *Art.* **360**, as follows:

For Georgia pine,	$i = 3 \cdot 15$
" locust,	$i = 2 \cdot 69$
" oak,	$i = 1 \cdot 65$
" spruce,	$i = 1 \cdot 87$
" white pine,	$i = 1 \cdot 55$
" hemlock,	$i = 1 \cdot 49$

362.—Rules giving the Values of c, l, b and d.—Taking formula (*143.*) we derive by inversions the following rules, namely:

The distance from centres, $$c = \frac{ibd^3}{l^3} \qquad (144.)$$

The length, $$l = \sqrt[3]{\frac{ibd^3}{c}} \qquad (145.)$$

The breadth, $$b = \frac{cl^3}{id^3} \qquad (146.)$$

The depth, $$d = \sqrt[3]{\frac{cl^3}{ib}} \qquad (147.)$$

363.—Example—Distance from Centres.—At what distance from centres should 3×12 inch Georgia pine beams of average quality, 24 feet long, be placed in a dwelling-house floor?

Here we have $i = 3 \cdot 15$, $b = 3$, $d = 12$ and $l = 24$; and by formula (*144.*)

$$c = \frac{3 \cdot 15 \times 3 \times 12^3}{24^3} = 1 \cdot 181$$

or the distance c should be about $14\frac{1}{4}$ inches.

364.—Example—Length.—Of what length may average quality white pine beams 3×10 inches square be used, when placed 16 inches from centres?

In this case $i = 1 \cdot 55$, $b = 3$, $d = 10$ and $c = 1\frac{1}{3}$; and formula (*145.*) gives

$$l = \sqrt[3]{\frac{1 \cdot 55 \times 3 \times 10^2}{1\frac{1}{3}}} = \sqrt[3]{3487 \cdot 5} = 15 \cdot 165$$

or these beams may be used 15 feet 2 inches long between bearings.

365.—Example—Breadth.—In floor beams 20 feet long and 12 inches deep, of oak of average quality, placed one foot from centres: What should be the breadth?

Here, $c = 1$, $l = 20$, $d = 12$ and $i = 1 \cdot 65$. With formula (*146.*), therefore, we have

$$b = \frac{1 \times 20^3}{1 \cdot 65 \times 12^3} = 2 \cdot 806$$

The breadth should be nearly $2\frac{7}{8}$, or say 3 inches.

366.—Example—Depth.—What should be the depth of spruce beams of average quality when 3 inches broad and 20 feet long, and placed 20 inches from centres? The symbols in this case are $c = 1\frac{2}{3}$, $l = 20$, $b = 3$, and $i = 1 \cdot 87$; and by formula (*147.*) we find

$$d = \sqrt[3]{\frac{1\frac{2}{3} \times 20^3}{1 \cdot 87 \times 3}} = 13 \cdot 345$$

or the depth required is $13\frac{3}{8}$ inches. Beams 3×13 could be used, provided the distances apart from centres were correspondingly decreased. The new distance would be (*form. 144.*) $18\frac{1}{2}$ instead of 20 inches.

367.—Floor Beams for Stores.—The several values of i for dwellings and assembly rooms, as given in *Art.* **361**, will be appropriate also for stores for light goods, because timbers apportioned by the rules having these values of i, will bear a load of 200 pounds per superficial foot before their deflection will reach the limit of elasticity.

For first-class stores—those intended for wholesale business, as that of dry-goods—the values of i, as above given, are too large. The proper values for this constant may be derived as below.

368.—Floor Beams of First-class Stores.—The load upon the floors of first-class stores may be taken at 250 pounds per superficial foot, and the deflection at 0·04 of an inch per foot lineal (see *Arts.* **313** and **314**). Beams proportioned by these requirements will bear a load of about $3 \times 250 = 750$ pounds per foot before the deflection will reach the limit of elasticity. With 250 as the loading, and, say 25 pounds (*Art.* **99**) for the weight of the materials of construction, we have $f = 275$.

Formula (*141.*), modified in accordance herewith, putting $r = 0 \cdot 04$, becomes

$$5 \times 275 cl^3 = 8 \times 0 \cdot 04 Fbd^3$$

$$cl^3 = \frac{F}{4296\frac{7}{8}} bd^3 \qquad (148.)$$

369.—Rule for Beams of First-class Stores.—Reducing the above constant, $\frac{F}{4296\frac{7}{8}}$, for six of the more common woods of average quality, and putting the symbol k for the results, we have for

Georgia pine,	$k = 1 \cdot 37$
Locust,	$k = 1 \cdot 18$
Oak,	$k = 0 \cdot 72$
Spruce,	$k = 0 \cdot 81$
White pine,	$k = 0 \cdot 67$
Hemlock,	$k = 0 \cdot 65$

With this symbol k, the rule for floor beams of first-class stores is reduced to this simple form,

$$cl^2 = kbd^2 \qquad (149.)$$

370.—Values of *c*, *l*, *b* and *d*.—By proper inversions, we obtain from formula (*149.*), rules for the several values required, thus:

The distance from centres, $$c = \frac{kbd^2}{l^2} \qquad (150.)$$

The length, $$l = \sqrt[2]{\frac{kbd^2}{c}} \qquad (151.)$$

The breadth, $$b = \frac{cl^2}{kd^2} \qquad (152.)$$

The depth, $$d = \sqrt[2]{\frac{cl^2}{kb}} \qquad (153.)$$

371.—Example—Distance from Centres.—In a first-class store: How far from centres should floor beams of Georgia pine of an average quality be placed, when said beams are 4×12, and 20 feet long between bearings?

In this example, we have $k = 1 \cdot 37$, $b = 4$, $d = 12$ and $l = 20$. Then by formula (*150.*)

$$c = \frac{1 \cdot 37 \times 4 \times 12^2}{20^2} = 1 \cdot 184$$

or the distance from centres is $1 \cdot 184$ feet, equal to about $14\frac{1}{4}$ inches.

372.—Example—Length.—At what length may 4×10 inch beams of average oak be used in the floors of a first-class store, when placed 12 inches from centres? Here we have $k = 0.72$, $b = 4$, $d = 10$ and $c = 1$; and by formula (*151.*)

$$l = \sqrt[3]{\frac{0.72 \times 4 \times 10^2}{1}} = 14.23$$

or the length should be 14 feet 3 inches.

373.—Example—Breadth.—The floor beams in a first-class store are to be 20 feet long and 14 inches deep, of white pine of average quality. When placed 12 inches from centres, what should be their breadth? Taking formula (*152.*) we have, as values of the symbols, $c = 1$, $l = 20$, $k = 0.67$ and $d = 14$; and

$$b = \frac{1 \times 20^3}{0.67 \times 14^2} = 4.35$$

The breadth should be $4\frac{3}{8}$ inches.

374.—Example—Depth.—What should be the depth, in a first-class store, of spruce beams, of average quality, 4 inches thick and 16 feet long, and placed 14 inches from centres?

In this case, we have $c = 1\frac{1}{6}$, $l = 16$, $k = 0.81$ and $b = 4$. Therefore, by formula (*153.*)

$$d = \sqrt{\frac{1\frac{1}{6} \times 16^3}{0.81 \times 4}} = 11.383$$

or a depth of $11\frac{3}{8}$ inches.

375.—Headers and Trimmers.—In Chap. VII., in *Arts.* **143** to **158**, rules for headers and trimmers, based upon the resistance of the material to rupture, are given. These rules

contain the symbol a, which represents the number of times the weight to be carried is contained in the breaking weight.

The value of this symbol may be assigned at any quantity not less than that which is given for it in Table XX., and, when made so great that the deflection shall not exceed 0·03 of an inch per foot of the length, the rules referred to will be proper for use for headers and trimmers for the floors of dwellings and assembly rooms.

376.—Strength and Stiffness—Relation of Formulas.— The value of a, the symbol for safety, may be determined from the following considerations:

Taking formula (*113.*), which is

$$F = \frac{Wl^3}{bd^3\delta}$$

and substituting G for W we have

$$Gl^3 = Fbd^3\delta$$

A comparison of the constants for rupture (B) and for elasticity (F) shows that

$$B : F :: 1 : m = \frac{F}{B}$$

or

$$Bm = F$$

and putting rl equal to δ we have, by substitution,

$$Gl^3 = Bmbd^3rl$$

$$Gl^2 = Bmbd^3r$$

$$\frac{Gl^2}{dmr} = Bbd^2$$

We have by formula (*10.*), *Art.* **36**,

$$B = \frac{Wl}{bd^2}$$

or

$$Wl = Bbd^2$$

Comparing this value of Bbd^2 with that above, we have

$$Wl = \frac{Gl^2}{dmr}$$

In this formula, G is the weight which may be carried by the beam, with a deflection per foot of the length equal to r; and W is the breaking weight. Putting these symbols in a proportion, we have

$$G : W :: 1 : a = \frac{W}{G}$$

or

$$Ga = W$$

Substitute for W this value of it, and we obtain

$$Gal = \frac{Gl^2}{dmr}$$

$$a = \frac{l}{dmr} = \frac{l}{d\frac{F}{B}r}$$

$$a = \frac{Bl}{Fdr} \qquad (154.)$$

377.—Strength and Stiffness—Value of a, in Terms of B and F.—The values of B and F (*form. 154.*) are found in Table XX., and $r = 0{\cdot}03$. The ratio $\frac{l}{d}$ (l in feet and d in inches) cannot be exactly determined until the length and depth have been established. An approximation may be

assumed, however, for a preliminary calculation, and then, if found to err materially, it may be taken more nearly correct in a final calculation. In all ordinary cases, the ratio $\frac{l}{d}$ will be found nearly equal to $\frac{17}{10} = 1 \cdot 7$. Taking this value in formula (*154.*) we have

$$a = \frac{56\frac{2}{3}B}{F} \qquad (155.)$$

378.—Example.—Let us apply this in the use of formula (*21.*), namely:

$$Wal = Bbd^2$$

What weight may be carried at the middle of a Georgia pine beam of average quality, 3×10 inches $\times 17$ feet, so as to deflect it no more than would be proper for the floors of a dwelling?

Here $b = 3$, $d = 10$, $a = \frac{56\frac{2}{3}B}{F}$, $F = 5900$ and $l = 17$;

therefore

$$W = \frac{B \times 3 \times 10^2}{\frac{56\frac{2}{3}B}{F} \times 17} = \frac{5900 \times 3 \times 10^2}{56\frac{2}{3} \times 17}$$

$$W = \frac{1770000}{963\frac{1}{3}} = 1837 \cdot 4$$

379.—Test of the Rule.—To test the accuracy of the result just found, the same problem may be solved by formula (*113.*),

$$F = \frac{Wl^3}{bd^3\delta}$$

from which we have, when $\delta = rl$, and substituting G for W,

$$Gl^2 = Fbd^3r$$

and

$$G = \frac{Fbd^3r}{l^2}$$

In this expression, in the above example, $F = 5900$, $b = 3$, $d = 10$, $l = 17$ and $r = 0{\cdot}03$; and hence

$$G = \frac{5900 \times 3 \times 10^2 \times 0{\cdot}03}{17^2} = \frac{531000}{289} = 1837{\cdot}4$$

380.—Rules for Strength and Stiffness Resolvable.— The result in the last article is the same as the one before found, and indeed could not be otherwise, since the one formula is derived directly from the other, and is readily resolvable into it; for if, in formula (*21.*),

$$Wal = Bbd^2$$

we substitute for a its equivalent as in formula (*154.*), we have

$$W \frac{Bl}{Fdr} l = Bbd^2$$

$$Wl^2 = Fbd^3 r$$

so that instead of computing the value of a for use in any particular case by formula (*155.*), we may introduce into the rule its value as given by (*154.*), and reduce to the lowest terms, as in the next article.

381.—Rule for the Breadth of a Header.—A rule for a header is given in formula (*27.*), *Art.* **145.** Substituting for a its value as in (*154.*), we have

$$\tfrac{1}{4} \frac{Bl}{Fdr} fng^2 = Bb(d-1)^2$$

In this expression, l and g are the same, both representing the length of the header, and the $(d-1)$ is put for the

effective depth, and is equal to the d of the first member; therefore, reducing, we have

$$\tfrac{1}{4} fng^2 = Fbr(d-1)^2$$

$$b = \frac{fng^2}{4Fr(d-1)^2} \qquad (156.)$$

which is a rule for the breadth of a header, based upon the resistance to flexure.

382.—Example of a Header for a Dwelling.—In a dwelling having spruce floor beams of an average quality, 10 inches deep: What would be the required breadth of a header of the same material, 10 feet long, carrying tail beams 12 feet long?

The values of the symbols are, $f = 90$ (*Art.* **115**), $n = 12$, $g = 10$, $F = 3500$ (Table XX.), $r = 0{\cdot}03$ and $d = 10$; and

$$b = \frac{90 \times 12 \times 10^2}{4 \times 3500 \times 0{\cdot}03 \times 9^2} = 3{\cdot}527$$

or the required breadth is $3\tfrac{1}{2}$ inches full.

383.—Example of a Header in a First-class Store.—In a first-class store, where the beams are 14 inches deep, what is the required breadth of a header of Georgia pine of average quality, 16 feet long, and carrying tail beams 17 feet long?

Here $f = 275$, $r = 0{\cdot}04$ (*Art.* **368**), $n = 17$, $g = 16$, $F = 5900$ (Table XX.) and $d = 14$; and by formula (*156.*),

$$b = \frac{275 \times 17 \times 16^2}{4 \times 5900 \times 0{\cdot}04 \times 13^2} = 9{\cdot}233$$

The breadth should be $9\tfrac{1}{4}$ inches.

384.—Carriage Beam with One Header.—(See *Art.* **389.**) In *Art.* **150** a rule (*form. 29.*) is given for this case, based upon the resistance of the material to rupture. As with a header (*Art.* **381**), so here, the rule given may be resolved into one depending upon the resistance to deflection.

Taking formula (*29.*), and for a substituting its value as per formula (*154.*), we have

$$\frac{Bl}{Fdr}f\left(\tfrac{1}{4}cl^2 + gn^2\frac{m}{l}\right) = Bbd^2$$

$$f\left(\tfrac{1}{4}cl^2 + gn^2m\right) = Fbd^2r \qquad (157.)$$

which is the required rule.

385.—Carriage Beam with One Header, for Dwellings.—In this rule, putting $f = 90$ and $r = 0{\cdot}03$, we obtain

$$3000\left(\tfrac{1}{4}cl^2 + gn^2m\right) = Fbd^2$$

$$b = \frac{3000\left(\tfrac{1}{4}cl^2 + gn^2m\right)}{Fd^2} \qquad (158.)$$

which is a rule for carriage beams with one header, in dwellings and assembly rooms. (See *Art.* **389.**)

386.—Example.—What should be the breadth, in a dwelling, of a carriage beam of average quality white pine, 20 feet long by 12 inches deep, and carrying a header 16 feet long at a point 5 feet from one end? The floor beams among which this carriage beam is placed are set at 16 inches from centres.

Here $c = 1\frac{1}{3}$, $l = 20$, $g = 16$, $n = 15$, $m = 5$, $F = 2900$ and $d = 12$; and by formula (*158.*)

$$b = \frac{3000\left[\left(\frac{1}{4} \times 1\frac{1}{3} \times 20^2\right) + \left(16 \times 15^2 \times 5\right)\right]}{2900 \times 12^2} = 12{\cdot}372$$

The breadth should be $12\frac{3}{8}$ inches.

387.—Carriage Beam with One Header, for First-class Stores.—If in formula (*157.*) we take the value of f equal to 275, and of r equal to 0·04, we shall then have

$$6875\,(\tfrac{1}{4}cl^2 + gn^2m) = Fbd^2$$

$$b = \frac{6875\,(\tfrac{1}{4}cl^2 + gn^2m)}{Fd^2} \qquad (159.)$$

which is the required rule (see *Art.* **389**).

388.—Example.—Of what breadth, in a first-class store, should be a Georgia pine carriage beam of average quality, 25 feet long, and carrying at 6 feet from one end a header 16 feet long; the floor beams being 15 inches deep, and placed 15 inches from centres?

Here $c = 1\tfrac{1}{4}$, $l = 25$, $g = 16$, $n = 19$, $m = 6$, $d = 15$ and $F = 5900$; and formula (*159.*) becomes

$$b = \frac{6875\,[(\tfrac{1}{4} \times 1\tfrac{1}{4} \times 25^2) + (16 \times 19^2 \times 6)]}{5900 \times 15^2} = 13 \cdot 651$$

or the breadth required is $13\tfrac{5}{8}$ inches.

389.—Carriage Beam with One Header, for Dwellings—More Precise Rule.—The rules above given (*157.*, *158.* and *159.*) are *not strictly* correct: they give a slight excess of material (see *Art.* **241**).

The rule shown in formula (*86.*),

$$4a\frac{mn}{l}\,(A' + \tfrac{1}{2}U) = Bbd^2$$

*is accurate,** and should be the one employed in special cases

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* Except when h is less than n (*Art.* **240**). In this case the result is slightly in excess, but so slightly that the difference is unimportant.

in which a costly material is used. Substituting for a in this formula, its value, as in formula (*154.*), we have

$$4\frac{Bl}{Fdr}\frac{mn}{l}(A' + \tfrac{1}{2}U) = Bbd^2$$

$$4mn(A' + \tfrac{1}{2}U) = Fbd^2r \qquad (160.)$$

in which A' is the concentrated load, and U the uniformly distributed load. Formula (*160.*) may be modified, in the case of a carriage beam, by using for these symbols their values, thus:

From *Arts.* **92** and **150**, $A' = \frac{1}{4}fng$, and $U = \frac{1}{4}cfl$, and hence

$$fmn(ng + cl) = Fbd^2r \qquad (161.)$$

which is a more precise general rule for a carriage beam carrying one header.

If, now, we put f equal to 90, and r equal to 0·03, we shall have

$$3000mn(ng + cl) = Fbd^2$$

$$b = \frac{3000mn(ng+cl)}{Fd^2} \qquad (162.)$$

which is a more precise rule for carriage beams with one header, in floors of dwellings and assembly rooms.

390.—Example.—Taking the example given in *Art.* **386**, we have $m = 5$, $n = 15$, $g = 16$, $c = 1\frac{1}{3}$, $l = 20$, $F = 2900$ and $d = 12$; and, in formula (*162.*)

$$b = \frac{3000 \times 5 \times 15\,(\overline{15 \times 16} + \overline{1\frac{1}{3} \times 20})}{2900 \times 12^2} = 11{\cdot}973$$

showing that by this, the more exact rule, the breadth should be 12 inches, while by the former rule it was determined to be $12\frac{5}{8}$ inches.

391.—Carriage Beam with one Header, for First-class Stores—More Precise Rule.—Modifying formula (*161.*), by putting 275 for f, and 0·04 for r, we have

$$6875mn(ng+cl) = Fbd^2$$

$$b = \frac{6875mn(ng+cl)}{Fd^2} \qquad (163.)$$

which is the more precise rule required.

392.—Example.—Applying this rule to the example given in *Art.* **388**, we find, $m = 6$, $n = 19$, $g = 16$, $c = 1\frac{1}{4}$, $l = 25$, $F = 5900$ and $d = 15$; and hence

$$b = \frac{6875 \times 6 \times 19(\overline{19 \times 16} + \overline{1\frac{1}{4} \times 25})}{5900 \times 15^2} = 13 \cdot 195$$

giving the breadth, by this more precise rule, at $13\frac{1}{4}$ inches. This is nearly half an inch less than by the former rule, which gave for the breadth, 13·651, or $13\frac{2}{3}$ inches nearly.

393.—Carriage Beam with Two Headers and Two Sets of Tail Beams, for Dwellings, etc.—Formula (*32.*) in *Art.* **155** gives the relations of the symbols referring to a case in which a carriage beam has to carry two headers, with two sets of tail beams. From this formula we have

$$b = \frac{af}{Bd^2}[m\frac{g}{l}(mn+s^2) + \frac{1}{4}cl^2]$$

If in this equation the value of a, as in formula (*154.*), be substituted, there results

$$b = \frac{Blf}{Bd^2Fdr}\left(\frac{gm(mn+s^2)}{l} + \frac{1}{4}cl^2\right)$$

$$b = \frac{f}{Fd^2r}[gm(mn+s^2) + \frac{1}{4}cl^2] \qquad (164.)$$

which is a general rule for these cases.

Putting $f = 90$ and $r = 0{\cdot}03$, we have

$$b = \frac{3000}{Fd^2}\left[gm\,(mn + s^2) + \tfrac{1}{4}cl^2\right] \qquad (165.)$$

which is a rule for a carriage beam, carrying two headers, with two sets of tail beams, in the floor of a dwelling or assembly room. (See *Arts.* **402**, **405**, **415** and **417**.)

394.—Example.—Under rule (*165.*) take the example given in *Art.* **156**, in which $F = 5900$, $d = 14$, $g = 12$, $c = 1\frac{1}{2}$ and $l = 25$. For m and s there are given 5 and 15, and taking m as the larger, $m = 15$, $n = 10$, $s = 5$ and $r = 20$; so that (*165.*) becomes

$$b = \frac{3000}{5900 \times 14^2}\left[\overline{12 \times 15\,(\overline{15 \times 10} + 5^2)} + \overline{\tfrac{1}{4} \times 1\tfrac{1}{2} \times 25^2}\right] = 6{\cdot}923$$

or the breadth should be 7 inches.

395.—Carriage Beam with Two Headers and Two Sets of Tail Beams, for First-class Stores.—If, in formula (*164.*), f be put at 275 and r at $0{\cdot}04$, we shall have

$$b = \frac{6875}{Fd^2}\left[gm\,(mn + s^2) + \tfrac{1}{4}cl^2\right] \qquad (166.)$$

which is a rule for a carriage beam carrying two headers, with two sets of tail beams, in a first-class store (see *Arts.* **402**, **407** and **417**).

396.—Example.—Referring to the same example (*Art.* **156**) we have $F = 5900$, $d = 14$, $g = 12$, $m = 15$, $n = 10$, $s = 5$, $c = 1\frac{1}{2}$ and $l = 25$; and the formula is

$$b = \frac{6875}{5900 \times 14^2}\left[\overline{12 \times 15\,(\overline{15 \times 10} + 5^2)} + \overline{\tfrac{1}{4} \times 1\tfrac{1}{2} \times 25^2}\right] = 15{\cdot}865$$

or the breadth should be $15\frac{7}{8}$ inches.

397.—Carriage Beam with Two Headers and One Set of Tail Beams.—Formula (*34.*), in *Art.* **157**, is a rule for a carriage beam with two headers, carrying but one set of tail beams. Substituting, in this formula, for a its value (*form. 154.*) $\frac{Bl_1}{Fdr}$, we have

$$\frac{Bfl}{Fdr}\left[\frac{jg}{l}m(n+s)+\tfrac{1}{4}cl^2\right]=Bbd^2$$

from which

$$b=\frac{f}{Fd^2r}\left[jgm(n+s)+\tfrac{1}{4}cl^2\right] \qquad (167.)$$

which is a general rule for a carriage beam carrying two headers, with but one set of tail beams, with a given rate of deflection. (See *Arts.* **402, 409, 411, 419** and **421.**)

398.—Carriage Beam with Two Headers and One Set of Tail Beams, for Dwellings.—If, in formula (*167.*), f be put at 90 and r at 0·03, we shall have

$$b=\frac{3000}{Fd^2}\left[jgm(n+s)+\tfrac{1}{4}cl^2\right] \qquad (168.)$$

a rule for a carriage beam with two headers, carrying only one set of tail beams, in a dwelling or assembly room. (See *Arts.* **402, 409, 411, 419** and **421.**)

399.—Example.—Let it be required to find, under this rule, the breadth of a carriage beam 20 feet long, of spruce of average quality; said beam carrying two headers, each 12 feet long, with tail beams 11 feet long between them, leaving an opening 4 feet wide on one side, and another 5 feet wide on the other side. The beams among which this carriage beam is placed are 12 inches deep and 16 inches from centres.

For the symbols we have, $F = 3500$, $d = 12$, $j = 11$, $g = 12$, $c = 1\frac{1}{8}$ and $l = 20$. Having for m and s the values 4 and 5, we make m equal to the larger one, and therefore $m = 5$, $n = 15$ and $s = 4$. These values substituted in formula (*168.*) produce

$$b = \frac{3000\,\left[11 \times 12 \times 5\,(15+4) + \frac{1}{4} \times 1\frac{1}{8} \times 20^2\right]}{3500 \times 12^2} = 7 \cdot 543$$

The breadth should be, say $7\frac{1}{2}$ inches.

400.—Carriage Beam with Two Headers and One Set of Tail Beams, for First-class Stores.—If, in formula (*167.*), we put 275 for f and 0·04 for r, we shall have

$$b = \frac{6875}{Fd^2}\left[jgm\,(n+s) + \tfrac{1}{4}cl^2\right] \qquad (169.)$$

which is a rule for carriage beams carrying two headers, with one set of tail beams between them, in a first-class store. (See *Arts.* **402**, **409** and **413**.)

401.—Example.—What should be the breadth, under this rule, of a carriage beam of average quality Georgia pine, 25 feet long, with two headers each 20 feet long, carrying tail beams 10 feet long between them? The tail beams are so located that there is an opening 10 feet wide at the left-hand end, and one 5 feet wide at the right-hand end. The tier of beams is 15 inches deep and placed 15 inches from centres.

Here $F = 5900$, $d = 15$, $j = 10$, $g = 20$, $c = 1\frac{1}{4}$ and $l = 25$. For the values of m and s we have 10 and 5; and 10 being the larger it follows that $m = 10$, $n = 15$ and $s = 5$; and by formula (*169.*),

$$b = \frac{6875\left[10 \times 20 \times 10\,(15+5) + \frac{1}{4} \times 1\frac{1}{4} \times 25^2\right]}{5900 \times 15^2} = 15 \cdot 496$$

or the breadth should be $15\frac{1}{2}$ inches.

402.—Carriage Beam with Two Headers and Two Sets of Tail Beams—More Precise Rules.—The rules for carriage beams given in *Arts.* **393** to **401** are drawn from formulas which are but close approximations to the truth. The resulting dimensions are always in excess slightly of the true amounts, and the rules therefore are safe.

The rule embodied in formula (*92.*), however, is deduced from exact premises, and its results are precise.

If for a its value (*form. 154.*) be substituted in formula (*92.*), we shall have

$$4\frac{Bl}{Fdr}[\tfrac{1}{4}cfht + b' + \frac{a'-b'}{d'}(h-s)] = Bbd^2$$

$$b = \frac{4l}{Fd^2r}[\tfrac{1}{4}cfht + b' + \frac{a'-b'}{d'}(h-s)] \qquad (170.)$$

and, as auxiliary thereto,

$$a' = fg\frac{m}{4l}(mn+s^2) \qquad (171.)$$

$$b' = fg\frac{s}{4l}(rs+m^2) \qquad (172.)$$

$$h = \tfrac{1}{2}l + \frac{a'-b'}{\tfrac{1}{2}cd'f} \qquad (173.)$$

When h is equal to or exceeds n, then n is to be substituted for h, and the portion

$$b' + \frac{a'-b'}{d'}(h-s)$$

of formula (*170.*) equals a' (see *Art.* **248**), and the formula itself reduces to

$$b = \frac{4l}{Fd^2r}(\tfrac{1}{4}cfmn + a')$$

Substituting for a' its value (*form. 171.*) we have

$$b = \frac{4l}{Fd'^3r}[\tfrac{1}{4}cfmn + fg\frac{m}{4l}(mn + s^2)]$$

$$b = \frac{4flm}{Fd'^3r}[\tfrac{1}{4}cn + \frac{g}{4l}(mn + s^2)]$$

$$b = \frac{fm}{Fd'^3r}[cnl + g(mn + s^2)] \qquad (174.)$$

We have here, in formula (*170.*), a general rule, and in formula (*174.*), a rule, general when h equals or exceeds n, for a carriage beam carrying two headers, with two sets of tail beams, with a given deflection.

403.—Example—h less than n.—Let it be shown, under these rules, what should be the breadth of a carriage beam of spruce of average quality, 20 feet long and 12 inches deep, carrying two headers each 12 feet long, so placed as to leave an opening 4½ feet wide; said opening being 7½ feet distant from one wall and 8 feet from the other.

The floor is to carry 100 pounds per superficial foot, with a deflection of 0·03 per foot, and the beams are placed 15 inches from centres.

Here we have $f = 100$, $g = 12$, $m = 8$, $l = 20$, $n = 12$, $s = 7\frac{1}{2}$, $r = 12\frac{1}{2}$, $c = 1\frac{1}{4}$, $d' = l - (m + s) = 20 - (8 + 7\frac{1}{2}) = 20 - 15\frac{1}{2} = 4\frac{1}{2}$, $F = 3500$ and $d = 12$.

Preliminary to finding the value of h we have to determine the values of a' and b'.

By formulas (*171.*) and (*172.*)

$$a' = \frac{100 \times 12 \times 8}{4 \times 20}(\overline{8 \times 12} + 7 \cdot 5^2) \quad = 18270$$

$$b' = \frac{100 \times 12 \times 7 \cdot 5}{4 \times 20}(\overline{12 \cdot 5 \times 7 \cdot 5} + 8^2) = 17746 \cdot 875$$

$$a' - b' = \quad 523 \cdot 125$$

From these and formula (*173.*) we have

$$h = 10 + \frac{523 \cdot 125}{\frac{1}{3} \times 1\frac{1}{4} \times 4 \cdot 5 \times 100} = 11 \cdot 86$$

So $h = 11 \cdot 86$, and since it is less than n (as n equals 12) is therefore to be retained; and we have (*form. 170.*)

$$b = \frac{4 \times 20}{3500 \times 12^2 \times 0 \cdot 03}\left[\frac{1}{4} \times 1\frac{1}{4} \times 100 \times 11 \cdot 86 \times 8 \cdot 14 + 17746 \cdot 875 + \frac{523 \cdot 125}{4 \cdot 5} \times (11 \cdot 86 - 7 \cdot 5)\right] = 9 \cdot 379$$

or the required breadth is $9\frac{3}{8}$ inches.

404.—Example—h greater than n.—What should be the breadth of a white pine carriage beam 20 feet long, 12 inches deep, and carrying two headers 10 feet long—one located at 9 feet from one wall and the other at 6 feet from the other wall; the floor to carry 100 pounds per foot superficial, with a deflection of 0·03 of an inch per foot lineal, and the beams to be placed 15 inches from centres?

Here $f = 100$, $F = 2900$, $d = 12$, $r = 0 \cdot 03$, $c = 1\frac{1}{4}$, $l = 20$ and $g = 10$. Comparing m and s we have $m = 9$, $n = 11$ and $s = 6$.

Proceeding as in the last article, we find that h exceeds n, therefore, according to *Art.* **402**, we have formula (*174.*) appropriate to this case; from which

$$b = \frac{100 \times 9}{2900 \times 12^2 \times 0 \cdot 03}[(1\tfrac{1}{4} \times 11 \times 20) + 10(9 \times 11 + 6^2)] = 9 \cdot 728$$

or the breadth should be $9\frac{3}{4}$ inches.

405.—Carriage Beam with Two Headers and Two Sets of Tail Beams, for Dwellings—More Precise Rule.—If, in formula (*174.*), $f = 90$ and $r = 0{\cdot}03$, we shall have

$$b = \frac{3000m}{Fd^3}[cnl + g(mn + s^2)] \qquad (175.)$$

which is a *precise* rule for carriage beams carrying two headers, with two sets of tail beams, in dwellings and assembly rooms. (See *Arts.* **393** and **402.**)

406.—Example.—An example under this rule may be had in that given in *Art.* **404**; in which we have $F = 2900$, $d = 12$, $c = 1\frac{1}{4}$, $l = 20$, $g = 10$, $m = 9$, $n = 11$ and $s = 6$. Then by formula (*175.*)

$$b = \frac{3000 \times 9}{2900 \times 12^3}[(1\tfrac{1}{4} \times 11 \times 20) + 10(\overline{9 \times 11} + 6^2)] = 8{\cdot}755$$

or the breadth should be $8\frac{3}{4}$ inches.

407.—Carriage Beam with Two Headers and Two Sets of Tail Beams, for First-class Stores—More Precise Rule.—If, in formula (*174*), $f = 275$ and $r = 0{\cdot}04$, we shall have

$$b = \frac{6875m}{Fd^3}[cnl + g(mn + s^2)] \qquad (176.)$$

which is a *precise* rule for carriage beams carrying two headers, with two sets of tail beams, in first-class stores. (See *Arts.* **395** and **402.**)

408.—Example.—What should be the breadth, under this rule, of a carriage beam of Georgia pine of average quality, 23 feet long, 14 inches deep, carrying two headers each 17 feet long, with tail beams on one side 7 feet long, and on the other 10 feet long; the beams being placed 14 inches from centres?

Here $F = 5900$, $d = 14$, $c = 1\frac{1}{8}$, $l = 23$ and $g = 17$. Taking the larger of the two, 10 and 7, for m, we have $m = 10$, $n = 13$, and $s = 7$; and by formula (*176.*)

$$b = \frac{6875 \times 10}{5900 \times 14^2}[(1\tfrac{1}{8} \times 13 \times 23) + 17\,(\overline{10 \times 13} + 7^2)] = 14 \cdot 404$$

the breadth should be, say $14\frac{1}{2}$ inches.

409.—Carriage Beam with Two Headers and One Set of Tail Beams—More Precise Rule.—In a case where there are two openings in the floor, one at each wall, then the two headers carry but one set of tail beams, and these are between the headers. The load at each header is the same; and when g equals the length of header, j the length of tail beams, and f the load per superficial foot, then the load at each end of each header is

$$W = \tfrac{1}{4}fgj$$

and the expression for the load at one point, as in *Art.* **153**, $\frac{m}{l}(Wn + Vs)$, becomes $\frac{Wm}{l}(n + s)$, and therefore (*Art.* **243**)

$$a' = \frac{fgjm}{4l}(n + s) \qquad (177.)$$

and

$$b' = \frac{fgjs}{4l}(r + m) \qquad (178.)$$

In the case under consideration, these two expressions are auxiliary to formula (*170.*), in the place of those given in formulas (*171.*) and (*172.*), and with h equal to, or exceeding n, formula (*170.*) becomes

$$b = \frac{4l}{Fd^2r}(\tfrac{1}{2}cfmn + a')$$

Substituting for a' its value, as in formula (*177.*), we have

$$b = \frac{4l}{Fd^2r'}\left[\tfrac{1}{4}cfmn + \frac{fgjm}{4l}(n+s)\right]$$

$$b = \frac{fm}{Fd^2r'}\left[cnl + gj(n+s)\right] \qquad (179.)$$

which is a *precise* rule for carriage beams carrying two headers, with one set of tail beams, and with a given rate of deflection. (See *Arts.* **397**, **398** and **402**.)

410.—Example.—What should be the breadth of a carriage beam of locust of average quality, 16 feet long and 8 inches deep, carrying two headers of 8 feet length, with one set of tail beams 7 feet long between them, so placed as to leave an opening of 6 feet width at one wall, and another of 3 feet at the other? The floor beams are placed 15 inches from centres, and are to carry 90 pounds per superficial foot, with a deflection of 0·04 of an inch per foot lineal.

We have from this statement $f = 90$, $m = 6$, $n = 10$, $l = 16$, $r = 13$, $s = 3$, $c = 1\tfrac{1}{4}$, $F = 5050$, $d = 8$, $r' = 0{\cdot}04$, $g = 8$ and $j = 7$.

To test the value of h we have, preliminary thereto, formula (*177.*), which gives

$$a' = \frac{90 \times 8 \times 7 \times 6}{4 \times 16} \times \overline{10 + 3} = 6142{\cdot}5$$

and, formula (*178.*),

$$b' = \frac{90 \times 8 \times 7 \times 3}{4 \times 16} \times \overline{13 + 6} = 4488{\cdot}75$$

$$a' - b' = 1653{\cdot}75$$

Then, by formula (*178.*),

$$h = \overline{\tfrac{1}{2} \times 16} + \frac{1653 \cdot 75}{\frac{1}{2} \times 1\frac{1}{4} \times 7 \times 90} = 12 \cdot 2$$

As $n = 10$, h exceeds n. We must, therefore, substitute n for h; and by formula (*179.*) we have

$$b = \frac{90 \times 6}{5050 \times 8^2 \times 0 \cdot 04}\left[\left(1\tfrac{1}{4} \times 10 \times 16\right) + \left(8 \times 7 \times \overline{10 + 3}\right)\right] = 4 \cdot 845$$

or the breadth should be $4\frac{7}{8}$, say 5 inches.

411.—Carriage Beam with Two Headers and One Set of Tail Beams, for Dwellings—More Precise Rule.—If, in formula (*179.*), $f = 90$ and $r' = 0 \cdot 03$, we shall have

$$b = \frac{3000m}{Fd^2}\left[cnl + gj(n+s)\right] \qquad (180.)$$

which is a *precise* rule (in cases where h exceeds n) for carriage beams carrying two headers, with one set of tail beams, in a dwelling or assembly room. (See *Arts.* **398**, **402** and **409.**)

412.—Example.—What should be the breadth, in a dwelling, of a carriage beam of spruce of average quality, 18 feet long and 10 inches deep, carrying two headers of 12 feet length, with a set of tail beams between them 7 feet long? The headers are placed so as to leave an opening of 8 feet on one side and 3 feet on the other, and the beams are set 15 inches from centres.

Here $f = 90$, $g = 12$, $j = 7$, $m = 8$, $n = 10$, $s = 3$, $r = 15$, $l = 18$, $F = 3500$, $d = 10$, $r' = 0 \cdot 03$ and $c = 1\frac{1}{4}$.

Preliminary to seeking the value of h we find, by formulas (*177.*) and (*178.*),

$$a' = \frac{90 \times 12 \times 7 \times 8}{4 \times 18}(10+3) = 10920$$

$$b' = \frac{90 \times 12 \times 7 \times 3}{4 \times 18}(15+8) = 7245$$

$$a' - b' = 3675$$

Now, by formula (*173.*),

$$h = \overline{\tfrac{1}{2} \times 18} + \frac{3675}{\frac{1}{2} \times 1\frac{1}{2} \times 7 \times 90} = 18 \cdot 333$$

But $n = 10$; therefore n is to be used in the place of h, and formula (*180.*) is the proper one to use in this example. This latter formula gives us

$$b = \frac{3000 \times 8}{3500 \times 10^3}[(1\tfrac{1}{2} \times 10 \times 18) + (12 \times 7 \times \overline{10+3})] = 9 \cdot 031$$

Thus the breadth should be 9 inches; or the beam be 9×10 inches.

413.—Carriage Beam with Two Headers and One Set of Tail Beams, for First-class Stores—More Precise Rule.— If, in formula (*179.*), $f = 275$ and $r = 0 \cdot 04$, we shall have

$$b = \frac{6875m}{Fd^3}[cnl + gj(n+s)] \qquad (181.)$$

which is a *precise* rule, when h exceeds n, for a carriage beam carrying two headers, with one set of tail beams, in a first-class store. (See *Arts.* **400** and **402.**)

414.—Example.—The example given in *Art.* **412** may be used to exemplify this rule, excepting the depth, which we will put at 14 inches instead of 10.

Formulas (*180.*) and (*181.*) are alike, with the exception of the numerical constant. The result found in *Art.* **412**, $b = 9 \cdot 031$, multiplied and divided to correct the constant, will give the result required in this case. The constant 6875 is to take the place of 3000, and the depth 14 is to replace 10. With these changes, we have

$$b = 9 \cdot 031 \times \frac{6875}{3000} \times \frac{1000}{2744} = 7 \cdot 542$$

or the breadth should be $7 \cdot 54$; say $7\frac{1}{2}$ inches.

415.—Carriage Beam with Two Headers, Equidistant from Centre, and Two Sets of Tail Beams—Precise Rule.—In case the opening in the floor be at the middle, leaving tail beams of equal length on either side, then the moments of the two concentrated loads upon the carriage beam are equal, or $a' = b'$ and, in formula (*170.*),

$$\frac{a' - b'}{d'}(h - s) = \frac{0}{d'}(h - s) = 0$$

and the formula itself becomes

$$b = \frac{4l}{Fd^2r}(\tfrac{1}{4}cfht + b') \qquad (182.)$$

in which b' represents the combined effect of the two loads, as acting at the location of either of them.

This effect is shown (*Art.* **153**) to be

$$b' = W\frac{mn}{l} + V\frac{ms}{l} = \frac{m}{l}(Wn + Vs)$$

In the case under consideration, $W = V$ and $m = s$, and therefore

$$b' = W\frac{m}{l}(n+m) = W\frac{ml}{l} = Wm$$

Now, W represents the weight concentrated in one end of one of the headers. The load on a header is $\frac{1}{2}fgm$, and the load at one end of the header is $\frac{1}{4}fgm$; therefore

$$b' = \frac{1}{4}fgm^2$$

and formula (*182.*) becomes

$$b = \frac{4l}{Fd^3r}(\frac{1}{4}cfht + \frac{1}{4}fgm^2)$$

$$b = \frac{fl}{Fd^3r}(cht + gm^2)$$

By formula (*173.*)

$$h = \frac{1}{2}l + \frac{a'-b'}{\frac{1}{2}cd'f}$$

and since in this case $a' - b' = 0$

$$h = \frac{1}{2}l = t \qquad \text{and}$$

$$cht = \frac{1}{2}cl \times \frac{1}{2}l = \frac{1}{4}cl^2 \qquad \text{and therefore}$$

$$b = \frac{fl}{Fd^3r}(\frac{1}{4}cl^2 + gm^2) \qquad (183.)$$

which is a *precise* rule for carriage beams carrying two headers, equidistant from the centre, with two sets of tail beams, and with a given rate of deflection. (See *Arts.* **393**, **395** and **402.**)

416.—Example.—Under this rule, what should be the breadth of a Georgia pine carriage beam of average quality, 20 feet long and 12 inches deep, to carry two headers each

12 feet long; the headers so placed as to leave an opening 6 feet wide in the middle of the width of the floor? The floor beams are set 16 inches from centres, and are to carry 200 pounds per foot superficial, with a deflection of 0·04 of an inch per foot lineal.

Here $l = 20$, $m = \frac{l - d'}{2} = \frac{20 - 6}{2} = 7$; $g = 12$, $d = 12$,

$c = 1\frac{1}{8}$, $F = 5900$, $f = 200$ and $r = 0{\cdot}04$; and by formula (*183.*)

$$b = \frac{200 \times 20}{5900 \times 12^2 \times 0{\cdot}04}\left[\left(\tfrac{1}{4} \times 1\tfrac{1}{8} \times 20^2\right) + \left(12 \times 7^2\right)\right] = 7{\cdot}075$$

or the breadth should be $7\frac{1}{8}$ inches.

417.—Carriage Beams with Two Headers, Equidistant from Centre, and Two Sets of Tail Beams, for Dwellings and for First-class Stores—Precise Rules.—If, in formula (*183.*), $f = 90$ and $r = 0{\cdot}03$, we shall have

$$b = \frac{3000l}{Fd^2}\left(\tfrac{1}{4}cl^2 + gm^2\right) \qquad (184.)$$

which is a *precise* rule for carriage beams carrying two headers, equidistant from the centre, with two sets of tail beams, in a dwelling or assembly room. (For an example, see *Art.* **418.**) But if, instead, $f = 275$ and $r = 0{\cdot}04$, then we shall have

$$b = \frac{6875l}{Fd^2}\left(\tfrac{1}{4}cl^2 + gm^2\right) \qquad (185.)$$

which is a *precise* rule for carriage beams carrying two headers, equidistant from the centre, with two sets of tail beams, in a first-class store. (See *Arts.* **393, 395** and **402.**)

418.—Examples.—Formulas (*184.*) and (*185.*) are alike, except in the numerical coefficient. One example will therefore suffice for an exemplification of the two. Let it be required to show what, in a dwelling, should be the breadth of a carriage beam, 20 feet long and 12 inches deep, of average quality of spruce, carrying two headers 10 feet long; these headers being so placed as to leave at the middle of the width of the floor an opening 8 feet wide. The beams are to be placed 16 inches from centres.

Here we have $l = 20$, $m = 6$, $g = 10$, $d = 12$, $c = 1\frac{1}{3}$ and $F = 3500$; and by formula (*184.*)

$$b = \frac{3000 \times 20}{3500 \times 12^3}\left[\left(\tfrac{1}{4} \times 1\tfrac{1}{3} \times 20^2\right) + \left(10 \times 6^2\right)\right] = 4 \cdot 894$$

or the breadth should be, say $4\frac{7}{8}$ inches.

For a first-class store this carriage beam, if of Georgia pine, would be required to be $6 \cdot 653$, say $6\frac{5}{8}$ inches broad. This result is found by eliminating the two constants 3000 and 3500 in the above and replacing them by those required by the new conditions, namely, 6875 and 5900. Doing this, we find

$$b = 4 \cdot 894 \times \frac{6875}{3000} \times \frac{3500}{5900} = 6 \cdot 653$$

419.—Carriage Beam with Two Headers, Equidistant from Centre, and One Set of Tail Beams—Precise Rule.— In some cases the wells or openings are at the wall on each side, and the tail beams at the middle of the floor. In this arrangement, if j equals the length of the tail beams, $\frac{1}{4}fgj$ will equal the load at the end of one header.

By *Art.* **415**, $b' = Wm$, from which

$$b' = Wm = \tfrac{1}{4}fgjm$$

and formula (*182.*) becomes

$$b = \frac{4l}{Fd^3r}\left(\tfrac{1}{4}cfht + \tfrac{1}{4}fgjm\right)$$

and since (*Art.* **415**) $h = t = \frac{1}{4}l$, therefore

$$\tfrac{1}{4}cht = \tfrac{1}{16}cl^2$$

By substituting this in the above,

$$b = \frac{4l}{Fd^3r}\left(\tfrac{1}{16}cfl^2 + \tfrac{1}{4}fgjm\right)$$

$$b = \frac{fl}{Fd^3r}\left(\tfrac{1}{4}cl^2 + gjm\right) \qquad (186.)$$

which is a *precise* rule for carriage beams, carrying two headers, equidistant from the centre, with one set of tail beams, the rate of deflection being given. (See *Arts.* **397**, **398**, **402**, **409** and **411**.)

420.—Example.—What should be the breadth of a carriage beam of hemlock of average quality, 16 feet long and 11 inches deep, carrying two headers, each 10 feet long, placed equidistant from the centre of the width of the floor, and having between them one set of tail beams 6 feet long? The floor beams, placed 15 inches from centres, are to carry 100 pounds per foot superficial, with a deflection of 0·035 of an inch per foot lineal.

Here we have $l = 16$, $m = 5$, $g = 10$, $j = 6$, $d = 11$, $c = 1\frac{1}{4}$, $F = 2800$, $f = 100$ and $r = 0{\cdot}035$; and by formula (*186.*)

$$b = \frac{100 \times 16}{2800 \times 11^3 \times 0{\cdot}035}\left[\left(\tfrac{1}{4} \times 1\tfrac{1}{4} \times 16^2\right) + (10 \times 6 \times 5)\right] = 4{\cdot}661$$

or the breadth should be $4\frac{5}{8}$ inches.

421.—Carriage Beams with Two Headers, Equidistant from Centre, and One Set of Tail Beams, for Dwellings and for First-class Stores—Precise Rules.—If, in formula (*186.*), $f = 90$ and $r = 0{\cdot}03$, then we shall have

$$b = \frac{3000l}{Fd^3}(\tfrac{1}{4}cl^2 + gjm) \qquad (187.)$$

which is a *precise* rule for carriage beams, carrying two headers, with one set of tail beams between them, at the middle of the floor, in a dwelling or assembly room.

For an example, see *Art.* **422.**

But if, instead of these, $f = 275$ and $r = 0.04$, we shall have

$$b = \frac{6875l}{Fd^3}(\tfrac{1}{4}cl^2 + gjm) \qquad (188.)$$

which is a *precise* rule for carriage beams, carrying two headers, with one set of tail beams between them, at the middle of the floor, in a first-class store.

422.—Example.—Formulas (*187.*) and (*188.*) are alike, except in the numerical coefficient. One example will suffice to show the application of both.

Take one coming under formula (*187.*), and in which $l = 20$, $m = 6$, $g = 10$, $j = 8$, $d = 12$, $c = 1\frac{1}{8}$ and $F = 3500$. Then, by the formula,

$$b = \frac{3000 \times 20}{3500 \times 12^3}[(\tfrac{1}{4} \times 1\tfrac{1}{8} \times 20^2) + (10 \times 8 \times 6)] = 6{\cdot}085$$

or the breadth should be $6\frac{1}{8}$ inches nearly.

423.—Beam with Uniformly Distributed and Three Concentrated Loads, the Greatest Strain being Outside.—In *Art.* **256**, formula (*96.*) is a general rule for this case, but

based upon the resistance to *rupture*. This rule may be modified so that it shall be based upon the resistance to *flexure*. To this end let a, in formula (*96.*), be substituted by its value in formula (*154.*), $\frac{Bl}{Fdr}$, and we have

$$b = \frac{4m}{Fd^2r}\left(\tfrac{1}{2}Un + A'n + B's + C'v\right) \qquad (189.)$$

which is a rule, based upon the resistance to *flexure*, for a beam uniformly loaded, and also carrying three concentrated loads, the largest of which is *not* between the other two.

424.—Example.—What ought to be the breadth of a beam of Georgia pine of average quality, 20 feet long and 12 inches deep, carrying an equally distributed load of 4000 pounds, together with three concentrated loads, namely, 7000 pounds at 7 feet from the right-hand end, 4000 pounds at 7 feet from the left-hand end, and 3000 pounds at 3 feet from the same end. (See *Art.* **264.**) Allotting the symbols to accord with the arrangement required under rule (*189.*), (the largest strain, as in *Fig.* 55, *not* between the other two), we have $U = 4000$, $A' = 7000$, $B' = 4000$, $C' = 3000$, $l = 20$, $m = 7$, $n = 13$, $s = 7$, $v = 3$, $d = 12$ and $F = 5900$, and let $r = 0{\cdot}04$; and by formula (*189.*)

$$b = \frac{4 \times 7}{5900 \times 12^2 \times 0{\cdot}04}\left[(\tfrac{1}{2} \times 4000 \times 13) + (7000 \times 13) + (4000 \times 7) + (3000 \times 3)\right] = 10{\cdot}574$$

or the breadth should be $10\tfrac{5}{8}$ inches.

425.—Carriage Beam with Three Headers, the Greatest Strain being at Outside Header.—If, in formula (*97.*), (*Art.* **258**), we substitute for a its value, $\frac{Bl}{Fdr}$ (*form.* *154.*), we shall have

$$b = \frac{mf}{Fd^3r}[cnl + g(mn + s^2 - v^2)] \qquad (190.)$$

which is a rule, based upon the resistance to flexure, for carriage beams carrying three headers, with two sets of tail beams, so located (as in *Figs.* 54 and 55) that the header at which there is the greatest strain shall *not* be between the other two headers.

426.—Example.—What should be the breadth of a carriage beam of Georgia pine of average quality, 20 feet long and 12 inches deep, carrying three headers 15 feet long, two of them, for a light-well 6 feet wide, located centrally as to the width of the floor, and the third header, at the side of an opening for a stairway 3 feet wide at one of the walls? The floor beams, placed 15 inches from centres, are to carry 200 pounds per superficial foot, with a deflection of 0·04 of an inch per foot lineal. (See *Art.* **264**.)

Allotting the symbols as in *Fig.* 55, we have $l = 20$, $m = 7$, $n = 13$, $s = 7$, $v = 3$, $g = 15$, $d = 12$, $c = 1\frac{1}{4}$, $F = 5900$, $f = 200$ and $r = 0 \cdot 04$; and by formula (*190.*) we have

$$b = \frac{7 \times 200}{5900 \times 12^3 \times 0 \cdot 04}[(1\tfrac{1}{4} \times 13 \times 20) + 15(\overline{7 \times 13} + 7^2 - 3^2)] = 7 \cdot 861$$

or the breadth should be $7\frac{7}{8}$ inches.

427.—Carriage Beam with Three Headers, the Greatest Strain being at Outside Header, for Dwellings.—If, in formula (*190.*), $f = 90$ and $r = 0 \cdot 03$, we shall have

$$b = \frac{3000m}{Fd^3}[cnl + g\,(mn + s^2 - v^2)] \qquad (191.)$$

which is a rule, based upon the resistance to flexure, for carriage beams in dwellings and assembly rooms, to carry three headers, with two sets of tail beams, so located that (as in *Fig.* 55) the header at which there is the greatest strain shall *not* be between the other two.

For an example, see *Art.* **429.**

428.—Carriage Beam with Three Headers, the Greatest Strain being at Outside Header, for First-class Stores.—If, in formula (*190.*), $f = 275$ and $r = 0 \cdot 04$, we shall have

$$b = \frac{6875m}{Fd^3}[cnl + g\,(mn + s^2 - v^2)] \qquad (192.)$$

which is a rule, based upon the resistance to flexure, for carriage beams in first-class stores, to carry three headers, with two sets of tail beams, so located that (as in *Fig.* 55) the header at which there is the greatest strain shall *not* be located between the other two.

429.—Examples.—Formulas (*191.*) and (*192.*) are alike, except in the numerical coefficient, which, in the rule for dwellings and assembly rooms, is 3000, while for first-class stores it is 6875. An example under one rule will serve to illustrate the other, by a simple substitution of the proper coefficient.

As an example under rule (*191.*): What should be the

breadth, in a dwelling, of a carriage beam of white pine of average quality, 20 feet long and 12 inches deep, carrying three headers 12 feet long, so placed as to provide an opening 4 feet wide for a stairs at one wall, and a light-well 6 feet wide at the middle of the width of the floor? The floor beams are placed 16 inches from centres. (See *Art.* **264**.)

Allotting the symbols as in *Fig.* 55 we have $l = 20$, $m = 7$, $n = 13$, $s = 7$, $v = 4$, $g = 12$, $d = 12$, $c = 1\frac{1}{4}$ and $F = 2900$; and by formula (*191.*)

$$b = \frac{3000 \times 7}{2900 \times 12^2}[(1\tfrac{1}{4} \times 13 \times 20) + 12\,(\overline{7 \times 13} + 7^2 - 4^2)] = 7 \cdot 688$$

or the breadth should be $7\frac{3}{4}$ inches.

This is the breadth when of white pine, and in a dwelling. If, instead, it be required of Georgia pine, and for a first-class store, then the breadth just obtained, treated by the proper constant and numerical coefficient, and at the same time relieved from those applying to the previous case, will be

$$b = 7 \cdot 688 \times \frac{6875}{3000} \times \frac{2900}{5900} = 8 \cdot 660$$

or the breadth, when of Georgia pine, and for a first-class store, should be $8\frac{5}{8}$ inches.

430.—Beams with Uniformly Distributed and Three Concentrated Loads, the Greatest Strain being at Middle Load.—In *Art.* **262** a rule is given for beams uniformly loaded, and also carrying three concentrated loads, the middle one of which produces the greatest strain. This rule is based upon the resistance to *rupture.* It may be modified to

depend upon the resistance to *flexure* by substituting, in formula (*99.*), for a its value $\frac{Bl}{Fdr}$ (*form. 154.*), thus

$$b = \frac{4}{Fd^3r}[m(\tfrac{1}{2}Un + A'n + B's) + C'nv] \qquad (193.)$$

which is a rule, based upon resistance to flexure, for beams carrying a uniform load (U) and three concentrated loads (A', B' and C'), the middle one of which produces the greatest strain of the three, as in *Fig.* 56.

431.—Example.—What should be the breadth of a beam of Georgia pine of average quality, 20 feet long and 14 inches deep, and carrying 4000 pounds uniformly distributed, 6000 pounds at 4 feet from one end, 6000 pounds at 9 feet from the same end, and 7000 pounds at 6 feet from the other end; with a deflection of 0·04 of an inch per lineal foot? (See *Art.* **264**.)

Assigning the symbols as per figure, we have, $U = 4000$, $A' = 6000$, $B' = 7000$, $C' = 6000$, $l = 20$, $m = 9$, $n = 11$, $s = 6$, $v = 4$, $d = 14$, $F = 5900$ and $r = 0{\cdot}04$; and by formula (*193.*),

$$b = \frac{4}{5900 \times 14^3 \times 0{\cdot}04}[9(\overline{\tfrac{1}{2} \times 4000 \times 11} + \overline{6000 \times 11} + \overline{7000 \times 6}) + (6000 \times 11 \times 4)] = 8{\cdot}858$$

or the breadth should be $8\tfrac{7}{8}$ inches.

432.—Carriage Beam with Three Headers, the Greatest Strain being at Middle Header.—If, in formula (*106.*), (*Art.* **267**), there be substituted for a its value $\frac{Bl}{Fdr}$ (*form. 154.*), we shall have

$$b = \frac{f}{Fd^3r}[m(cnl + gs^2) + gn(m^2 - v^2)] \qquad (194.)$$

which is a rule, based upon resistance to flexure, for carriage beams carrying three headers and two sets of tail beams, so placed (as in *Fig.* 56) that of the strains produced at the headers, the greatest shall be at the header which is between the other two.

433.—Example.—What should be the breadth of a carriage beam, 20 feet long and 12 inches deep, of Georgia pine of average quality, carrying three headers 14 feet long, so placed as to provide a stair opening 4 feet wide at one wall, and a light-well 5 feet wide 6 feet from the other wall? The floor beams, placed 15 inches from centres, are to carry 200 pounds per foot superficial, with a deflection of 0·04 of an inch per foot lineal. (See *Art.* **264**.)

Assigning the symbols as per *Fig.* 56 we have, $l = 20$, $m = 9$, $n = 11$, $s = 6$, $v = 4$, $g = 14$, $d = 12$, $c = 1\frac{1}{4}$, $F = 5900$, $f = 200$ and $r = 0·04$; and by formula (*194.*)

$$b = \frac{200}{5900 \times 12^3 \times 0·04}\left[9(\overline{1\tfrac{1}{4} \times 11 \times 20} + \overline{14 \times 6^2}) + (14 \times 11 \times \overline{9^2 - 4^2})\right] = 8·348$$

or the breadth should be, say $8\frac{3}{8}$ inches.

434.—Carriage Beam with Three Headers, the Greatest Strain being at Middle Header, for Dwellings.—If, in formula (*194.*), $f = 90$ and $r = 0·03$, we shall have

$$b = \frac{3000}{Fd^3}\left[m(cnl + gs^2) + gn(m^2 - v^2)\right] \qquad (195.)$$

which is a rule, based on resistance to flexure, for carriage beams in dwellings and assembly rooms, to carry three headers, with two sets of tail beams relatively placed as in *Fig.* 56, so that, of the three strains produced at the headers, the greatest shall be at the header which is between the other two. (For an example, see *Art.* **436**.)

435.—Carriage Beam with Three Headers, the Greatest Strain being at Middle Header, for First-class Stores.—If, in formula (*194.*), $f = 275$ and $r = 0{\cdot}04$, then we shall have

$$b = \frac{6875}{Fd^3}\,[m(cnl + gs^2) + gn(m^2 - v^2)] \qquad (196.)$$

which is a rule, based on resistance to flexure, for carriage beams in first-class stores, to carry three headers, with two sets of tail beams relatively placed as in *Fig.* 56, so that, of the three strains produced at the headers, the greatest shall be at the header which is between the other two.

436.—Example.—Formulas (*195.*) and (*196.*) being alike, except in the numerical coefficient, a single example will suffice to illustrate them.

In a dwelling, what should be the breadth of a carriage beam of oak of average quality, 20 feet long and 12 inches deep, to carry three headers 15 feet long, with two sets of tail beams, so placed as to provide a stair opening 4 feet wide at one wall, and a light-well 7 feet wide, distant 5 feet from the other wall? The beams are to be placed 15 inches from centres. (See *Art.* **264.**)

Arranging the symbols in the order in which they appear in *Fig.* 56, we have, $l = 20$, $m = 8$, $n = 12$, $s = 5$, $v = 4$, $g = 15$, $d = 12$, $c = 1\frac{1}{4}$ and $F = 3100$; and, by formula (*195.*),

$$b = \frac{3000}{3100 \times 12^3}[8(\overline{1\tfrac{1}{4} \times 12 \times 20} + \overline{15 \times 5^2}) + (15 \times 12 \times \overline{8^2 - 4^2})] = 7{\cdot}863$$

or the breadth should be, say $7\frac{7}{8}$ inches.

QUESTIONS FOR PRACTICE.

437.—In a dwelling: What should be the depth of white pine beams of average quality; they being 18 feet long and 3 inches broad, placed 18 inches from centres, and allowed to deflect 0·03 of an inch per foot?

438.—In a first-class store: What should be the breadth of the floor beams of spruce of average quality, 19 feet long, 13 inches deep, placed 13 inches from centres, and with a deflection of 0·04 of an inch per foot?

439.—In a dwelling: What ought to be the breadth of a header of white pine of average quality, 14 feet long and 13 inches deep, carrying one end of a set of tail beams 15 feet long, and with a rate of deflection of 0·03 of an inch per foot?

440.—In the floor of an assembly room, in which the beams are 15 inches from centres: What should be the breadth of a carriage beam of spruce of average quality, 20 feet long and 12 inches deep, carrying one header 13 feet long, located at 5 feet from one end? The deflection allowable is 0·03 of an inch per foot.

441.—In the floor of a first-class store, where the beams are 15 inches deep and set 14 inches from centres: What should be the breadth of a carriage beam 24 feet long, of Georgia pine of average quality, carrying two headers

16 feet long, located, one at 9 feet from one end, and the other at 7 feet from the other end, with two sets of tail beams? The deflection is 0·04 of an inch per foot.

442.—In the floor of a first-class store, with beams 16 inches deep placed 15 inches from centres: What should be the breadth of a carriage beam of Georgia pine of average quality, 26 feet long, and carrying three headers 18 feet long, located as in *Fig.* 54, one at 4 feet from one wall, another at 8 feet from the same wall, and the third at 8 feet from the other wall? The deflection to be 0·04 of an inch per foot.

CHAPTER XVIII.

BRIDGING FLOOR BEAMS.*

ART. **443.—Bridging Defined.**—Bridging is a system of bracing floor beams. Small struts are cut to fit between each pair of beams, and secured by nails or spikes; as shown in *Fig.* 65. The effect of this bracing is decidedly

FIG. 65.

beneficial in sustaining any *concentrated* weight upon a floor. The beam immediately beneath the weight is materially as-

* The principles upon which this chapter is based the author first made public in an article which appeared in the *Scientific American*, July 26th, 1873, entitled "On Girders and Floor Beams—The Effect of Bridging."

sisted, through these braces, by the beams on each side of it. It is customary to insert rows of cross-bridging at every five to eight feet in the length of the beams.

It is the usual practice, where the ceiling of a room is plastered, to attach the plastering laths to *cross-furring*, or narrow strips of boards crossing the beams at right angles, and nailed to their bottom edge. These strips are set at, say 12 inches from centres, and when firmly nailed to the beams act as a tie to sustain the lateral thrust of the bridging struts. The floor plank at the top serve a like purpose.

444.—Experimental Test.—To test the effect of bridging, about three years since I constructed a model, and subjected it to pressure. It was made upon a scale of $1\frac{1}{2}$ inches to the foot, or $\frac{1}{8}$ of full size, and represented a floor of seven beams placed 16 inches from centres, each beam being 3×10 inches and $14\frac{2}{3}$ feet long. These beams were connected by two rows of cross-bridging, and secured against lateral movement by strips representing floor plank and ceiling boards, which were nailed on top and beneath. There were four strips at each row of bridging, two above and two below.

Before putting these beams in position in the model, I submitted each beam to a separate test, and ascertained that to deflect it one tenth of an inch required from 37 to 40 pounds, or on the average $38\frac{1}{3}$ pounds.

With the model completed, the beams being bridged, it required a pressure of $155\frac{3}{4}$ pounds applied at the centre of the middle beam to deflect it as before, one tenth of an inch. And while this pressure deflected the central beam to this extent, the beam next adjoining on each side was deflected 0·0808 of an inch, the ones next adjoining these were each deflected 0·0617 of an inch, while the two outside beams

were each depressed 0·0425 of an inch. Had there been more than seven beams, and all bridged together, the effect would doubtless have been still better.

As the result of this test of the effect of bridging, we have one beam sustaining $155\frac{3}{4}$ pounds with the same deflection that was produced by $38\frac{1}{3}$ pounds before bridging, or an increase of $117\frac{5}{12}$ pounds; an addition of more than three times the amount borne by the unbridged beam.

445.—Bridging—Principles of Resistance.—The assistance contributed by the adjacent beams to a beam under pressure may be computed, but preliminary thereto we have these considerations, namely:

First.—The deflections of a beam are (within the limits of elasticity) in proportion to the weights producing the deflections. Thus, if one hundred pounds deflect a beam one tenth of an inch, two hundred pounds will deflect it two tenths of an inch. From which, knowing the deflection of a beam, we can compute the resistance it offers.

Second.—The resistance thus offered, being at a distance from the beam suffering the direct pressure, is not so effectual as it would be were it in direct opposition to the pressure. It is diminished in proportion to its distance from that beam.

446.—Resistance of a Bridged Beam.—Based upon the two preceding considerations, we will construct a rule by which to measure the increase of resistance derived from the adjacent beams through their connection by cross-bridging.

Let *Fig.* 66 represent the cross-section of a tier of floor

beams connected by cross-bridging, in which C is the location of a concentrated weight, AB the distance on one side of the weight to which the deflecting influence acting

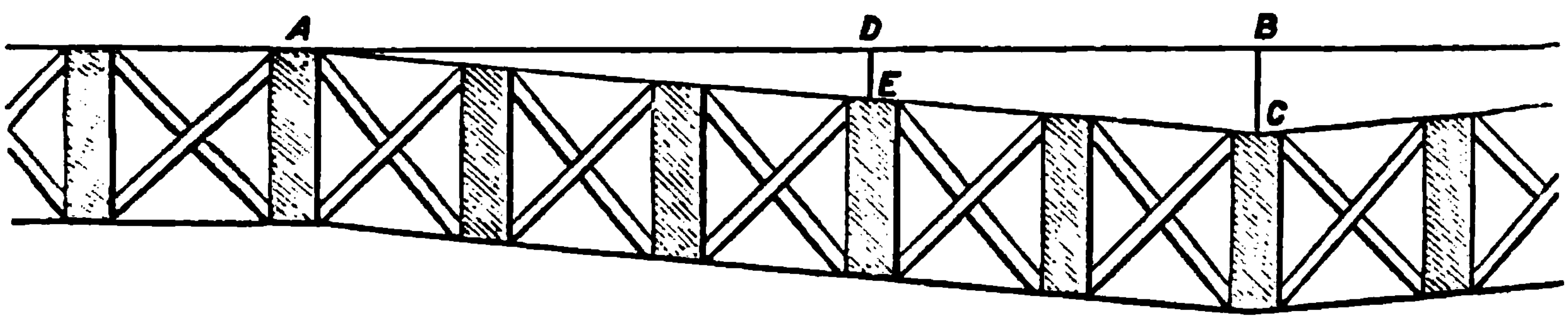

FIG. 66.

through the cross-bridging is extended, BC the deflection at the weight, and DE the deflection of one of the beams E, caused by the weight at C. The triangles ABC and ADE are similar, and their sides are in proportion. Putting p for AB, m for AD, a' for BC, and b' for DE, we have

$$AB : BC :: AD : DE$$

$$p : a' :: m : b' = a'\frac{m}{p}$$

This is the measure of the deflection at E, or at any one of the beams the distance of which from A is equal to m, and, since the deflections are as the weights producing them, therefore b', the deflection at E, measures the strain there, when a' measures that at C.

It is required, however, to know not only the resistance offered by each beam, but also what weight r, acting at C, would be required to overcome this resistance. The line AB (or p) may be considered to serve as a lever, having its fulcrum at A. The weight r, at B, acting in the line BC, is opposed at D by b', the resistance of the beam at E, acting in the line ED, with the leverage m. The weight r will act with the moment rp, and b' will resist with the moment $b'm$. Putting these moments in equilibrium, we have $b'm = rp$, or $r = b'\frac{m}{p}$.

In this, substituting for b' its value as above found, we have

$$r = a'\frac{m}{p} \times \frac{m}{p}$$

$$r = a'\frac{m^2}{p^2} \qquad (197.)$$

This weight r represents the effect at C of the resistance to deflection of any beam whose distance from A is equal to m, and where a' equals the load borne by the beam at B, and p is put for the distance AB.

447.—Summing the Resistances.—Let the distance from centres between the floor beams be represented by c, and the number of spaces from A to any beam, as, for example, that at D, by n; then $m = nc$, and substituting this value for m in (*197.*) we have

$$r = a'\frac{n^2c^2}{p^2}$$

$$r = n^2\frac{a'c^2}{p^2} \qquad (198.)$$

In this expression, a', c^2 and p^2 are constants, or quantities which remain constant for the several values of r which are to be obtained from the resistances of the several beams. For convenience, put t for $\frac{a'c^2}{p^2}$ and then

$$r = n^2t \qquad (199.)$$

With this expression, the various values of r may be obtained and grouped together. In doing this, we have, for the first beam from A, $n = 1$; for the second, $n = 2$; for

the third, $n = 3$, and so on to the middle or point of greatest depression. Therefore the whole resistance will be

$$R' = t + 2^2t + 3^2t + 4^2t + \text{etc.}$$

$$R' = t\,(1 + 4 + 9 + 16 + \text{etc.})$$

This gives the resistance on one side of the point C. The beams on the other side afford a like resistance; and the sum of the two resistances will be

$$R = 2t\,(1 + 4 + 9 + 16 + \text{etc.})$$

$$R = 2\,\frac{a'c^2}{p^2}\,(1 + 4 + 9 + 16 + \text{etc.}) \qquad (200.)$$

448.—Example.—When a concentrated weight deflects six beams on each side of it, they being placed 16 inches from centres: What will be the amount of resistance to deflection offered by the twelve beams, the beam upon which the weight rests being capable of sustaining alone, unaided by the adjoining beams, 1000 pounds?

Here $a' = 1000$, $c = 1\frac{1}{8}$ and $p = 7 \times 1\frac{1}{8} = 9\frac{1}{8}$. Therefore, by formula (*200.*),

$$R = \frac{2 \times 1000 \times 1\frac{1}{8}^2}{9\frac{1}{8}^2}\,(1 + 4 + 9 + 16 + 25 + 36) = 3714\cdot 3$$

This 3714 pounds is the resistance offered by the twelve beams, through the means of bridging, and is nearly four times the amount that the centre beam, unaided by the bridging, would carry with a like deflection. The combined resistance of all the beams would be $3714 + 1000 = 4714$ pounds.

449.—Assistance Derived from Cross-bridging.—Just how many beams on each side will be affected, and by their resistance contribute in aiding the beam at C, will depend upon circumstances. The bridging will be effective in resisting deflection in proportion to the elevation of the angle at which the bridging pieces are placed, which will be directly as the depth of the beams and inversely as their distance apart. It will also be in proportion to the faithfulness with which the work of bridging is executed. From these considerations, and from the experiment of *Art.* **444**, we conclude that, in well-executed work, we shall have

$$p = \frac{d}{c}$$

An equally distributed load upon a floor beam is represented (*Art.* **92**) by cfl. A load at the centre of the beam producing an equal effect will be $\frac{5}{8}$ of this, or $\frac{5}{8}cfl$. The symbol a' (*form. 200.*) represents the load at the middle of a floor beam, and therefore

$$a' = \tfrac{5}{8}cfl$$

These values of p and a' may be substituted for these symbols in formula (*200.*), and the result will be

$$R = 2\,\frac{\frac{5}{8}cflc^2}{\left(\frac{d}{c}\right)^2}(1 + 4 + 9 + \text{etc.}) \qquad \text{or,}$$

$$R = \frac{5c^4fl}{4d^2}(1 + 4 + 9 + \text{etc.}) \qquad (201.)$$

In this rule R equals the additional resistance to a concentrated weight on a beam, obtained from adjacent beams through the cross-bridging.

450.—Number of Beams Affording Assistance.—The value of p, as above, is $\frac{d}{c}$. The symbol n being put for the number of spaces on each side of the beam sustaining the concentrated weight, over which this weight exerts an influence; or p, the distance AB of *Fig.* 66; and c for the distance apart from centres at which the beams are placed; then, $p = \frac{d}{c} = nc$; from which we have

$$n = \frac{d}{c^2} \qquad (202.)$$

To apply this rule: How many beams on each side of a concentrated weight would contribute towards sustaining it, when they are 12 inches deep, and 16 inches from centres?

Here we have $d = 12$ and $c = 1\frac{1}{3}$, and therefore

$$n = \frac{12}{1\frac{1}{3}^2} = 6\frac{3}{4} \text{ say } 7 \text{ spaces.}$$

In seven spaces, six beams will be affected.

451.—Bridging Useful in Sustaining Concentrated Weights.—The results shown in *Art.* 448 illustrate the advantage of cross-bridging in resisting concentrated weights, and show the importance of always having floor beams bridged, and the work faithfully executed. The advantage, however, of cross-bridging inheres only in the case of *concentrated* weights. For, although in the example of *Art.* 448, the 13 beams sustained by their united resistance a concentrated weight of 4714 pounds, yet it will be observed that this is not the limit of their power, for they are each capable of sustaining 1000 pounds placed at the middle, or together, 13,000 pounds; nearly three times the previous amount.

452.—Increased Resistance Due to Bridging.—A useful application of the results of this investigation is found in determining the amount of concentrated weight which may be borne upon a floor beam. As an example: In a dwelling with well-bridged floor beams of an average quality of white pine, 3×10 inches, and 16 feet long, what concentrated weight may be safely sustained at the middle of one of them?

The distance from centres at which these beams should be placed is had by formula (*144.*), *Art.* **362**,

$$c = \frac{ibd^3}{l^3} = \frac{1 \cdot 55 \times 3 \times 10^3}{16^3} = 1 \cdot 135$$

the value of i being taken as found in *Art.* **361**.

With the above value, $c = 1 \cdot 135$, we may, by formula (*202.*), find the distance to which the effect of the weight extends on each side, thus:

$$n = \frac{d}{c^2} = \frac{10}{1 \cdot 135^2} = \frac{10}{1 \cdot 289} = 7 \cdot 762$$

say 8 spaces, or 7 beams. The symbols of formula (*201.*), applied in this case, will be as follows: $c = 1 \cdot 135$, $f = 90$, $l = 16$ and $d = 10$, and the squares in the parenthesis extend to 7 places. Therefore

$$R = \frac{5 \times \overline{1 \cdot 135}^2 \times 90 \times 16}{4 \times 10^2}(1 + 4 + 9 + 16 + 25 + 36 + 49)$$

$$R = 18 \times \overline{1 \cdot 135}^2 \times 140$$

The product of these factors, one of them being raised to a high power, will best be obtained by logarithms, thus:

$$\begin{array}{lrl}
\text{Log. } 1\cdot135 & = & 0\cdot0549959 \\
 & & \underline{\quad\quad\quad 5} \\
 & & 0\cdot2749795 \\
\text{Log. } 18 & = & 1\cdot2552725 \\
\text{Log. } 140 & = & \underline{2\cdot1461280} \\
4746\cdot6 & = & 3\cdot6763800
\end{array}$$

The product of the factors, or the value of R, is therefore equal to 4746·6 pounds. This is the *increased* resistance.. The resistance offered by the beam upon which the weight is laid equals (*Art.* **449**)

$$\tfrac{5}{8}cfl = \frac{5 \times 1\cdot135 \times 90 \times 16}{8} = 1021\cdot5$$

$$\text{To this, adding the increase} = \underline{4746\cdot6}$$

$$\text{we have} \quad 5768\cdot1$$

as the total resistance to a concentrated load at the middle of the beam, when assisted by 7 beams on each side by cross-bridging.

CHAPTER XIX.

ROLLED-IRON BEAMS.

ART. **453.—Iron a Substitute for Wood.**—When the beams composing a floor are of wood, they are of *rectangular* form in cross-section. Investigations into the philosophy of the transverse strain, by which the importance of *depth* was developed, led to the use of beams of which the rectangle of cross-section was narrow and high. Owing to the liability, in wooden beams as generally used, of destruction by conflagration and by other causes, iron was introduced as a substitute. The greater cost of this material over that of wood, made it important, now more than ever, to give to the beam that shape which should prove the strongest.

454.—Iron Beam—Its Progressive Development.—In the use of iron as a floor beam, economical considerations reduced the breadth until the beam became weak *laterally*. To remedy this defect, metal was added at the top and bottom in the form of horizontal plates, and these were connected to the thin vertical beam by angle irons as in *Fig.* 67; the whole forming what is known as the *plate* beam or girder. This expedient served not only to stiffen the thin vertical beam laterally, but added *very greatly* to its ab-

FIG. 67.

solute strength. The added material had been placed just where it would do the greatest possible good; at a point far removed from the neutral axis of the beam.

455.—Rolled-Iron Beam—Its Introduction.—The increase of strength obtained in the plate beam (*Fig.* 67) was so great that it became popular. To supply the demand, iron manufacturers, at great expense, made rolls similar to those for making railroad iron, by which they were enabled to furnish beams (*Fig.* 68) rolled out in one piece, with all the best features of the plate beam, and which could be much more readily and cheaply made. Owing to the large cost of the rolls, only a very few sizes were at first made, but these few only increased the demand. The manufacturer, thus encouraged, made rolls for other sizes, and thus the number of beams was increased, until now we have them in great variety, from 4 to 15 inches high.*

FIG. 68.

456.—Proportions between Flanges and Web.—These beams, as usually made, have the top and bottom plates, or flanges, of the same form and size. In wrought-iron the resistances to *rupture*, by compression and by tension, are not equal. When the load upon the beam, however, is not so large as to strain the metal beyond the limits of *elasticity*,

* There were exhibited at the Centennial Exposition at Philadelphia, by the Union Iron Co., of Buffalo, a 15 inch beam 52 feet in length, and a 9 inch beam 80 feet long. This is believed to be the limit reached in American manufacture at the present time. The English and Germans, however, are rolling them larger. A German exhibit in Machinery Hall contained beams from Burbach half a metre (19·69 inches) high by 15 metres (49·21 feet) long.

then it resists both compression and tension equally well, and hence the propriety of having the top and bottom flanges equally large.

The manifest advantage of having the material accumulated at a distance from the neutral axis, has led to putting as much as possible of the area of the whole section into the flanges, and thereby reducing the web or vertical part to the smallest practical thickness. The web is required to maintain the connection between the top and bottom flanges, and to resist the shearing effects of the load. In rolled-iron beams, as usually made, the thickness of the web is more than sufficient to resist these strains.

457.—The Moment of Inertia Arithmetically Considered. —For the intelligent use of the rolled-iron beam as a substitute for the wooden beam in floors, as well as for other uses, the rules already given need modification.

The resistance of a beam to flexure or bending is termed its *moment of inertia*. This is represented in symbolic formulas by the letter I. In formula (*111.*), (*Art.* **300**), the coefficient $\frac{1}{12}$ and the symbols bd^3 represent the moment of inertia, and I, its symbol, may be substituted for them, thus:

$$\delta = \frac{PN^3}{\frac{1}{12}rbd^3} = \frac{PN^3}{rI}$$

The moment of inertia for any cross-section is equal to the sum of the products of each particle of the area of the cross-section, into the square of its distance from the neutral axis.* For example: in a beam with a cross-section of the I form, a horizontal line drawn through the centre of area of the cross-section will be the neutral line for strains within

* Rankine's Applied Mechanics, Art. 573.

the limits of clasticity. Let the area be divided into a large number of small areas. Then, for the portion of the figure above the neutral line, multiply each of these small areas by the square of its vertical distance above the neutral line, and the sum of these products will equal the moment of inertia for the upper half of the section. A like process will give the moment of inertia for the lower half. The two in this case will be equal, and their sum is the moment of inertia for the whole section. The result thus obtained will not be exact, but will approach accuracy in proportion to the smallness of the parts into which the area of the cross-section is divided.

458.—Example A.—As an illustration, let $ABCD$, in *Fig.* 69, represent the cross-section of a beam; MN, drawn through the middle of the height AD, being the neutral axis; and let the lines EF, GH, IJ, KL, OP, QR, and ST divide the area $ABMN$ into twenty equal parts. The four squares in each horizontal row are equally distant from the neutral line MN, and may therefore be taken together. Suppose each of these squares to measure 2×2 inches, then the area of each will be 4 inches, and of the four in each horizontal row will be $4 \times 4 = 16$ inches area. The distances from the neutral line to the centre of each square will be as follows:

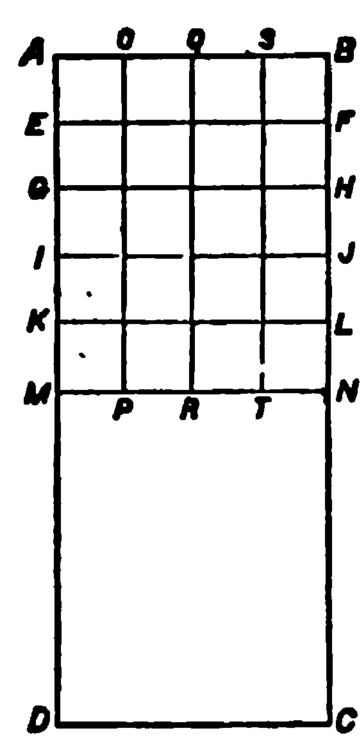

FIG. 69.

In the first row, $D = 1$
" " second " $D = 3$
" " third " $D = 5$
" " fourth " $D = 7$
" " fifth " $D = 9$

Their moment of inertia will be as follows:

In the first row,	$I_1 = 16 \times 1^2 = 16 \times 1$
" " second "	$I_2 = 16 \times 3^2 = 16 \times 9$
" " third "	$I_3 = 16 \times 5^2 = 16 \times 25$
" " fourth "	$I_4 = 16 \times 7^2 = 16 \times 49$
" " fifth "	$I_5 = 16 \times 9^2 = 16 \times 81$

and their sum $I = 16(1+9+25+49+81) = 16 \times 165 = 2640.$

459.—Example B.—If we subdivide each of the squares in *Fig.* 69, and take the sum of the products as before, the result will be larger and nearer the truth. For example: divide each of the squares into four equal parts, each one inch square. There will be eight of these parts in a row, and ten rows. The area of each row will be $8 \times 1 = 8$, and their distances from the neutral line will be $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, $\frac{11}{2}$, $\frac{13}{2}$, $\frac{15}{2}$, $\frac{17}{2}$ and $\frac{19}{2}$ respectively. The moments of inertia will be as follows:

In the first row,	$I_1 = 8 \times (\frac{1}{2})^2 = 8 \times \frac{1}{4} = 2 \times 1$
" second "	$I_2 = 8 \times (\frac{3}{2})^2 = 8 \times \frac{9}{4} = 2 \times 9$
" third "	$I_3 = 8 \times (\frac{5}{2})^2 = 8 \times \frac{25}{4} = 2 \times 25$
" fourth "	$I_4 = 8 \times (\frac{7}{2})^2 = 8 \times \frac{49}{4} = 2 \times 49$
" fifth "	$I_5 = 8 \times (\frac{9}{2})^2 = 8 \times \frac{81}{4} = 2 \times 81$
" sixth "	$I_6 = 8 \times (\frac{11}{2})^2 = 8 \times \frac{121}{4} = 2 \times 121$
" seventh "	$I_7 = 8 \times (\frac{13}{2})^2 = 8 \times \frac{169}{4} = 2 \times 169$
" eighth "	$I_8 = 8 \times (\frac{15}{2})^2 = 8 \times \frac{225}{4} = 2 \times 225$
" ninth "	$I_9 = 8 \times (\frac{17}{2})^2 = 8 \times \frac{289}{4} = 2 \times 289$
" tenth "	$I_{10} = 8 \times (\frac{19}{2})^2 = 8 \times \frac{361}{4} = 2 \times 361$

which is equal to twice the sum of the series of

$$1+9+25+ \text{etc.}$$

or,

$$I = 2 \times 1330 = 2660$$

This result exceeds in amount the previous one (2640).

460.—Example C.—If the eighty squares of this last trial be each subdivided into four equal parts, the whole cross-section will contain $4 \times 80 = 320$ parts; there will be twenty rows, with sixteen in each row; the area of each part will be $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$; and the perpendicular distance from the neutral line to the centres of these 320 parts will be:

In the first	row,	$\frac{1}{4}$
" second	"	$\frac{3}{4}$
" third	"	$\frac{5}{4}$
" fourth	"	$\frac{7}{4}$

and so on, each distance being a fraction having 4 for a denominator, and for a numerator one of the arithmetical series of the odd numbers 1, 3, 5, 7, 9, 11, etc., to 39. The moment of inertia will be the sum of the products, as follows:

In the first row, $I_1 = 16 \times \frac{1}{4} \times (\frac{1}{4})^2$
" second " $I_2 = 16 \times \frac{1}{4} \times (\frac{3}{4})^2$
" third " $I_3 = 16 \times \frac{1}{4} \times (\frac{5}{4})^2$ etc.

These are equal to:

In the first row, $I_1 = 16 \times \frac{1}{4} \times \frac{1}{16} \times 1^2 = \frac{1}{4} \times 1^2$
" second " $I_2 = 16 \times \frac{1}{4} \times \frac{1}{16} \times 3^2 = \frac{1}{4} \times 3^2$
" third " $I_3 = 16 \times \frac{1}{4} \times \frac{1}{16} \times 5^2 = \frac{1}{4} \times 5^2$ etc.

Thus the sum of all the products will be equal to a quarter of the sum of the squares of the arithmetical series of the odd numbers 1, 3, 5, 7, 9, 11, etc., to 39.

Selecting the squares of these numbers from a table of squares, we find their sum to equal 10,660, and then, as above,

$$I = \frac{1}{4} \times 10660 = 2665$$

461.—Comparison of Results.—We have now the three results, 2640, 2660, and 2665, gradually increasing as the number of parts into which the sectional area is divided increases, and tending towards the true amount, to which it can only arrive when the parts become infinitely small and infinite in number. To compute these by the arithmetical method would be impossible, but by the calculus it is exceedingly simple and direct. The formula for the moment of inertia, as generally used, is complicated, but for a rectangular section in a horizontal beam, subject to limited vertical pressure, is simple.

462.—Moment of Inertia, by the Calculus—Preliminary Statement.—Let $ABCD$ in *Fig.* 70 represent the rectangular cross-section of a beam; let MN be the neutral line, and the two lines at EF be drawn parallel to MN. Let the breadth of the section EF equal y, and the perpendicular distance from the neutral line to the lower line EF equal x. The two parallel lines at EF may be taken at any distance, x, from the neutral line. This distance is variable; x is a variable representing any and every distance possible on the line GH, from zero to its full length. It is always the distance from the line MN to y, the lower line at EF, wherever y be taken. The vertical distance between the two lines at EF is termed dx, which means the differential of x, or the difference in the length of x when slightly increased by the movement of y farther from MN. This augmentation, dx, is taken infinitesimally small.

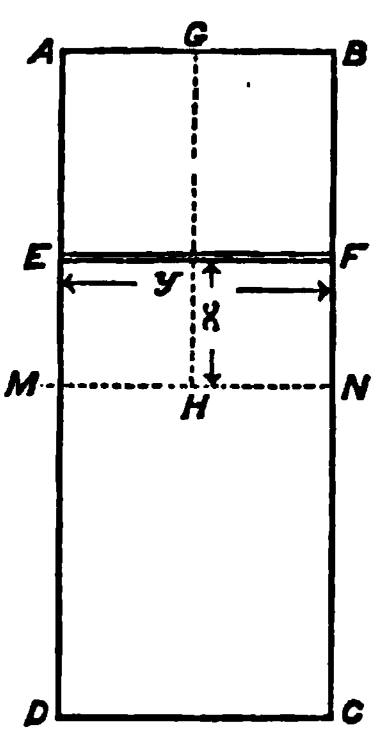

FIG. 70.

Now the area of the space between the two lines at EF will be the product of its length by its height, or $y \times dx$.

463.—Moment of Inertia, by the Calculus.—The moment of inertia is equal (*Art.* **457**) to the sum of the products of each particle of the area of the cross-section, into the square of its distance from the neutral axis. In the last article, the expression ydx represents the area of the infinitesimally small space at the lines *EF*, *Fig.* 70. The distance from this small area to the neutral axis is x, and the square of the distance is x^2; therefore x^2ydx equals the area into the square of its distance, equals the moment of inertia of the small area ydx; or, the *differential* of the moment of the area of the whole figure *ABMN*. This differential is expressed thus,

$$dI = x^2ydx \qquad (203.)$$

This expression represents the moment of only one of the infinitesimal parts into which the area *ABMN* is supposed to be divided. To obtain the moment of the whole area, it is requisite to add together the moments of all the infinitesimal parts; or, to obtain from the differential (*form.* *203.*) its integral. The rule for this is,* "Add one to the index of the variable, and divide by the index thus increased and by the differential of the variable." Applying this rule to formula (*203.*) it becomes

$$I = \tfrac{1}{3}yx^3$$

This is in its general form. To make it definite, we have $y = b$, the breadth; and x, at its maximum, equals $\tfrac{1}{2}d$, half the depth. These values substituted for y and x, we have

$$\tfrac{1}{3}yx^3 = \tfrac{1}{3}b\,(\tfrac{1}{2}d)^3$$

$$I = \tfrac{1}{24}bd^3 \qquad (204.)$$

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* Ritchie, Dif. and Integ. Calculus, p. 21.

This result is the moment of inertia for the upper half of the section of the beam, and represents the resistance to compression. The resistance to tension in the lower half of the beam is (under the circumstances of the case we are considering) an equal amount; hence for the two we have*

$$I = 2 \times \tfrac{1}{24} bd^3$$

$$I = \tfrac{1}{12} bd^3 \qquad (205.)$$

464.—Application and Comparison.—This formula gives the value of the moment of inertia for the whole section; for the two parts, one above and the other below the neutral line. To obtain the value of the part above the line, for comparison with the results obtained in *Arts.* **458** to **460**, we take formula (*204.*)

$$I = \tfrac{1}{24} bd^3$$

in which b is the breadth and d the depth of the beam. The section of beam given in *Art.* **458**, *Fig.* 69, was proposed to be 8 inches broad and 20 inches high, or $AB = b = 8$ and $AD = d = 20$. With these figures in the formula, we have

$$I = \tfrac{1}{24} \times 8 \times 20^3 = 2666\tfrac{2}{3}$$

This is the *exact* amount. In the three trials of *Arts.* **458** to **460**, we had the approximate values 2640, 2660 and 2665. In the last trial, in which the parts were small and numerous, the result was a close approximation.

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* Moseley, Am. Ed. by Mahan, Art. 362.

465.—Moment of Inertia Graphically Represented.—The two processes, arithmetical and by the calculus, are graphically represented in *Fig.* 71, in which the area of the figure contained within the straight lines *OB* and *AB* and the curved line *OA*, is the correct area by the calculus, to which the sum of the squares of the arithmetical progression 1, 3, 5, 7, 9 and 11 closely approximates. Here x and y, indicating the distances along the axes *OX* and *OY*, are co-ordinates to points in the curve, as *A*, *C*, *D*, *E*, etc., these points being midway in the difference between the sides of each two contiguous squares. The values of y for these points are 2, 4, 6, 8, 10 and 12; a difference between each two consecutive values equal to 2. The consecutive ordinates x are 1, 4, 9, 16, 25 and 36; or 1^2, 2^2, 3^2, 4^2, 5^2 and 6^2.

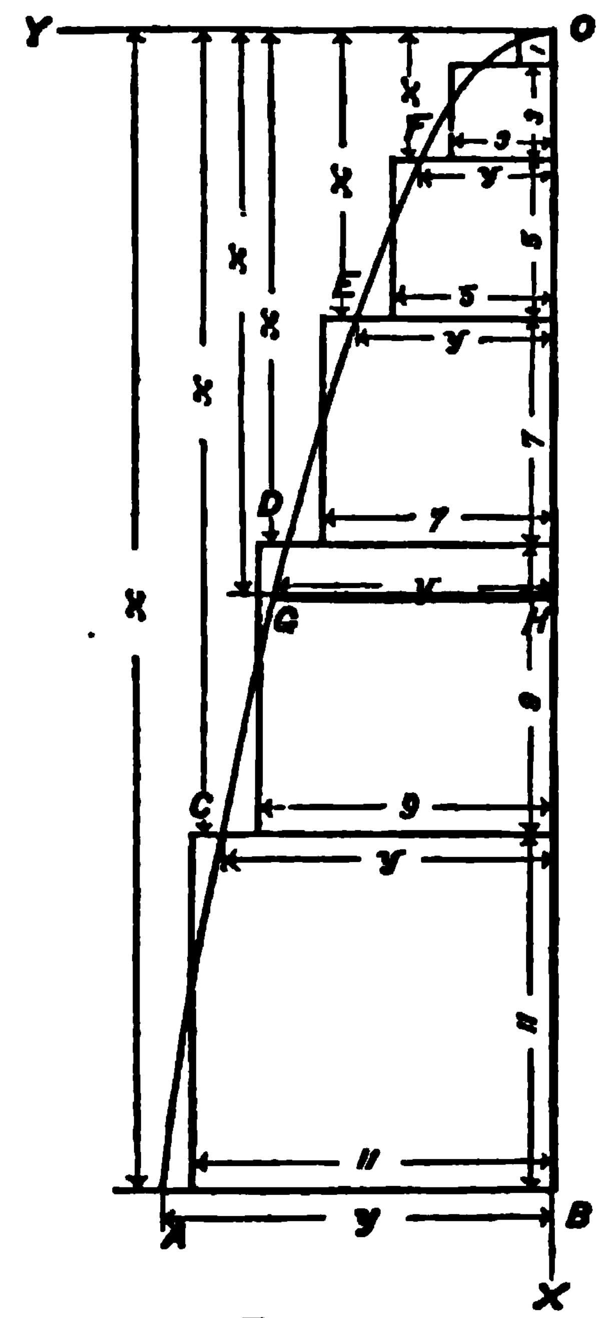

FIG. 71.

Comparing these values ot y and x in each pair, we have

In the first pair,	$y = 2$	and	$x = 1 = 1^2$
" " second "	$y = 4$	"	$x = 4 = 2^2$
" " third "	$y = 6$	"	$x = 9 = 3^2$
" " fourth "	$y = 8$	"	$x = 16 = 4^2$
" " fifth "	$y = 10$	"	$x = 25 = 5^2$
" " sixth "	$y = 12$	"	$x = 36 = 6^2$

From this, the relation between y and x is readily seen to be represented by the following expressions:

$$\left(\frac{y}{2}\right)^2 = x = \tfrac{1}{4}y^2$$

$$y^2 = 4x \qquad (206.)$$

466. — Parabolic Curve — Area of Figure.—The expression just obtained is the equation to the curve, and this curve is a parabola, with $p = 2$, or $y^2 = 2px$.* By formula (*206.*) any number of points in the curve may be found, and the curve itself drawn through them. Also, by it and by the rules of the calculus, the area of the figure inclosed between the curved line and the two lines AB and BO may be found. To this end, let the narrow space included between the two lines GH, drawn perpendicular to OB from H to G (a point in the curve), be a small portion of the area of the whole figure; dx, the distance between the two lines, being exceedingly small. The area of this narrow space will be the product of its length by its breadth, or $y \times dx$. The differential of formula (*206.*), the equation to the curve,† is

$$2ydy = 4dx$$

$$\tfrac{1}{2}ydy = dx$$

Multiplying both sides by y gives

$$\tfrac{1}{2}y^2dy = ydx$$

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* Robinson's Conic Sections and Analytical Geometry, 1863, p. 50.

† Ritchie's Dif. and Integ. Calculus, p. 20.

which equals the differential of the area as above shown. The integral of this value of ydx is, by the rule (*Art.* **463**),

$$\tfrac{1}{2}\int y^2 dy = \tfrac{1}{6}y^3$$

or the area

$$A = \tfrac{1}{6}y^3. \qquad (207.)$$

This is the area of the figure bounded by the curved line OA and the straight lines AB and BO.

467.—Example.—The example given in *Art.* **460** may be taken to show an application of the last formula. The number of horizontal rows of parts into which the area is there divided is 20, and the last number of the arithmetical series is 39. By an examination of *Fig.* 71, it will be seen that AB, the base of the figure, is equal to the side of the last square plus unity. Therefore, $39 + 1 = 40$ is the base of the area proposed in *Art.* **460**, or $y = 40$. From the discussion in that article, it appears that the small squares considered are each $\frac{1}{4}$ of unity in area, from which the area of the figure in that case is found to be one quarter of the sum of the squares of the arithmetical series; or, by formula (*207.*),

$$A = \tfrac{1}{4} \times \tfrac{1}{6}y^3 = \tfrac{1}{24}y^3$$

To apply this result to the present case, where $y = 40$, we have

$$A = \tfrac{1}{24} \times 40^3 = 2666\tfrac{2}{3}$$

the same result as in *Art.* **464**.

468.—Moment of Inertia—General Rule.—That formula (*207.*) may be general in its application, we need to find a proper coefficient.

Let the beam, instead of being 8 inches wide, as in *Fig.* 69, be only one inch wide, and let the portion above the neutral line be divided by horizontal lines into any number of equal parts. Put n for the number of parts, and t for the thickness of each part. The area of each part will be $1 \times t = t$ inches, and the several distances from the neutral line to the centre of each part will be, respectively, $\frac{1}{2}t$, $\frac{3}{2}t$, $\frac{5}{2}t$, $\frac{7}{2}t$, etc., to the last, which will be $\frac{2n-1}{2}t$.

Now, the moment of inertia of each part being its area into the square of the distance to its centre of gravity, therefore the several moments will be as follows:

In the first piece,	$t\left(1 \times \frac{t}{2}\right)^2 = 1^2 \times \frac{1}{4}t^3$
" " second "	$t\left(3 \times \frac{t}{2}\right)^2 = 3^2 \times \frac{1}{4}t^3$
" " third "	$t\left(5 \times \frac{t}{2}\right)^2 = 5^2 \times \frac{1}{4}t^3$
" " fourth "	$t\left(7 \times \frac{t}{2}\right)^2 = 7^2 \times \frac{1}{4}t^3$
" " last "	$t\left((2n-1)\frac{t}{2}\right)^2 = (2n-1)^2 \times \frac{1}{4}t^3$

The sum of these will be

$$S = \frac{1}{4}t^3 \left[1^2 + 3^2 + 5^2 + 7^2 + \cdots\cdots (2n-1)^2\right] \quad (208.)$$

But the sum of the series $1^2 + 3^2 + 5^2 +$ etc., is the area of the parabolic figure (*Fig.* 71), and has been found to be equal to $\frac{1}{3}y^3$ (*form. 207.*)

Now y, when at its maximum, coincides with the base AB of *Fig.* 71, and is equal to the side of the last square plus unity. As above, the side of the last square is $2n - 1$, from which $y = 2n$, and

$$\tfrac{1}{6}y^3 = \tfrac{1}{6}2^3n^3 = \tfrac{8}{6}n^3$$

and therefore formula (*208.*) becomes

$$S = \tfrac{1}{4}t^3 \times \tfrac{8}{6}n^3$$

$$S = \tfrac{1}{3}t^3n^3 \qquad (209.)$$

which is a rule for ascertaining correctly the moment of inertia for a beam one inch broad.

469.—Application.—To show the application of the above, take the example of *Art.* **458**, where the number of slices is 5 and the thickness is 2, and we have, by the use of formula (*209.*),

$$S = \tfrac{1}{3} \times 2^3 \times 5^3 = 333\tfrac{1}{3}$$

The formula gives the result for a beam one inch broad. The beam in *Art.* **458** is 8 inches broad. Therefore, for the full amount we have

$$8 \times 333\tfrac{1}{3} = 2666\tfrac{2}{3}$$

Again, take the example of *Art.* **460**, where $t = \frac{1}{2}$ and $n = 20$, and we find as the result

$$S = \tfrac{1}{3} \times (\tfrac{1}{2})^3 \times 20^3 = 333\tfrac{1}{3}$$

and

$$8 \times 333\tfrac{1}{3} = 2666\tfrac{2}{3}$$

Thus in both cases we have the same result as that obtained directly by the calculus. If b, for the breadth, be added to formula (*209.*) we shall have the complete rule, thus:

$$I = \tfrac{1}{3}bt^3n^3$$

and since tn equals the height above the neutral line, equals $\frac{d}{2}$, the half of the depth of the beam,

$$t^2n^2 = (tn)^2 = (\tfrac{1}{2}d)^2 = \tfrac{1}{8}d^2$$

and this value of t^2n^2 substituted for it in the above equation, gives

$$I = \tfrac{1}{3}b \times \tfrac{1}{8}d^3$$

$$I = \tfrac{1}{24}bd^3$$

This is for one half the beam. For the whole beam we have twice this amount, or

$$I = \tfrac{1}{12}bd^3$$

the same as found directly by the calculus in formula (*205.*)

470. — Rolled-Iron Beam — Moment of Inertia — Top Flange.—An expression for the moment of inertia appropriate to rolled-iron beams of the I form of section (*Fig.* 68) may be obtained directly from the formula (*205.*) for the rectangular section. In *Fig.* 72, showing the cross-section required, b equals the breadth of the beam, or the width of the top and bottom flanges, and t equals the width or thickness of the web; b minus t equals $b_{,}$, d equals the entire height of the section, and $d_{,}$ the height between the flanges. MN is the neutral line drawn at half height.

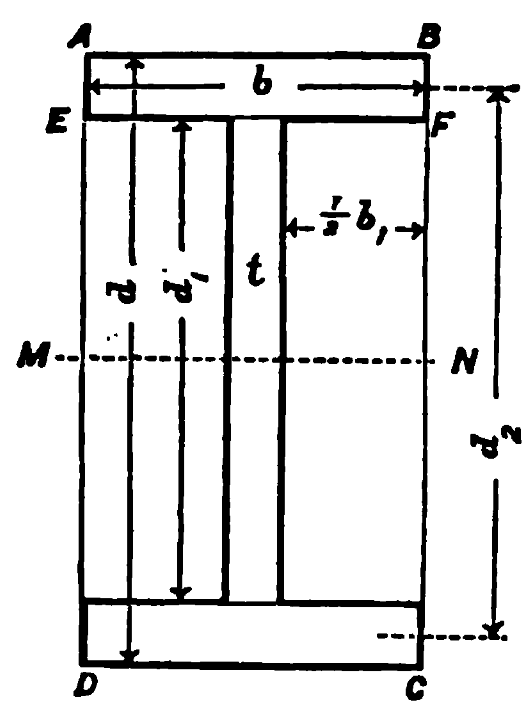

FIG. 72.

By formulas (*203.*) and (*204.*) the moment of inertia for the part above the neutral axis is

$$I = \int yx^2dx$$

If this be applied so that $x = \frac{1}{2}d$, the result ($\frac{1}{24}bd^3$), as in (*204.*), is the moment for the rectangle *ABMN*. Again, if it be applied with $x = \frac{1}{2}d_{,,}$ the result ($\frac{1}{24}bd_{,}^3$) will be the moment for the rectangle *EFMN*. Now, if the latter result be subtracted from the former, the remainder will be the moment for the area *ABEF*, the upper flange, or

$$I_f = \tfrac{1}{24}bd^3 - \tfrac{1}{24}bd_{,}^3$$

$$I_f = \tfrac{1}{24}b(d^3 - d_{,}^3) \qquad (210.)$$

471.—Rolled-Iron Beam—Moment of Inertia—Web.— Formula (*210.*) is the moment of inertia for the top flange. The moment of inertia for the upper half of the web is that due to a rectangle having for its breadth $y = t$, and for its height $x = \frac{1}{2}d_{,,}$ and by *Art.* **463**,

$$I = \tfrac{1}{3}yx^3 = \tfrac{1}{3}t(\tfrac{1}{2}d_{,})^3$$

$$I_w = \tfrac{1}{24}td_{,}^3$$

and since $t = b - b_{,,}$ therefore

$$I_w = \tfrac{1}{24}(b - b_{,})d_{,}^3 \qquad (211.)$$

472.—Rolled-Iron Beam—Moment of Inertia—Flange and Web.—Formula (*211.*) is the moment of inertia for the upper half of the web. Added to formula (*210.*), the sum, representing the moment of inertia for all of the beam above the neutral line, will be

$$I_s = \tfrac{1}{24}b(d^3 - d_{,}^3) + \tfrac{1}{24}(b - b_{,})d_{,}^3$$

$$I_s = \tfrac{1}{24}(bd^3 - bd_{,}^3 + bd_{,}^3 - b_{,}d_{,}^3)$$

$$I_s = \tfrac{1}{24}(bd^3 - b_{,}d_{,}^3) \qquad (212.)$$

473.—Rolled-Iron Beam—Moment of Inertia—Whole Section.—Formula (*212.*) is the moment of inertia for that half of the rolled-iron beam which is located above the neutral line. The moment for the portion below the line will be equal in amount; and therefore, for the moment of the entire section, we have twice the amount of formula (*212.*) or

$$I = \tfrac{1}{12}(bd^3 - b_, d_,^3) \qquad (213.)$$

474.—Rolled-Iron Beam—Moment of Inertia—Comparison with other Formulas.—Formula (*213.*) is the same as that given by Professor Rankine* and others, and is in general use. Canon Moseley† gives an expression which is complicated. Mr. Edwin Clark, in his valuable work on the Britannia and Conway Tubular Bridges, Vol. I., p. 247, gives the formula

$$I = \tfrac{1}{12}d_2^2(6a + a_,) \qquad (214.)$$

in which d_2 is the distance between the centres of gravity of the top and bottom flanges, a is the area of the top or bottom flange, and $a_,$ is the area of the web. This is more simple than the common formula (*213.*), but is not exact. It is only an approximation. Its relation to the true formula will now be shown.

From formula (*213.*) we have, multiplying by 12,

$$12I = bd^3 - b_, d_,^3$$

Of these symbols we have (*Fig.* 72, putting $h = AE$),

$$d = d_2 + h, \quad b_, = b - t \quad \text{and} \quad d_, = d_2 - h$$

By substitution, we now have

$$12I = b(d_2 + h)^3 - (b - t)d_,^3$$

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* Rankine's Applied Mechanics, pp. 316 and 317.

† Moseley's Mech. of Eng., Am. Ed. by Mahan, Art. 504.

and since $(b-t)d_{,}^3 = bd_{,}^3 - td_{,}^3 = bd_{,}^3 - a_{,}d_{,}^2$ (putting $a_{,}$ for the area of the web); and since $d_{,} = d_2 - h$, therefore we have

$$12I = b(d_2+h)^3 - [b(d_2-h)^3 - a_{,}d_{,}^2]$$

$$12I = b(d_2+h)^3 - b(d_2-h)^3 + a_{,}d_{,}^2$$

$$12I = b[(d_2+h)^3 - (d_2-h)^3] + a_{,}d_{,}^2$$

Then we have

$$(d_2+h)^3 = d_2^3 + 3d_2^2h + 3d_2h^2 + h^3$$

$$(d_2-h)^3 = d_2^3 - 3d_2^2h + 3d_2h^2 - h^3$$

$$(d_2+h)^3 - (d_2-h)^3 = \quad 0 + 6d_2^2h + 0 + 2h^3$$

Substituting these in the above, we have

$$12I = b(6d_2^2h + 2h^3) + a_{,}d_{,}^2$$

The area of the top flange equals $bh = a$, therefore

$$12I = 6a(d_2^2 + \tfrac{1}{3}h^2) + a_{,}d_{,}^2 \qquad \text{or,}$$

$$I = \tfrac{1}{12}[6a(d_2^2 + \tfrac{1}{3}h^2) + a_{,}d_{,}^2] \qquad (215.)$$

In Mr. Clark's formula, (*214.*), we have

$$I = \tfrac{1}{12}d_2^2(6a + a_{,}) \qquad \text{or,}$$

$$I = \tfrac{1}{12}(6ad_2^2 + a_{,}d_2^2)$$

Comparing this with the reduction of the common formula as just found [*form.* (*215.*)], the difference is readily seen to be, that while in the one the quantities a and $a_{,}$ are each multiplied by the factor d_2^2, in the other the factor for a is $(d_2^2 + \frac{1}{3}h^2)$ and that for $a_{,}$ is $d_{,}^2$.

475.—Rolled-Iron Beam—Moment of Inertia—Comparison of Results.—To show, by an application, the difference in the results obtained by the two formulas (*214.*) and (*215.*), let it be required to find the moment of inertia for a rolled-iron beam 12 inches high and 4 inches broad, and in which the top and bottom flanges are one inch thick, and the web one half inch thick. Here we have $d = 12$, $d_{,} = 10$, $d_{\text{a}} = 11$, $t = \frac{1}{2}$, $b = 4$, $b_{,} = 3\frac{1}{2}$, $a = 4 \times 1 = 4$, $a_{,} = 10 \times \frac{1}{2} = 5$ and $h = 1$; and by formula (*214.*) we have

$$I = \tfrac{1}{12} \times 11^2 \times (\overline{6 \times 4} + 5) = 292\tfrac{5}{12}$$

The value by formula (*215.*) is

$$I = \tfrac{1}{12}\left[\overline{6 \times 4 \times (11^2 + \tfrac{1}{3} \times 1^2)} + (5 \times 10^2)\right] = 284\tfrac{2}{3}$$

The value by the common formula, (*213.*), is

$$I = \tfrac{1}{12}[(4 \times 12^3) - (3 \cdot 5 \times 10^3)] = 284\tfrac{2}{3}$$

Thus we have by either of the two formulas (*213.*) or (*215.*) the exact value, $I = 284\frac{2}{3}$, while by formula (*214.*) the value obtained is $I = 292\frac{5}{12}$.

476.—Rolled-Iron Beam—Moment of Inertia—Remarks.—When, in a rolled-iron beam, the top and bottom flanges are comparatively thin, the difference between $d_{,}$ and d_{a} will be small, and in consequence the value of I as derived by formula (*214.*) will differ but little from the truth. This formula, therefore, for such cases, is a near approximation, and for some purposes may be useful; but formula (*215.*), and that from which it is derived, (*213.*), are exact in their results, and should be used in preference to formula (*214.*) in all important cases.

477.—Reduction of Formula—Load at Middle.—The expression (*213.*), then, is that which is proper for the moment of inertia for rolled-iron beams—namely:

$$I = \tfrac{1}{12}(bd^3 - b_1d_1^3)$$

In *Art.* **303**, formula (*115.*), we have

$$12F = \frac{Wl^3}{I\delta}$$

This is for a beam supported at each end, with the load in pounds at the middle, the length in feet and the other dimensions in inches. F is a constant, which, from an average of experiments (*Art.* **701**) upon rolled-iron beams, has been ascertained to be 62,000. The value of I, the moment of inertia, has been computed, for many of the sizes of beams in use, by formula (*213.*), and will be found in Table XVII.

We have, therefore, from (*115.*)

$$12 \times 62000 = \frac{Wl^3}{I\delta}$$

$$744000 = \frac{Wl^3}{I\delta} \qquad (216.)$$

478.—Rules—Values of W, l, δ and I.—Rule (*216.*) is for a load at the middle of a rolled-iron beam. The values of the several symbols in (*216.*) may be had by transpositions, as follows:

The weight, $W = \dfrac{744000I\delta}{l^3}$ (*217.*)

" length, $l = \sqrt[3]{\dfrac{744000I\delta}{W}}$ (*218.*)

" deflection, $\delta = \dfrac{Wl^3}{744000I}$ (*219.*)

" moment of inertia, $I = \dfrac{Wl^3}{744000\delta}$ (*220.*)

479.—Example—Weight.—Formula (*217.*) is a rule by which to find the weight in pounds which may be carried at the middle of a rolled-iron beam, with a given deflection. As an example: What weight may be carried at the middle of a 9 inch 90 pound beam, 20 feet long between bearings, with a deflection of one inch?

Here we have $\delta = 1$, $l = 20$ and (from Table XVII.) $I = 109 \cdot 117$; and, by the formula,

$$W = \frac{744000 \times 109 \cdot 117 \times 1}{20^3} = 10147 \cdot 881$$

or the weight to be carried equals 10,148 pounds; or say 5 net tons.

480.—Example—Length.—Formula (*218.*) is a rule by which to find the length at which a beam may be used when required to carry at the middle a given load, with a given deflection. For example: To what length may a Buffalo 6 inch 50 pound rolled-iron beam be used, when required to carry 5000 pounds at the middle, with a deflection of $\frac{3}{10}$ of an inch?

Here $I = 29 \cdot 074$ (from Table XVII.), $\delta = 0 \cdot 3$ and $W = 5000$; and, by the formula,

$$l = \sqrt[3]{\frac{744000 \times 29 \cdot 074 \times 0 \cdot 3}{5000}} = 10 \cdot 908$$

or the length may be 10 feet 11 inches.

481.—Example—Deflection.—Formula (*219.*) is a rule for finding the deflection in a rolled-iron beam, when carrying at the middle a given load. As an example: What deflection will be caused in a Phœnix 9 inch 70 pound beam 20 feet long, by a load of 7500 pounds at the middle?

Here $W = 7500$, $l = 20$ and (from Table XVII.) $I = 92 \cdot 207$; and, by the formula,

$$\delta = \frac{7500 \times 20^3}{744000 \times 92 \cdot 207} = 0 \cdot 87461$$

or the deflection will be $\frac{7}{8}$ of an inch.

482.—Example—Moment of Inertia.—In formula (*220.*) we have a rule by which to ascertain the moment of inertia of a rolled-iron beam, laid on two supports, and carrying a load at the middle. To exemplify the rule: Which of the beams in Table XVII. would be proper to carry 10,000 pounds at the middle, with a deflection of one inch; the length between the bearings being twenty feet?

Here $W = 10000$, $l = 20$ and $\delta = 1$, and by the formula,

$$I = \frac{10000 \times 20^3}{744000 \times 1} = 107 \cdot 527$$

or the required moment of inertia is $107 \cdot 527$. The nearest amount to this in Table XVII. is $107 \cdot 793$, pertaining to the Phœnix 9 inch 84 pound beam. This beam, therefore, would be the one required.

483.—Load at Any Point—General Rule.—The rules just given are for cases where the loads are at the middle. Rules for loads at any other place in the length will now be developed.

Formula (*23.*) is

$$4 Wa \frac{mn}{l} = Bbd^2$$

If bd^2 be multiplied by $\frac{12d}{12d}$ its value will not be changed, and there will result

$$bd^2 = \frac{12bd^3}{12d} = \frac{12 \times \frac{1}{12}bd^3}{d} = \frac{12I}{d} \text{ [see } form. \text{ (205.)]}$$

and formula (*23.*) becomes

$$4Wa\frac{mn}{l} = B\frac{12I}{d}$$

By formula (*154.*), in *Art.* **376**,

$$a = \frac{Bl}{Fdr}$$

and as $rl = \delta$, or $r = \frac{\delta}{l}$, therefore

$$a = \frac{Bl}{Fd\frac{\delta}{l}} = \frac{Bl^2}{Fd\delta}$$

For a in the above, substituting this value, we have

$$4W\frac{Bl^2}{Fd\delta} \times \frac{mn}{l} = B\frac{12I}{d}$$

$$4BWlmn = B\frac{12I}{d}Fd\delta$$

$$4Wlmn = 12FI\delta \qquad (221.)$$

484.—Load at Any Point on Rolled-Iron Beams.—The moment of inertia, I, in formula (*221.*) is [*form.* (*205.*)], $I = \frac{1}{12}bd^3$ for a rectangular beam. For a tube, or for a beam of the **I** form, it is, by formula (*213.*),

$$I = \tfrac{1}{12}(bd^3 - b_1d_1^3)$$

If in (*221.*) we substitute for I this value of it, we have

$$4Wlmn = F\delta(bd^3 - b_1d_1^3) \qquad (222.)$$

This is a rule for rolled-iron beams supported at each end and carrying a load at any point in the length, with a given deflection; and in which W is the weight in pounds, m and n the distances from the load to the two supports, and m plus n equals l equals the length; m, n and l all being in feet; δ is the deflection, b and d are the breadth and depth of the beam, $b_{,}$ and $d_{,}$ the breadth and depth of the part which is wanting of the solid bd (*Art.* **470**); δ, b, d, $b_{,}$ and $d_{,}$ all being in inches; and F is the constant for rolled iron (Table XX.).

485.—Load at Any Point on Rolled-Iron Beams of Table XVII.—The value of F is 62,000. If it be substituted for F in (*221.*) we shall have

$$4Wlmn = 12 \times 62000 I\delta$$

$$4Wlmn = 744000 I\delta$$

$$Wlmn = 186000 I\delta$$

$$W = \frac{186000 I\delta}{lmn} \qquad (223.)$$

which is a rule for ascertaining the weight which may be carried, with a given deflection, at any point in the length of any of the rolled-iron beams of Table XVII.

486.—Example.—What weight may be carried on a Paterson $12\frac{1}{4}$ inch 125 pound rolled-iron beam, 25 feet long between bearings, at 10 feet from one of the bearings, with a deflection of $1 \cdot 5$ inches?

Here we have $\delta = 1 \cdot 5$, $m = 10$, $n = 15$, $l = 25$ and $I = 292 \cdot 05$ (from Table XVII.); and hence

$$W = \frac{186000 \times 292 \cdot 05 \times 1\frac{1}{2}}{25 \times 10 \times 15} = 21728 \cdot 52$$

or the weight allowable is, say 21,730 pounds.

487.—Load at End of Rolled-Iron Lever.—In formula (*115.*) we have

$$12F = \frac{Wl^2}{I\delta} \qquad \text{or,}$$

$$Wl^2 = 12FI\delta$$

This expression is for a beam supported at each end and loaded at the middle. In a *lever* the strains will be the same when the weight and length are each just one half those in a beam supported at each end. Hence if for W we take $2P$, and for l take $2n$, P being the weight at the end of a lever and n the length of the lever, we shall have, by substitution in the above,

$$2P \times \overline{2n}^2 = 12FI\delta$$

$$16Pn^2 = 12FI\delta$$

$$16Pn^2 = F\delta(bd^3 - b_1d_1^3) \qquad (224.)$$

and

$$Pn^2 = \tfrac{3}{4}FI\delta \qquad (225.)$$

and further, since $F = 62000$ (Table XX.), therefore

$$Pn^2 = 46500I\delta$$

$$P = \frac{46500I\delta}{n^2} \qquad (226.)$$

which is a rule for ascertaining the weight which may be supported at the free end of a lever, with a given deflection, the lever being made of any one of the rolled-iron beams of Table XVII.

488.—Example.—Let it be required to show the weight which may be sustained at the free end of a Trenton $15\frac{3}{16}$ inch 150 pound rolled-iron beam, firmly imbedded in a wall, and projecting therefrom 20 feet; the deflection not to exceed 2 inches.

Here $I = 528 \cdot 223$ (Table XVII.), $\delta = 2$ and $n = 20$; and by formula (*226.*)

$$P = \frac{46500 \times 528 \cdot 223 \times 2}{20^2} = 6140 \cdot 59$$

or the weight which may be carried is 6140 pounds.

489.—Uniformly Distributed Load on Rolled-Iron Beam.—By formula (*115.*) we have

$$Wl^2 = 12FI\delta$$

This is for a load at the middle of a beam. Let U represent an equally distributed load; then $\frac{5}{8}U$ will have an effect upon the beam equal to the concentrated load W, (*Art.* **340**), and hence, substituting this value,

$$\tfrac{5}{8}Ul^2 = 12FI\delta$$

$$\tfrac{5}{8}Ul^2 = F\delta(bd^3 - b_1d_1^3) \qquad (227.)$$

By Table XX. $F = 62000$, and the formula reduces to

$$Ul^2 = 1190400I\delta$$

$$U = \frac{1190400I\delta}{l^2} \qquad (228.)$$

which is a rule for ascertaining the amount of weight, equally distributed, which, with a given deflection, may be borne upon any of the rolled-iron beams of Table XVII.

490.—Example.—What weight, uniformly distributed, may be sustained upon a Buffalo $10\frac{1}{2}$ inch 105 pound rolled-iron beam, 25 feet long between bearings, with a deflection of $\frac{3}{4}$ of an inch?

Here $I = 175 \cdot 645$ (Table XVII.), $\delta = 0 \cdot 75$ and $l = 25$; and therefore, by (*228.*),

$$U = \frac{1190400 \times 175 \cdot 645 \times 0 \cdot 75}{25^2} = 10036 \cdot 21$$

or the weight uniformly distributed is 10,036 pounds.

491.—Uniformly Distributed Load on Rolled-Iron Lever.—A rule for a lever loaded at the free end is given in formula (*225.*),

$$Pn^2 = \tfrac{1}{4} FI\delta$$

When a load concentrated at the free end of a lever is equal to $\frac{3}{8}$ of a load uniformly distributed over the length of the lever, the effects are equal. (*Art.* **347.**)*

If U equals the load equally distributed, and P the load concentrated at the free end, then $\frac{3}{8}U = P$, and substituting this value for P in formula (*225.*) gives

$$\tfrac{3}{8} Un^2 = \tfrac{1}{4} FI\delta$$
$$6 Un^2 = 12 FI\delta$$
$$6 Un^2 = F\delta (bd^3 - b_1 d_1^3) \qquad (229.)$$

Putting for F its value 62,000, and reducing, we have

$$Un^2 = 124000 I\delta$$

$$U = \frac{124000 I\delta}{n^2} \qquad (230.)$$

which is a rule for ascertaining the load, uniformly distributed, which may be sustained upon any of the rolled-iron beams of Table XVII., with a given deflection, when used as a lever.

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* Rankine, Applied Mechanics, p. 329.

492.—Example.—What weight, uniformly distributed, may be sustained upon a Trenton 6 inch 40 pound rolled-iron beam, used as a lever, and projecting 10 feet from a wall in which it is firmly imbedded; the deflection not exceeding $\frac{3}{8}$ of an inch?

Here $I = 23 \cdot 761$, $\delta = \frac{3}{8}$ and $n = 10$; and by (*230.*)

$$U = \frac{124000 \times 23 \cdot 761 \times \frac{3}{8}}{10^3} = 1964 \cdot 24$$

or the weight will be 1965 pounds.

493.—Components of Load on Floor.—When rolled-iron beams are used as floor beams, they have to sustain a compound load. This load may be considered as composed of three parts, namely:

First: The superincumbent load, or load proper;

Second: The weights of the materials within the spaces between the beams, and of the covering; and,

Third: The weight of the beams themselves.

494.—The Superincumbent Load.—This will be in proportion to the use to which the floor is to be subjected. If for the storage of merchandise, the weight will vary according to the weight of the particular merchandise intended to be stored. *Warehouses* are sometimes loaded heavily, and for these each case needs special computation. For general purposes, such as our first-class stores are intended for, the load may be taken at 250 pounds per superficial foot (*Art.* **368**). A portion of the floor may in some cases be loaded heavier than this, but as there is always a considerable part kept free for passage ways, 250 pounds per foot will in general be found ample to cover the heavier loads on floors of this class.

On the floors of assembly rooms, banks, insurance offices, dwellings, and of all buildings in which the floors are likely to be covered with people, the weight may be taken at 66, or say 70 pounds per foot; 66 pounds being the weight of a crowd of people (*Art.* **114**).

495.—The Materials of Construction—Their Weight.—These (not including the iron beam) will differ in accordance with the plan of construction. As usually made, with brick arches, concrete filling, and wooden floor laid on strips bedded in the concrete, this weight will not differ much from 70 pounds per superficial foot, and, in general, it may be taken at this amount.

496.—The Rolled-Iron Beam—Its Weight.—The difference in the weight of rolled-iron beams is too great to permit the use in the rule of a definite amount, taken as an average. To represent this weight, therefore, we shall have to make use of a symbolic expression.

Let y equal the weight of the beam in pounds per lineal yard, and c equal the distance in feet between the centres of two adjacent beams. Then $\frac{1}{3}y$ will equal the weight of the beam per lineal foot; and this divided by c will give, as a quotient,

$$m = \frac{y}{3c} \qquad (231.)$$

equals the weight of beam per superficial foot of the floor.

497.—Total Load on Floors.—Putting together the three weights, as above, we have the total weight per superficial foot as follows:

For the floors of dwellings, assembly rooms, banks, etc.,

the superincumbent load is	70 pounds;
the brick arches, concrete, etc., equal	70 "
and the rolled-iron beams equal	$\frac{y}{3c}$ "

These amount in all to

$$f = 140 + \frac{y}{3c} \qquad (232.)$$

For the floors of first-class stores,

the superincumbent load is	250 pounds;
the brick arches, concrete, etc., equal	70 "
and the rolled-iron beams equal	$\frac{y}{3c}$ "

or, in all,

$$f = 320 + \frac{y}{3c} \qquad (233.)$$

498.—Floor Beams—Distance from Centres.—In formula (*228.*) U stands for the weight uniformly distributed over the length of the beam. When f is taken to represent the total load in pounds per superficial foot of the floor, c the distance apart in feet between the centres of two adjacent beams, and l the length of the beam in feet, then

$$U = fcl$$

Substituting for U in formula (*228.*) its value as here shown, we have

$$fcl = \frac{1190400 I\delta}{l^3} \qquad (234.)$$

When r represents the rate of deflection per foot lineal of the beam, we have $rl = \delta$, equals the whole deflection. Substituting for δ in formula (*234.*) this equivalent value we have

$$fcl = \frac{1190400Irl}{l^3}$$

$$fc = \frac{1190400Ir}{l^2} \qquad (235.)$$

Again; for f substituting its value as in (*232.*), we have

$$\left(140 + \frac{y}{3c}\right)c = \frac{1190400Ir}{l^2}$$

$$140c + \frac{y}{3} = \frac{1190400Ir}{l^2}$$

$$140c = \frac{1190400Ir}{l^2} - \frac{y}{3}$$

$$c = \frac{8502\frac{6}{7}Ir}{l^2} - \frac{y}{420} \qquad (236.)$$

which is a rule for ascertaining the distance apart from centres between rolled-iron beams, in the floors of assembly rooms, banks, etc., with a given rate of deflection.

499.—Example.—It is required to show at what distance from centres, Paterson $10\frac{1}{2}$ inch 105 pound rolled-iron beams, 25 feet long, should be placed in the floors of a bank, in which the rate of deflection is fixed at 0·035 of an inch.

Here we have $I = 191{\cdot}04$ (Table XVII.), $r = 0{\cdot}035$, $l = 25$ and $y = 105$; and by (*236.*)

$$c = \frac{8502\frac{6}{7} \times 191{\cdot}04 \times 0{\cdot}035}{25^2} - \frac{105}{420} = 3{\cdot}39$$

or the distance from centres should be, say 3 feet 4⅜ inches.

500.—Floor Beams—Distance from Centres—Dwellings, etc.—If the rate of deflection be fixed, and at 0·03 (*Art.* 314), then formula (*236.*), so modified, becomes

$$c = \frac{255 \cdot 0\tfrac{7}{8} \times I}{l^3} - \frac{y}{420} \qquad (237.)$$

which is a rule for ascertaining the distance apart from centres of rolled-iron beams, in the floors of assembly rooms, banks, etc., with a rate of deflection fixed at 0·03 of an inch per foot lineal of the beam.

501.—Example.—What distance apart from centres should Buffalo 12½ inch 125 pound rolled-iron beams 25 feet long be placed, in the floor of an assembly room?

Here $I = 286 \cdot 019$ (Table XVII.), $l = 25$ and $y = 125$; and by formula (*237.*)

$$c = \frac{255 \cdot 0\tfrac{7}{8} \times 286 \cdot 019}{25^3} - \frac{125}{420} = 4 \cdot 37$$

or the distance from centres should be 4⅜ feet, or 4 feet 4½ inches.

The distances from centres of various sizes of beams have been computed by formula (*237.*), and the results are recorded in Table XVIII.

502.—Floor Beams—Distance from Centres.—If in formula (*235.*) we substitute for f' its value in (*233.*) we shall have

$$\left(320 + \frac{y}{3c}\right)c = \frac{1190400Ir}{l^3}$$

$$320c + \frac{y}{3} = \frac{1190400Ir}{l^3}$$

$$320c = \frac{1190400Ir}{l^3} - \frac{y}{3}$$

$$c = \frac{1190400Ir}{320l^3} - \frac{y}{320 \times 3}$$

$$c = \frac{3720Ir}{l^3} - \frac{y}{960} \qquad (238.)$$

This is a rule for ascertaining the distance apart from centres between rolled-iron beams, in floors of first-class stores, with a given rate of deflection.

503.—Example.—At what distance apart should Phœnix 15 inch 150 pound beams 25 feet long be placed, with a rate of deflection of $r = 0{\cdot}045$?

Here we have $I = 514{\cdot}87$ (Table XVII.), $r = 0{\cdot}045$, $l = 25$ and $y = 150$; and in formula (*238.*)

$$c = \frac{3720 \times 514{\cdot}87 \times 0{\cdot}045}{25^3} - \frac{150}{960} = 5{\cdot}36$$

or the distance required is $5{\cdot}36$ feet, or 5 feet $4\frac{1}{2}$ inches.

504.—Floor Beams—Distance from Centres—First-class Stores.—If the rate of deflection be fixed, and at $0{\cdot}04$ of an inch (*Arts.* **313, 314** and **368**), then formula (*238.*) becomes

$$c = \frac{148{\cdot}8I}{l^3} - \frac{y}{960} \qquad (239.)$$

which is a rule for ascertaining the distance apart from centres of rolled-iron beams, in floors of first-class stores, with a rate of deflection fixed at 0·04 of an inch per foot lineal of the beam.

505.—Example.—At what distance apart should Buffalo $12\frac{1}{4}$ inch 180 pound rolled-iron beams 20 feet long be placed, in a first-class store?

Here $I = 418{\cdot}945$ (Table XVII.), $l = 20$ and $y = 180$; and, by the above formula,

$$c = \frac{148{\cdot}8 \times 418{\cdot}945}{20^3} - \frac{180}{960} = 7{\cdot}60$$

or the distance from centres should be $7{\cdot}6$ feet, or 7 feet $7\frac{1}{4}$ inches nearly.

The distances from centres, as per formula (*239.*), have been computed for rolled-iron beams of various sizes, and the results are recorded in Table XIX.

506.—Floor Arches—General Considerations.—If the spaces between the iron floor beams be filled with brick arches and concrete, as in *Art.* **495**, care is necessary that these arches be constructed with very hard whole brick of good shape, be laid without mortar, in contact with each other, and that the joints be all well filled with best cement grout and be keyed with slate. As to dimensions, the arch when well built need not be over four inches thick for spans of seven or eight feet, except for about a foot at each springing, where it should be eight inches thick, and where care should be taken to form the skew-back quite solid and at right angles to the line of pressure.

In order to economize the height devoted to the floor, it

is desirable to make the versed sine or rise of the arch small. But there is a limit, beyond which a reduction of the rise will cause so great a strain that the material of which the bricks are made will be rendered liable to crushing. Experiments have shown that this limit of rise is not much less than 1¼ inches per foot width of the span, and in practice it is found to be safe to make the rise 1⅛ inches per foot.

507.—Floor Arches—Tie-Rods.—The lateral thrust exerted by the brick arches may be counteracted by tie-rods of iron. The arches, if made with a small rise, will differ but little in form from the parabolic curve. Let *Fig.* 73 represent one half of the arch and tie-rod. Draw the lines AD and DC tangent to the points A and C. Then $AE = EB$*

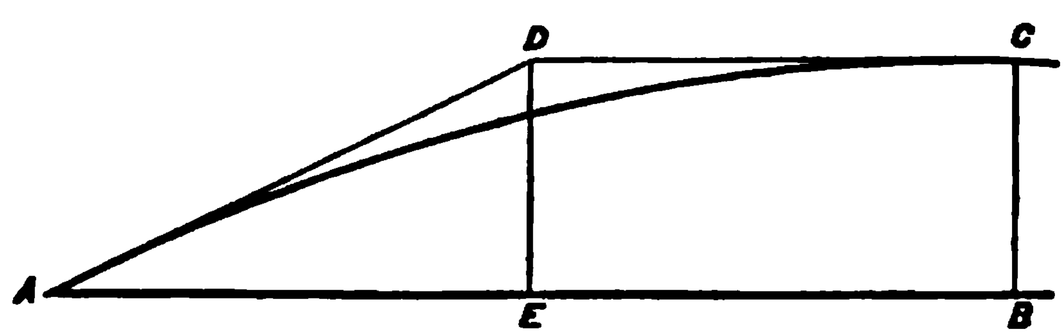

FIG. 73.

equals ¼ of the span, or $\frac{1}{4}s$, and $DE = BC$ equals the versed sine, or height of the arch. If DE, by scale, be equal to the load upon the half arch AC, then AE equals the horizontal strain; or

$$DE : AE :: \tfrac{1}{2}U : H$$

$$v : \tfrac{1}{4}s :: \tfrac{1}{2}U : H$$

$$H = \frac{Us}{8v} \qquad (240.)$$

in which U is the load, in pounds, and s is the span and v the versed sine, both in feet. To resist this strain

* Tredgold's Elementary Principles of Carpentry, Art. 57 and Fig. 28.

the rod must contain the requisite amount of metal. The ultimate tensile strength of wrought-iron may be taken at an average of 55,000 pounds per inch. Owing, however, to defects in material and in workmanship (such, for instance, as an oblique bearing, which, by throwing the strain out of the axis and along one side of the rod, would materially increase the destructive effect of the load), the metal should be trusted with not over 9000 pounds per inch. If a represent the area of the tie-rod in inches, then

$$9000a = H$$

Substituting this value of H in formula (*240.*) we have

$$9000a = \frac{Us}{8v} \qquad (241.)$$

For U we may put its equivalent, which is the load per foot multiplied by the superficial area of the floor sustained by the rod, or

$$U = cfs$$

c being the distance from centres between the rods, and s the span of the arch, both in feet, and f the weight of the brick-work and the superimposed load, in pounds, or $70 + q$. If the arch be made to rise $1\frac{1}{2}$ inches per foot of width, or $\frac{1}{8}$ of the span, then $8v = s$, and formula (*241.*) becomes

$$a = \frac{70 + q}{9000} cs \qquad (242.)$$

Putting q, the superimposed load, at seventy pounds, we have

$$a = \frac{140}{9000} cs$$

$$a = 0 \cdot 01\tfrac{5}{9} \times cs \qquad (243.)$$

which is a rule for the area, in inches, of a tie-rod in a bank, office building, or assembly room floor.

If q be put equal to 250 pounds, then

$$a = \frac{320}{9000} cs$$

$$a = 0 \cdot 03\tfrac{5}{9} \times cs \qquad (244.)$$

which is a rule for the area, in inches, of a tie-rod in the floor of a first-class store.

For general use, the *diameter*, rather than the area, of the tie-rod is desirable. We have as the area of any rod,

$$a = \cdot 7854 d^2$$

and therefore $\cdot 7854 d^2 = 0 \cdot 01\tfrac{5}{9} \times cs$

and $$d = \sqrt{0 \cdot 0198 cs} \qquad (245.)$$

which is a rule for banks, etc.; and

$$d = \sqrt{0 \cdot 04527 cs} \qquad (246.)$$

which is a rule for first-class stores.

508.—Example.—In a first-class store, with beams 20 feet long, and arches 6 feet span: What is the required diameter of tie-rods?

Here $s = 6$, and if there are to be, say two rods in the length of each arch, then $c = 6\tfrac{2}{3}$, and therefore

$$d = \sqrt{0 \cdot 04527 \times 6\tfrac{2}{3} \times 6} = 1 \cdot 35$$

or the required rods are to be $1\tfrac{3}{8}$ inches diameter.

Tie-rods should be placed at or near the bottom flange, and so close together that the horizontal strain between them from the thrust of the arch shall not be greater than the bottom flange of the beam is capable of resisting.

509.—Headers.—In *Art.* **381** we have the expression

$$\tfrac{1}{4}fng^2 = Fbr(d-1)^2$$

a rule for a header of rectangular section. We have also in formula (*205.*)

$$I = \tfrac{1}{12}bd^3$$

or

$$12I = bd^3$$

Substituting this $12I$ for $b(d-1)^2$ in the above equation gives

$$\tfrac{1}{4}fng^2 = 12IFr$$

and

$$I = \frac{fng^2}{48Fr} \qquad (247.)$$

which is a rule for rolled-iron headers; and in which f is the load in pounds per superficial foot, n is the length of the tail beams having one end resting on the header, and g is the length of the header; n and g both being in feet.

510.—Headers for Dwellings, etc.—If in (*247.*) we substitute for f its value as per formula (*232.*), and for F its value 62,000 (Table XX.), and make $r = 0{\cdot}03$ (*Art.* **314**), we shall have

$$I = \frac{\left(140 + \frac{y}{3c}\right)ng^2}{48 \times 62000 \times 0{\cdot}03}$$

$$I = \frac{140 + \frac{y}{3c}}{89280}ng^2 \qquad (248.)$$

which is a rule for ascertaining the moment of inertia of a rolled-iron header, in a floor of an assembly room, bank, etc.; from which an inspection of Table XVII. will show the required header.

511.—Example.—In the floors of a bank, constructed of Buffalo $10\frac{1}{2}$ inch 105 pound beams, placed 4 feet from centres: What ought a header to be which is 20 feet long, and which carries tail beams 16 feet long?

Here $y = 105$, $c = 4$, $n = 16$ and $g = 20$; and by (*248.*)

$$I = \frac{140 + \frac{105}{12}}{89280} \times 16 \times 20^2 = 213 \cdot 262$$

or the beam should be of such size that its moment of inertia be not less than $213 \cdot 262$. By reference to Table XVII. we find the beam, the moment of inertia of which is next greater than this, to be the Trenton $10\frac{1}{2}$ inch 135 pound beam, for which $I = 241 \cdot 478$. This may be taken for the header, although it is stronger than needed. Instead of this one beam, however, we may use two of the Phœnix 9 inch 84 pound beams, bolted together; for of this latter beam $I = 107 \cdot 793$, and

$$2 \times 107 \cdot 793 = 215 \cdot 586$$

only a trifle more than $213 \cdot 262$, the result of the computation by formula (*248.*). But these two beams, although nearer the required strength, yet, when taken together, weigh 168 pounds per yard; while the $10\frac{1}{2}$ inch beam weighs but 135 pounds. On the score of economy, therefore, it is preferable to use the $10\frac{1}{2}$ inch beam.

512.—Headers for First-class Stores.—If, in formula (*247.*), for f, F and r, there be substituted their proper values, namely, $f = 320 + \frac{y}{3c}$ (*form. 233.*), $F = 62000$ and $r = 0 \cdot 04$, as in *Arts.* **367** and **368**, we shall have

$$I = \frac{320 + \frac{y}{3c}}{119040} ng^2 \qquad (249.)$$

which is a rule for rolled-iron headers in the floors of first-class stores.

As this expression is the same as (*248.*), excepting the numerical coefficients, the example of the last article will suffice to illustrate it, by simply substituting the coefficient

$$\frac{320+\frac{y}{3c}}{119040} \text{ in place of } \frac{140+\frac{y}{3c}}{89280}.$$

513.—Carriage Beam with One Header.—Formula (*161.*) is appropriate for a case of this kind, but it is for a beam of rectangular section. To modify it for use in this case, we have (*205.*) $I = \frac{1}{12}bd^3$; or $12I = bd^3$. Substituting for bd^3, in (*161.*), this value, we have

$$fmn(ng+cl) = 12IFr$$

$$I = \frac{fmn}{12Fr}(ng+cl) \qquad (250.)$$

which is a general rule for this case.

514.—Carriage Beam with One Header, for Dwellings, etc.—In formula (*250.*), putting for f its value $140+\frac{y}{3c}$ (*form. 232.*), for F its value 62,000 (Table XX.), and for r its value 0·03 (*Art.* 314), we have

$$I = \frac{\left(140+\frac{y}{3c}\right)mn}{12 \times 62000 \times 0{\cdot}03}(ng+cl)$$

$$I = \frac{\left(140+\frac{y}{3c}\right)mn}{22320}(ng+cl) \qquad (251.)$$

which is a rule for the moment of inertia of a rolled-iron carriage beam, with one header, in floors of assembly rooms, banks, etc. With the moment of inertia found by this rule, the required beam may be selected from Table XVII.

515.—Example.—In a dwelling floor of Paterson 9 inch 70 pound beams, 20 feet long and $2\frac{8}{10}$ feet from centres: Of what size should be a carriage beam which at 5 feet from one end carries a header 17 feet long, with tail beams 15 feet long?

Here $y = 70$, $c = 2 \cdot 8$, $m = 5$, $n = 15$, $g = 17$ and $l = 20$; and by (*251.*) we have

$$I = \frac{\left(140 + \frac{70}{3 \times 2 \cdot 8}\right) \times 5 \times 15}{22320} \times (\overline{15 \times 17} + \overline{2 \cdot 8 \times 20}) = 155 \cdot 012$$

or the moment of inertia required is $155 \cdot 012$.

By reference to Table XVII. we find $I = 154 \cdot 917$ as the moment of inertia of the 9 inch 125 pound Trenton beam, almost exactly the amount called for. If the construction of the floor permit the use of a beam $1\frac{1}{2}$ inches higher, then it would be preferable to use for this carriage beam one of the four $10\frac{1}{2}$ inch beams of the table; as these beams, although stronger than we require, are yet (being 20 pounds lighter) more economical.

516.—Carriage Beam with One Header, for First-class Stores.—If, in formula (*250.*), f be substituted by its value $320 + \frac{y}{3c}$ (*form. 233.*), F by its value 62,000 (Table XX.), and r by $0 \cdot 04$ (*Arts.* **367** and **368**), we shall have

$$I = \frac{\left(320 + \frac{y}{3c}\right) mn}{12 \times 62000 \times 0 \cdot 04}(ng + cl)$$

$$I = \frac{\left(320 + \frac{y}{3c}\right) mn}{29760}(ng + cl) \qquad (252.)$$

which is a rule for the moment of inertia for rolled-iron carriage beams, carrying one header, in first-class stores.

517.—Example.—Of what size, in a first-class store, should be a rolled-iron carriage beam 25 feet long, which carries at 5 feet from one end a header 20 feet long, with tail beams 25 feet in length; the tail beams being Trenton $12\frac{1}{4}$ inch 125 pound beams, placed $2\frac{5}{8}$ feet from centres?

Here $y = 125$, $c = 2\frac{5}{8}$, $m = 5$, $n = 20$, $g = 20$ and $l = 25$; and by formula (*252.*) we have

$$I = \frac{\left(320 + \frac{125}{3 \times 2\frac{5}{8}}\right) \times 5 \times 20}{29760} \times (\overline{20 \times 20} + \overline{2\tfrac{5}{8} \times 25}) = 526 \cdot 294$$

or the moment of inertia required is $526 \cdot 294$.

To supply the strength needed in this case, we may take one of the $10\frac{1}{2}$ inch 135 pound beams, with one of the $12\frac{1}{4}$ inch 125 pound beams; as these two bolted together will give a moment of inertia a trifle more than the computed amount. It will be more economical, however, to take two of the $12\frac{1}{4}$ inch 125 pound beams, since the weight of metal will be less, although the strength will be greater than required.

518.—Carriage Beam with Two Headers and Two Sets of Tail Beams.—Formula (*170*) contains the elements appropriate to this case, but is for beams of rectangular section. It is quite general in its application, although somewhat complicated. A more simple rule is found in formula (*174.*). This is not quite so general in application, but still sufficiently so to use in ordinary cases (see *Art.* **402**). In any event, the result derived from its use, if not accurate, deviates so slightly from accuracy that it may be safely taken. We will take, then, formula (*174.*) and modify it as required

for the present purpose. For bd^3 putting $12I$, its value (*form.* *205.*), we have

$$12IFr = fm\,[cnl + g(mn + s^2)]$$

$$I = \frac{fm}{12Fr}[cnl + g(mn + s^2)] \qquad (253.)$$

which is a general rule for the case above stated (see *Arts.* **153** and **243**).

519.—Carriage Beam with Two Headers and Two Sets of Tail Beams, for Dwellings, etc.—If, in formula (*253.*), $140 + \frac{y}{3c}$ be substituted for f (*form.* *232.*), 62,000 for F (Table XX.), and 0·03 for r (*Art.* **314**), then we have as a result

$$I = \frac{140 + \frac{y}{3c}}{22320} m\,[cnl + g(mn + s^2)] \qquad (254.)$$

which is a rule for the moment of inertia for this case as above stated (see *Arts.* **153** and **243**).

520.—Example.—In a dwelling having a floor of Paterson $10\frac{1}{2}$ inch 105 pound rolled-iron beams, 20 feet long, and placed 5·84 feet from centres: Which of the beams of Table XVII. would be appropriate for a carriage beam to carry two headers 16 feet long, one located 9 feet, and the other 15 feet, both from the same end of the carriage beam? (See *Arts.* **153** and **243**.)

Here the two headers are respectively 9 feet and 5 feet from the walls. The one 9 feet from its wall, being farther away than the other, will create the greater strain,

and therefore $m = 9$, $n = 11$, $r = 15$, $s = 5$, $l = 20$, $c = 5 \cdot 84$ and $y = 105$; and by formula (*254.*) we have

$$I = \frac{140 + \frac{105}{3 \times 5 \cdot 84}}{22320} \times 9 [(5 \cdot 84 \times 11 \times 20) + 16 (\overline{9 \times 11} + 5^2)] = 192 \cdot 428$$

or the required moment of inertia is $192 \cdot 428$. By reference to Table XVII., we find, as the nearest in amount to this, the Trenton or Paterson $10\frac{1}{2}$ inch 105 pound beam, of which $I = 191 \cdot 040$, and which will be the proper beam for this case.

521.—Carriage Beam with Two Headers and Two Sets of Tail Beams, for First-class Stores.—If, in formula (*253.*), there be substituted for F its value 62,000 (Tablè XX.), for f its value $320 + \frac{y}{3c}$ (*form. 233.*), and for r its value $0 \cdot 04$ (*Arts.* **367** and **368**), we shall have

$$I = \frac{320 + \frac{y}{3c}}{29760} m [cnl + g(mn + s^2)] \qquad (255.)$$

which is a rule for the moment of inertia required in this case, as above stated (see *Arts.* **153** and **243**).

522.—Example.—In a store having a floor of Trenton 15 inch 150 pound rolled-iron beams, 25 feet long and $4 \cdot 87$ feet from centres: What ought a carriage beam to be which carries two headers 20 feet long, one located 10 feet from one wall, and the other at 7 feet from the other wall?

Here the distances to the header more remote from its wall are to be called (see *Arts.* **153** and **243**) m and n.

Then $m = 10$, $n = 15$, $r = 18$, $s = 7$, $g = 20$, $l = 25$, $c = 4{\cdot}87$ and $y = 150$; and by formula (*255.*)

$$I = \frac{320 + \dfrac{150}{3 \times 4{\cdot}87}}{29760} \times 10\left[(4{\cdot}87 \times 15 \times 25) + 20\left(\overline{10 \times 15 + 7}^{\,2}\right)\right]$$
$$= 644{\cdot}359$$

or the moment required is $644{\cdot}359$. By an examination of Table XVII., we find that the moment of any one of the four 15 inch 200 pound beams is more than enough for this case, and its use more economical than any combination of other beams affording the requisite strength.

523.—Carriage Beam with Two Headers, Equidistant from Centre, and Two Sets of Tail Beams, for Dwellings, etc.—If for f, F, bd^3 and r, in formula (*183.*), their respective values be substituted, namely, $f = 140 + \dfrac{y}{3c}$ (*form. 232.*), $F = 62000$ (Table XX.), $bd^3 = 12I$ (*form. 205.*), and $r = 0{\cdot}03$ (*Art.* **314**); then formula (*183.*) becomes

$$I = \frac{140 + \dfrac{y}{3c}}{22320}\, l\left(\tfrac{1}{2}cl^2 + gm^2\right) \qquad (256.)$$

which is a rule for a rolled-iron carriage beam, carrying two headers equidistant from the centre, with two sets of tail beams, in assembly rooms, banks, etc.

524.—Example.—In an assembly room, having a floor of Buffalo $10\frac{1}{2}$ inch 105 pound rolled-iron beams, 20 feet long and $5{\cdot}35$ feet from centres: What ought a carriage beam to be which carries two headers 16 feet long, located equidistant from the centre of the width of the floor, with an opening between them 6 feet wide?

Here $c = 5 \cdot 35$, $y = 105$, $l = 20$, $g = 16$ and $m = 7$; therefore by formula (*256.*) we have

$$I = \frac{140 + \frac{105}{3 \times 5 \cdot 35}}{22320} \times 20 \left(\overline{\tfrac{1}{4} \times 5 \cdot 35 \times 20^2} + \overline{16 \times 7^2}\right) = 173 \cdot 198$$

By reference to Table XVII. we find that either of the four $10\frac{1}{2}$ inch 105 pound beams of the table is sufficiently strong to serve for the required carriage beam.

525.—Carriage Beam with Two Headers, Equidistant from Centre, and Two Sets of Tail Beams, for First-class Stores.—In formula (*183.*), if we substitute for f, F, bd^2 and r their respective values, as follows, $f = 320 + \frac{y}{3c}$ (*form.* *233.*), $F = 62000$ (Table XX.), $bd^2 = 12I$ (*form.* *205.*) and $r = 0 \cdot 04$ (*Arts.* **367** and **368**), we shall have

$$I = \frac{320 + \frac{y}{3c}}{29760} \, l \left(\tfrac{1}{4} c l^2 + g m^2\right) \qquad (257.)$$

which is a rule for a rolled-iron carriage beam carrying two headers equidistant from the centre, with two sets of tail beams, in first-class stores.

526.—Example.—In a first-class store, having a floor of Phœnix 15 inch 150 pound beams 25 feet long and $4 \cdot 75$ feet from centres: What ought a carriage beam to be which carries two headers 20 feet long, located equidistant from the centre of the width of the floor, with an opening between them 8 feet wide?

Here we have $y = 150$, $c = 4\cdot75$, $l = 25$, $g = 20$ and $m = 8\frac{1}{2}$; therefore formula (*257.*) becomes

$$I = \frac{320 + \dfrac{150}{3 \times 4\cdot75}}{29760} \times 25(\overline{\tfrac{1}{4} \times 4\cdot75 \times 25^2} + \overline{20 \times 8\tfrac{1}{2}^2}) = 607\cdot294$$

or the moment required is $607\cdot294$. Table XVII. shows that either of the 15 inch 200 pound beams is of sufficient strength to satisfy the requirements of this case.

527.—Carriage Beam with Two Headers and One Set of Tail Beams, for Dwellings, etc.—If, in formula (*179.*), we substitute for the symbols bd^3, f, F and r, their respective values, as follows, $bd^3 = 12I$ (*form.* *205.*), $f = 140 + \frac{y}{3c}$ (*form.* *232.*), $F = 62000$ (Table XX.) and $r' = 0\cdot03$ (*Art.* **314**), we shall have

$$I = \frac{140 + \dfrac{y}{3c}}{22320} m[cnl + gj(n+s)] \qquad (258.)$$

which is a rule for the moment of inertia of a rolled-iron carriage beam, carrying two headers with one set of tail beams, for floors of assembly rooms, banks, etc. (See *Arts.* **153** and **409.**)

528.—Example.—In a bank having a floor of Paterson $10\frac{1}{2}$ inch 105 pound rolled-iron beams, 20 feet long and $5\cdot84$ feet from centres: What ought a carriage beam to be which carries two headers 16 feet long, located one at 5 feet from one wall and the other at 6 feet from the other wall, the tail beams being between them?

Here (*Art.* **157**) m is to be put at the wider opening, hence $m = 6$, $n = 14$, $s = 5$, $l = 20$, $c = 5 \cdot 84$, $g = 16$, $j = l - (m + s) = 20 - 11 = 9$ and $y = 105$; and by formula (*258.*)

$$I = \frac{140 + \frac{105}{3 \times 5 \cdot 84}}{22320} \times 6 \left[\overline{5 \cdot 84 \times 14 \times 20} + \overline{16 \times 9 \times (14 + 5)}\right]$$
$$= 171 \cdot 550$$

or, the moment required is $171 \cdot 550$. Referring to Table XVII. we find that either of the $10\frac{1}{2}$ inch 105 pound beams will be suitable for this case.

529.—Carriage Beam with Two Headers and One Set of Tail Beams, for First-class Stores.—If, in formula (*258*), we substitute (as in *Art.* **525**) $\dfrac{320 + \frac{y}{3c}}{29760}$ for $\dfrac{140 + \frac{y}{3c}}{22320}$ we shall have

$$I = \frac{320 + \frac{y}{3c}}{29760} m \left[cnl + gj(n + s)\right] \qquad (259.)$$

which is a rule for the moment of inertia for a rolled-iron carriage beam, carrying two headers and one set of tail beams, in a first-class store.

530.—Example.—In a first-class store having a floor of Buffalo 15 inch 150 pound beams 25 feet long and $4\frac{1}{2}$ feet from centres: What ought a carriage beam to be which carries two headers 20 feet long, located, one at 5 feet from one wall, and the other at 8 feet from the other wall, with tail beams between them?

Here (*Art.* **157**) $m = 8$, $n = 17$, $s = 5$, $l = 25$, $g = 20$, $j = 25 - (5 + 8) = 12$, $c = 4\frac{1}{2}$ and $y = 150$; and by (*259.*)

$$I = \frac{320 + \dfrac{150}{3 \times 4\frac{1}{2}}}{29760} \times 8 \left[4\tfrac{1}{2} \times 17 \times 25 + 20 \times 12 \times (17 + 5)\right]$$

$$= 640 \cdot 193$$

which is the moment required. Either of the 15 inch 200 pound beams of Table XVII. will serve the present purpose.

531.—Carriage Beam with Three Headers, the Greatest Strain being at Outside Header, for Dwellings, etc.—As in *Fig.* 54, floor beams are sometimes framed with two openings, one for a stairway at the wall, and another for light at or near the middle of the floor. In this arrangement the carriage beams are required to sustain three headers. Formula (*190.*) in *Art.* **425** is appropriate to this case, but is adapted to a beam of rectangular section. Substituting for bd^3 its value $12I$ (*form.* *205.*), for f its value $140 + \frac{y}{3c}$ (*form.* *232.*), for F its value 62,000 (Table XX.), and for r its value 0·03 (*Art.* **314**), we have

$$I = \frac{140 + \dfrac{y}{3c}}{22320} m \left[cnl + g(mn + s^2 - v^2)\right] \qquad (260.)$$

which is a rule for the moment of inertia for a rolled-iron carriage beam carrying three headers, in an assembly room, bank, etc.; the headers placed, as in *Fig.* 54, so that the one causing the greatest strain shall *not* be between the other two. (See *Arts.* **252** to **254**.)

532.—Example.—In an assembly room having a floor of Trenton 9 inch 70 pound beams 20 feet long and 2·80 feet from centres: Of what size should be a carriage beam carrying, as in *Fig.* 54, three headers 15 feet long; two of them located at the sides of an opening 6 feet wide, which is placed at the middle of the width of the floor, and the other header located at 3 feet from one of the side walls?

As two of these headers are equidistant from the centre of the floor, the one carrying the longer tail beams will produce the greater strain upon the carriage beam (*Art.* **253**). The distances from this header, therefore, are to be designated by m and n (*Art.* **244**), while r and s are to represent the distances from the other, and v and u are to be the distances from the third header; the one at the stairway.

Here $m = 7$, $n = 13$, $s = 7$, $v = 3$, $l = 20$, $g = 15$, $c = 2 \cdot 8$ and $y = 70$; and by formula (*260.*) we have

$$I = \frac{140 + \frac{70}{3 \times 2 \cdot 8}}{22320} \times 7 \left[(2 \cdot 8 \times 13 \times 20) + 15 \left(\overline{7 \times 13} + 7^2 - 3^2 \right) \right]$$

$$= 125 \cdot 279$$

which is the required moment. An examination of Table XVII. shows that either of the 9 inch 125 pound beams will be more than sufficient for this case.

533.—Carriage Beam with Three Headers, the Greatest Strain being at Outside Header, for First-class Stores.—Here, with the headers located, as in *Fig.* 54, so that the one causing the greatest strain in the carriage beam shall *not* be between the other two, the rule is the same, with the excep-

tion of the coefficient, as in the case last presented (*form.* *260.*). Substituting therefore, in formula (*260.*), $\frac{320+\frac{y}{3c}}{29760}$ (see *form. 259.*) in place of $\frac{140+\frac{y}{3c}}{22320}$, we shall have

$$I = \frac{320+\frac{y}{3c}}{29760} m\,[cnl+g(mn+s^2-v^2)] \qquad (261.)$$

which is a rule for the moment of inertia required for a rolled-iron carriage beam carrying three headers, in a first-class store; the headers being placed, as in *Fig.* 54, so that the one carrying the greatest strain shall *not* be between the other two. (See *Arts.* **252** to **254**.)

The example given in *Art.* **532** will serve to illustrate this rule, for the two rules are alike except in the coefficient, as above explained.

534.—Carriage Beam with Three Headers, the Greatest Strain being at Middle Header, for Dwellings, etc.—If the headers be located as in *Fig.* 56, so that the header causing the greatest strain in the carriage beam shall be between the other two (*Arts.* **260** and **264**), then we have formula (*194.*) (in *Art.* **432**) appropriate to this case, except that it is for a beam of rectangular section. To modify it to suit our present purpose, we have only to substitute for bd^3, f, F and r, their respective values as in *Art.* **531**, and we have

$$I = \frac{140+\frac{y}{3c}}{22320}[m(cnl+gs^2)+gn(m^2-v^2)] \qquad (262.)$$

as a rule for the moment of inertia required for rolled-iron

carriage beams carrying three headers, in an assembly room, etc.; the headers so located that the one causing the greatest strain shall be between the other two. (See *Art.* **264**.)

535.—Example.—In a bank, having a floor of Phœnix $10\frac{1}{2}$ inch 105 pound rolled-iron beams, 20 feet long and placed $5 \cdot 59$ feet from centres: Of what size ought a carriage beam to be which carries three headers, 16 feet long, placed, as in *Fig.* 54, so that the opening in the floor at the wall shall be 4 feet wide, and the other opening 5 feet wide, and distant 6 feet from the other wall?

The middle header in this case being the one which causes the greatest strain in the carriage beam, the distances from it to the two walls are to be called m and n. (See *Arts.* **244** and **253**.) The header carrying the tail beams, one end of which rest upon the wall causing the next greatest strain, the distances from it to the walls are to be called r and s. The distances from the third header are v and u. We have, therefore, $m = 9$, $n = 11$, $s = 6$, $v = 4$, $l = 20$, $g = 16$, $y = 105$ and $c = 5 \cdot 59$; and by formula (*262.*) have

$$I = \frac{140 + \frac{105}{3 \times 5 \cdot 59}}{22320} \left[9\left(\overline{5 \cdot 59 \times 11 \times 20} + \overline{16 \times 6^2}\right) + 16\left(\overline{11 \times 9^2 - 4^2}\right)\right]$$

$$= 181 \cdot 465$$

or the required moment is $181 \cdot 465$. From the recorded moments in Table XVII. we find that the Phœnix $10\frac{1}{2}$ inch 105 pound beam is a trifle stronger than the required amount. The Trenton and Paterson $10\frac{1}{2}$ inch 105 pound beams are still stronger than this. Being of the same weight, either of the three named beams will serve the purpose.

536.—Carriage Beam with Three Headers, the Greatest Strain being at Middle Header, for First-class Stores.—Take a case where the header causing the greatest strain in the carriage beam occurs between the other two, as in *Fig.* 56. Formula (*262.*) is suitable for this case, except in its coefficient. To modify it to suit our purpose, let $\dfrac{320 + \dfrac{y}{3c}}{29760}$ in formula (*261.*) be substituted for $\dfrac{140 + \dfrac{y}{3c}}{22320}$ in formula (*262.*); and we have

$$I = \frac{320 + \dfrac{y}{3c}}{29760}[m(cnl + gs^2) + gn(m^2 - v^2)] \qquad (263.)$$

which is a rule for the moment of inertia for rolled-iron carriage beams carrying three headers, in first-class stores; the header causing the greatest strain being between the other two. (See *Art.* **264.**)

The example given in *Art.* **535** will be sufficient to illustrate this rule, as the two formulas are alike, except in their coefficients.

QUESTIONS FOR PRACTICE.

537.—What is the moment of inertia for a beam having a rectangular section?

538.—What is the moment of inertia for a beam of I section, or of the form of rolled-iron beams?

539.—Which of the beams of Table XVII. would be appropriate, when laid upon two supports 25 feet apart, to sustain 15,000 pounds at the middle, with a deflection of $\frac{3}{4}$ of an inch?

540.—What weight could be sustained at 10 feet from one end of a Trenton $10\frac{1}{2}$ inch 105 pound beam, 25 feet long between bearings, with a deflection of one inch?

541.—What weight uniformly distributed could be sustained upon a Buffalo 9 inch 90 pound beam, projecting as a lever 15 feet from a wall (in which one end is firmly imbedded), with a deflection of $\frac{1}{2}$ an inch?

542.—In the floors of a first-class store, constructed with Phœnix 12 inch 125 pound beams, $3\frac{1}{2}$ feet from centres: Which of the beams of Table XVII. ought to be used for a header 15 feet long, carrying one end of a set of tail beams 12 feet long?

543.—In the floor of a first-class store, constructed with 12 inch 125 pound beams $2\frac{1}{2}$ feet from centres: Which of the beams of Table XVII. ought to be used for a carriage beam 25 feet long between bearings, carrying, with 0·04 of an inch per foot deflection, a header 20 feet long, located at 7 feet from one end of the carriage beam, and carrying one end of a set of tail beams 18 feet long?

544.—In the floor of a first-class store, constructed of 15 inch 150 pound beams $4\frac{1}{2}$ feet from centres: What size should be a carriage beam 25 feet long, which carries two headers 19 feet long, one located at 9 feet from one wall, and the other at 8 feet from the other wall; the two headers having an opening between them?

545.—In the floor of a bank, constructed of $10\frac{1}{2}$ inch 105 pound beams 22 feet long, and placed 4 feet 4 inches from centres: Of what size should be a carriage beam which carries three headers, 16 feet long, and located, as in *Fig.* 56, so that one opening at the wall shall be 3 feet wide, and the other opening 6 feet wide, with a width of floor of 6 feet between the two openings?

CHAPTER XX.

TUBULAR IRON GIRDERS.

ART. **546.—Introduction of the Tubular Girder.**—During the construction of the great tubular bridges over the Conway River and the Menai Straits, Wales (1846 to 1850), engineers and architects were moved with new interest in discussions and investigations as to the possibilities of constructions involving transverse strains. Since the complete success of those justly celebrated feats of engineering skill, the tubular girder (*Fig.* 74), as also the plate girder (*Fig.* 67), and the rolled-iron beam (*Fig.* 68), all of which owe their utility to the same principle as that involved in the construction of the tubular girder, have become deservedly popular. They are now extensively used, not only by the engineer in spanning rivers for the passage of railway trains, but also by the architect in the lesser, but by no means unimportant, work of constructing floors over halls of the largest dimensions, without the use of columns as intermediate supports.

FIG. 74.

547.—Load at Middle—Rule Essentially the Same as that for Rolled-Iron Beams.—The capacity of tubular gir-

ders may be computed by the rules already given. For example: Formula (*216.*) affords a rule for a load at the middle of a rolled-iron beam, in which (*form. 213.*),

$$I = \frac{1}{12}(bd^3 - b_{,}d_{,}^3)$$

whereof b is the width of top or bottom flange, and $b_{,}$ equals b, less the thickness of the two upright parts, or webs; d is the entire depth, and $d_{,}$ is the depth, or height, in the clear between the top and bottom flanges. bd then is the area of the whole cross-section, measured over all, while $b_{,}d_{,}$ represents the area of the vacuity, or of so much of the cross-section as is *wanting* to make it a solid. The numerical coefficient in formula (*216.*) is based upon a value of F equal to 62,000, which is the amount derived from experiments on solid rolled-iron beams. For *built* beams, such as the tubular girder, F by experiment would prove to be less, but the formula (*216.*) may be used as given, provided that proper allowance be made in the flanges on account of the rivet holes; that is, taking instead of the actual breadths of the flanges only so much of them as remains uncut for rivets.

548.—Load at Any Point—Load Uniformly Distributed. —For a load at any point in the length of a beam, formula (*222.*) will serve, while for a load uniformly distributed, formula (*228.*) affords a rule. In general, any rule adapted to rolled-iron beams will serve for the tubular or plate girder, by taking as the areas of metal the uncut portion only.

549.—Load at Middle—Common Rule.—The rules just quoted are not those which are generally used for tubular beams. Preliminary to planning the Conway and Britannia

tubular bridges, the engineers tested several model tubes, and from them deduced the formula

$$W = \frac{a'dC}{l}$$

in which C is a constant, found to be equal to 80 when W represents gross tons. Changing W to pounds, we have

$$W = \frac{2240 \times 80 \times a'd}{l} = 179200\frac{a'd}{l}$$

This is for the breaking weight. Taking the safe weight as 9000 pounds per inch, or $\frac{1}{5}$ of the breaking weight, we have

$$\frac{179200}{5} = 35840$$

and, as an expression for the safe weight, the area of the bottom flange equals

$$a' = \frac{Wl}{35840d}$$

or, if instead of the above constant, 80, we put 80·357, we shall have our constant in round numbers, thus,

$$a' = \frac{Wl}{36000d} \qquad (264.)$$

which is a rule for the area of the bottom flange of a tubular girder, with the load at the middle; a' being in inches, l and d in feet, and W in pounds. This rule is identical with formula (*265.*), deduced in another manner.

550.—Capacity by the Principle of Moments.—Generally, the strength of tubular beams is ascertained by the principle of *moments* or *leverage*. Sufficient material must be

provided in the top flange to resist crushing, and in the bottom flange to resist tearing asunder, while the material in the web or upright part should be adequate to resist shearing.

551.—Load at Middle—Moments.—We will first consider the requirements in the flanges.

The *leverage*, or action of the power tending to break the beam, as also that of the resistance of the materials, is represented in *Figs.* 8 and 9. When the load upon a beam is concentrated at the middle, it acts with a power of half the weight into half the length of the beam (*Art.* **35**), and the tension thereby produced in the bottom flange is resisted by a leverage equal to the height of the beam; or, if d equals the height of the beam between the centres of gravity of the cross-sections of the top and bottom flanges, and T equals the amount of tension produced in the lower flange by the action of a weight W upon the middle of the beam, then

$$\tfrac{1}{2}W \times \tfrac{1}{2}l = dT$$

$$\tfrac{1}{4}Wl = dT$$

Again, if k equals the pounds per square inch of section with which the metal in the lower flange may be safely trusted, and a' equals the area in inches in the bottom flange, then $a'k = T$, and

$$\tfrac{1}{4}Wl = a'kd$$

$$a' = \frac{Wl}{4dk} \qquad (265.)$$

which is a rule for the area of the bottom flange of a tubular girder, loaded at the middle, and in which W and k are in pounds, a' is in inches, and d and l are in feet. (The

area of the top flange is to be made equal to that of the bottom flange. See *Art.* **456**.) If k be taken at 9000, as in *Art.* **549**, then $4k = 36000$, and formula (*265.*) becomes identical with formula (*264.*).

552.—Example.—What area of metal would be required in the bottom flange of a tubular girder 40 feet long and 3 feet high, to sustain at the middle 75,000 pounds; 9000 pounds being the weight allowed upon one inch of the wrought-iron of which the flanges are to be made?

Here $W = 75000$, $l = 40$, $d = 3$ and $k = 9000$; and we have, by formula (*265.*),

$$a' = \frac{75000 \times 40}{4 \times 3 \times 9000} = 27 \cdot 77$$

or the area equals $27\frac{3}{4}$ inches. This is the amount of metal in addition to that required for rivet holes.

553.—Load at Any Point.—A load concentrated at any point in the length of the beam acts with a leverage equal to $W\frac{mn}{l}$ (see *Art.* **56**), and the resistance is $Td = a'kd$; therefore

$$W\frac{mn}{l} = a'dk$$

$$a' = W\frac{mn}{dkl} \qquad (266.)$$

which is a rule for this case, as above stated, in which a' is in inches, W is in pounds, and m, n, d and l are in feet.

554.—Example.—What amount of metal would be required in the bottom flange of a tubular girder 50 feet long

and $3\frac{1}{2}$ feet high, to sustain a load of 50,000 pounds at 20 feet from one end, when $k = 9000$?

Here $W = 50000$, $m = 20$, $n = 30$, $d = 3\frac{1}{2}$, $k = 9000$ and $l = 50$; and, by formula (*266.*),

$$a' = \frac{50000 \times 20 \times 30}{3\frac{1}{2} \times 9000 \times 50} = 19{\cdot}05$$

or the area should have 19 inches of solid metal, uncut by rivet holes. The top flange should contain an equal amount. (See *Art.* **456**.)

555.—Load Uniformly Distributed.—For this load the effect at any point in the beam is equal to that of half the load, if concentrated at that point (see *Art.* **214**); or, from formula (*266.*),

$$a' = U\frac{mn}{2dkl} \qquad (267.)$$

which is a rule for the area of the bottom flange at any point in its length, and in which a' is in inches, U is in pounds, and m, n, d and l are in feet.

556.—Example.—In a tubular girder 50 feet long, $3\frac{1}{2}$ feet high, and loaded with an equably distributed load of 120,000 pounds: What should be the area of the bottom flange at the middle, and at each 5 feet of the length thence to each support, k being taken at 9000?

Here $U = 120000$, $d = 3\frac{1}{2}$, $k = 9000$ and $l = 50$; and by formula (*267.*) we have

$$a' = \frac{120000mn}{2 \times 3\frac{1}{2} \times 9000 \times 50} = 0{\cdot}038095mn$$

When $m = n = 25$, then

$$a' = 0{\cdot}038095 \times 25 \times 25 = 23{\cdot}81$$

or the area required in the bottom flange at mid-length is $23{\cdot}81$ inches.

When $m = 20$, then $n = 30$, and

$$a' = 0{\cdot}038095 \times 20 \times 30 = 22{\cdot}86$$

or the required area at 5 feet from the middle, either way, equals $22\frac{7}{8}$ inches.

When $m = 15$, then $n = 35$, and

$$a' = 0{\cdot}038095 \times 15 \times 35 = 20{\cdot}00$$

or, at 10 feet each side of the middle, the area should be 20 inches.

When $m = 10$, then $n = 40$, and

$$a' = 0{\cdot}038095 \times 10 \times 40 = 15{\cdot}24$$

or, at 15 feet each side of the middle, the area should be $15\frac{1}{4}$ inches.

When $m = 5$, then $n = 45$, and

$$a' = 0{\cdot}038095 \times 5 \times 45 = 8{\cdot}57$$

or, at 20 feet each side of the middle, the area should be $8\frac{5}{8}$ inches.

557.—Thickness of Flanges.—In the results of the example just given, it will be observed that the area of metal required in the flanges increases gradually from the points of support each way to the middle of the beam (see *Art.* **178**). In practice, this requirement is met by building up the flanges with laminas or plates of metal, lapping on according

to the computed necessary amount. In this process, the plates used are generally not less than ¼ of an inch thick. For an example, take the results just found. Adding, say ⅛ for rivet holes, and dividing the sum by the width of the girder, which we will call 12 inches, there results as the thickness of metal required,

at the middle,	2·31,	say	2½ inches;
" 5 feet from middle,	2·22,	"	2¼ "
" 10 " "	1·95,	"	2 "
" 15 " "	1·48,	"	1½ "
" 20 " "	0·83,	"	1 inch.

558.—Construction of Flanges.—The girder of the last article might be built with the two flanges in plates 12 inches wide, thus: Lay down first a plate one inch thick the whole length of the girder. (With an addition for supports on the walls, say $\frac{1}{10}$ of the length, or 2½ feet at each end, this plate would be 55 feet long.) Upon this place a plate ½ inch thick and 40 feet long; on this a plate ½ inch thick and 30 feet long; on this a plate ¼ inch thick and 20 feet long; and on this a plate ¼ inch thick and 10 feet long. The plates are all to extend to equal length each side of the middle of the girder, and to be well secured together by riveting. The longer plates, probably, will have to be in more than one piece in length. Where heading joints occur, a covering plate should be provided for the joint and riveted.

559.—Shearing Strain.—A sufficient area having been provided in the top and bottom flanges to resist the compressive and tensile strains, there will be needed in the *web* metal sufficient to resist only the shearing strain. This strain

is, theoretically, nothing at the middle of a beam uniformly loaded, but from thence increases by equal increments to each support, at which place it is equal to one half of the whole load (*Arts.* **172** and **174**). For example: In the case considered in *Art.* **556**, the beam, 50 feet long, carries 120,000 pounds uniformly distributed over its whole length; half of the load over half of the beam. At the centre, the shearing strain is nothing; at 5 feet from the centre, it is equal to $\frac{1}{5}$ of half the load, or is equal to 12,000 pounds; at 10 feet it is 24,000; at 15 feet it is 36,000; at 20 feet it is 48,000; and at 25 feet, or at the supports, it is 60,000 pounds, or half the whole load.

560.—Thickness of Web.—If G be put for the shearing stress, then

$$G = a'k'$$

in which a' is the area in inches of the web at the point of the stress, and k' is the effective resistance of wrought-iron to shearing, per inch area of cross-section. If t equals the thickness, and d the height of the web, then $a' = td$, and the above equation becomes

$$G = k'td$$

or

$$t = \frac{G}{dk'} \qquad (268.)$$

which is a rule for the thickness of the web, at any point in the length of the beam, and in which t and d are in inches.

561.—Example.—What should be the thickness of the web of the tubular girder considered in *Art.* **556**, computed

at every 5 feet in length of the girder? If k' be taken at 7000 pounds, it will be but little more than three quarters of 9000, the amount taken in tensile strain (*Art.* **173**),* and taking d at, say 38 inches, we have, by formula (*268.*),

$$t = \frac{G}{38 \times 7000} = \frac{G}{266000}$$

Therefore, when G equals 60,000 (*Art.* **559**), then

$$t = \frac{60000}{266000} = 0 \cdot 225$$

When G equals 48,000, then $t = \frac{48000}{266000} = 0 \cdot 180$. When G equals 36,000, then $t = \frac{36000}{266000} = 0 \cdot 135$. As these are the greater of the strains, and are all below the practical thickness in girders, it is not worth while to compute those at the remainder of the stations.

562.—Construction of Web.—From the results in the last article, it appears that in this case the web is required, of necessity, to be only a quarter of an inch thick in its thickest part, at the supports. With an increase of load, the thickness of the web would increase, for by the formula it is directly as the load.

The thickness of web just computed is the whole amount required in the two sides of the girder. In practice, it is found unwise to use plates less than a quarter of an inch thick. Following this custom, the two sides of the girder

* The resistance to shearing is generally taken at three quarters of the tensile strength (see Haswell's Engineers' and Mechanics' Pocket-Book, p. 485—Weisbach's Mechanics and Engineering, vol. 2, p. 77).

taken together would be half an inch thick, more than twice the amount of metal actually required. Hence it may justly be inferred that in similar cases the plate beam (*Fig.* 67) would be preferable to the tubular girder, as its web, being single, would require only half the metal that would be required in the two sides of the tubular girder. It is also preferable for the reason that it is more easily painted, and thus kept from corrosion. On the other hand, a tubular beam is stiffer laterally. In the construction of the web, as a precaution to prevent *buckling*, or contortion, it is requisite to provide uprights of **T** iron, at intervals of, say 3 feet on each side, to which the web is to be riveted.

563.—Floor Girder—Area of Flange.—If for U in formula (*267.*), there be substituted its value in a floor, $c'fl$, of which c' is the distance from centres between girders, or the width of floor sustained by the girder, l is the length of the girder between supports (both in feet), and f is the load per foot superficial upon the floor, including the weight of the materials of construction, then

$$a' = c'fl\frac{mn}{2dkl}$$

$$a' = f\frac{c'mn}{2dk} \qquad (269.)$$

which is a rule for the area of the bottom flange of a tubular girder, sustaining a floor, and in which a' is in inches and c', m, n and d are in feet.

564.—Weight of the Girder.—In estimating the load to be carried by a girder, the estimate must include the weight of the girder itself. It is desirable therefore to be able to

measure its weight approximately before its dimensions have been definitely fixed. The weight of a tubular girder will be in proportion to its area of cross-section (which will be approximately as the load it has to carry), and to its length (*form.* 265.); or, when U is the gross load to be carried, and l the length between bearings, then the weight of the girder between the bearings is

$$K = \frac{Ul}{n}$$

in which n is a constant, and U is the whole load, including that of all the materials of construction. The value of n, when derived from so large a structure as that of the tubular bridge over Menai Straits, is about 600, but from several examples of girders from 35 to 50 feet long, in floors of buildings, its value is found to be about 700. For our purpose, then, we have $n = 700$. If for U we put its equivalent $c'fl$, as in *Art.* **563**, then

$$K = \frac{c'fl^2}{700} \qquad (270.)$$

This is the weight of so much of the girder as occurs within the clear span between the supports.

565.—Weight of Girder per Foot Superficial of Floor.— The area of the floor supported by a girder is $c'l$. Dividing K by this, the quotient will be f', the weight of the girder per foot superficial of the floor, thus:

$$f' = \frac{K}{c'l} = \frac{\frac{c'fl^2}{700}}{c'l} = \frac{fl}{700}$$

Now f, the total load per foot superficial of the floor, com-

prises the superimposed load, the weight of the brick arches, etc., and the weight of the girder f'; and, putting m for the weight of all else save that of the girder, we have

$$f = m + f' \qquad \text{and, from the above,}$$

$$f' = \frac{fl}{700} = \frac{(m + f')l}{700}$$

$$f' = \frac{lm + f'l}{700}$$

$$700f' = lm + f'l$$

$$700f' - f'l = lm$$

$$f'(700 - l) = lm$$

$$f' = \frac{lm}{700 - l} \qquad (271.)$$

which is a rule for ascertaining the weight per foot superficial of the floor due to the tubular girder.

566.—Example.—A floor, the weight of which, including that of the superimposed load, is 140 pounds per superficial foot, is carried upon a girder 50 feet in length between its bearings. What additional amount per foot superficial should be added for the weight of the girder?

Here $l = 50$ and $m = 140$, and by (*271.*),

$$f' = \frac{140 \times 50}{700 - 50} = 10 \cdot 77$$

or the weight to be added for the girder is $10\frac{3}{4}$ pounds. Then $f = m + f' = 140 + 10\frac{3}{4} = 150\frac{3}{4}$ pounds.

567.—Total Weight of Floor per Foot Superficial, including Girder.—In the last article m represents the weight of one foot superficial of a floor, including the load to be car-

ried; also, f' represents the weight due to the girder; or, for the total load, $f = m + f'$. Using for f' its value as in formula (*271.*) we have

$$f = m + f' = m + \frac{lm}{700-l}$$

$$f = m\left(1 + \frac{l}{700-l}\right)$$

$$f = m\,\frac{700}{700-l}$$

and for m, taking its value as given in formula (*232.*), it being there represented by f,

$$f = \left(140 + \frac{y}{3c}\right)\frac{700}{700-l} \qquad (272.)$$

which is the value of f, the total load per superficial foot of the floors of assembly rooms, banks, etc., to be used in the calculation of tubular girders; and taking the value of m, as expressed in formula (*233.*) we have

$$f = \left(320 + \frac{y}{3c}\right)\frac{700}{700-l}, \qquad (273.)$$

which is the corresponding value of f for the floors of first-class stores.

568.—Girders for Floors of Dwellings, etc.—If in formula (*269.*), we substitute for f its value as in formula (*272.*), we shall have

$$a' = \left(140 + \frac{y}{3c}\right)\frac{700}{700-l} \times \frac{c'mn}{2dk} \qquad (274.)$$

which is a rule for the area of the bottom flange of a tubular girder, supporting the floor of an assembly room or bank, and in which a' is in inches, and c, l, c', m, n and d are in feet.

569.—Example.—In a floor of 9 inch 70 pound beams, 4 feet from centres: What ought to be the area of the bottom flange of a tubular girder 40 feet long between bearings, $2\frac{5}{8}$ feet deep, and placed 17 feet from the walls, or from other girders; the area of the flange to be ascertained at every five feet in length of the girder?

Here $y = 70$, $c = 4$, $l = 40$, $c' = 17$ and $d = 2\frac{5}{8}$.

Putting k at 9000 we have, by (*274.*),

$$a' = \left(140 + \frac{70}{3 \times 4}\right)\frac{700}{700-40} \times \frac{17}{2 \times 2\frac{5}{8} \times 9000} \times mn$$

$$a' = 0 \cdot 05478mn$$

The values of m and n are as follows:

At the middle,	$m = 20$	and	$n = 20$
" 5 feet from middle,	$m = 15$	"	$n = 25$
" 10 " " "	$m = 10$	"	$n = 30$
" 15 " " "	$m = 5$	"	$n = 35$

from which the values of a' are as follows:

At the middle,	$a' = 0 \cdot 05478 \times 20 \times 20 = 21 \cdot 91$
" 5 feet from middle,	$a' = 0 \cdot 05478 \times 15 \times 25 = 20 \cdot 54$
" 10 " " "	$a' = 0 \cdot 05478 \times 10 \times 30 = 16 \cdot 43$
" 15 " " "	$a' = 0 \cdot 05478 \times 5 \times 35 = 9 \cdot 59$

These are the areas of cross-section of the lower flange, at the respective points named. The top flange is to be of the same size. (See *Art.* **456.**)

570.—Girders for Floors of First-class Stores.—If, in formula (*274.*), 320 be substituted for 140 (see *form. 233.*), we shall have

$$a' = \left(320 + \frac{y}{3c}\right)\frac{700}{700-l} \times \frac{c'mn}{2dk} \qquad (275.)$$

which is a rule for the area of the bottom flange of a tubular girder in a first-class store. [The area of the upper flange should be made equal to that of the bottom flange (*Art.* **456**).]

As this rule is similar to (*274.*), the example given to illustrate that rule will suffice also for this.

571.—Ratio of Depth to Length, in Iron Girders.—In order that the requisite strength in tubular girders may be attained with a minimum of metal, the depth of a girder should bear a certain relation to the length. To deduce a rule for this ratio from mathematical considerations purely, is not an easy problem. Baker in his work on the Strength of Beams, p. 288, discusses the subject at some length. No more will be attempted here than to obtain a rule based upon some general considerations, and upon results tested and corrected by experience.

572.—Economical Depth.—In the construction of tubular girders for the floors of large buildings, it is found in practice to be unadvisable to use plates of a less thickness than one quarter of an inch. If each side of the girder be a quarter of an inch thick, then the least thickness for the *web* (using this term technically) is a half inch. This is more than is usually found necessary, in this class of girders, to resist shearing (*Art.* **562**). As the thickness is thus fixed, therefore the area of the web will be in proportion to its height, and consequently it is advisable, in so far as the web is concerned, to have the depth of the girder small; but, on the other hand, as the area of the flanges is inversely proportional to the depth (see *form. 265.*), a reduction of the flanges will require that the depth be increased. The cost of the girder is in proportion to its weight, which is in proportion

to its area of cross-section, and hence the desirability of making both as small as possible.

The area of the flanges is, by formula (*265.*), in proportion to $\frac{Wl}{dk}$, and, as before shown, the area of the web will be in proportion to its height; or the whole area will be in proportion to $\frac{Wl}{dk}+d$; and the problem is to find such a value of d as will make this expression a minimum. Putting the differential of this equation equal to zero, we find that the area of the cross-section of the beam will be a minimum when

$$d=\sqrt{\frac{Wl}{kx}}$$

in which x is a constant, to be derived from experience, and which, by an application of the formula to girders of this class, is, when the weight is equally distributed, found to be equal to 30. This reduces the formula to

$$d=\sqrt{\frac{Ul}{30k}} \qquad (276.)$$

and when for U its value $c'fl$ is substituted

$$d=l\sqrt{\frac{c'f}{30k}} \qquad (277.)$$

which is a rule for ascertaining the economical depth of a tubular girder; a rule useful in cases where the depth is not fixed by other considerations.

573.—Example.—In a floor where the girders are 50 feet long and placed 15 feet from centres, and where the total load per foot superficial is 155 pounds: What would be the most economical depth for the girders?

Here $l = 50$, $c' = 15$, $f = 155$ and $k = 9000$, equals the safe tensile power of wrought-iron; and by (*277.*)

$$d = 50 \times \sqrt{\frac{15 \times 155}{30 \times 9000}} = 4 \cdot 64$$

or the depth should be 4 feet 7⅝ inches. The depth may be found by this formula, and then the area of flanges by formula (*274.*) for assembly rooms, banks, etc.; or, by formula (*275.*) for first-class stores.

QUESTIONS FOR PRACTICE

574.—In a tubular girder 50 feet long, 3 feet 4 inches high, and loaded with 100,000 pounds at the middle: What ought to be the area of each of the top and bottom flanges, when the metal of which they are made may be safely trusted with 9000 pounds per inch?

575.—In the same girder: What should be the area of the top or bottom flange, if the load of 100,000 pounds be placed at 15 feet from one end, instead of at the middle of the beam?

576.—In a tubular girder 50 feet long, 40 inches high, and uniformly loaded with 200,000 pounds: What should be the area of the top and bottom flanges, at every five feet of the length of the girder?

577.—In the same girder: What ought to be the thickness of the web, at every five feet of the length of the girder. to effectually resist the shearing strain?

578.—In a tubular girder 40 feet long, 32 inches high, sustaining, with other girders and the walls, the floor of an assembly room, composed of 9 inch 70 pound beams 5 feet from centres, the girders being placed 16 feet from centres: What should be the area of each of the top and bottom flanges, at every five feet of the length, the metal in the flanges being such as may be safely trusted with 9000 pounds per inch?

579.—In a floor, where the depth of the tubular girders is not arbitrarily fixed, where the girders are 42 feet long and placed 17 feet from centres, and where the total load to be carried is 160 pounds per superficial foot: What would be the proper depth of the girders, putting the safe tensile strain upon the metal at 9000 pounds?

CHAPTER XXI.

CAST-IRON GIRDERS.

ART. **580.—Cast-Iron Superseded by Wrought-Iron.**—The means for the manufacture of rolled-iron beams (Chapter XIX.) have so multiplied within the last ten years that the cost of their production has been much reduced, and as a consequence this beam is now so extensively used as to have almost entirely superseded the formerly much used cast-iron beam or girder. Beams and girders of cast-iron, however, are still used in some cases, and it is well to know the proper rules by which to determine their dimensions. A few pages, therefore, will here be devoted to this purpose.

581.—Flanges—Their Relative Proportion.—In *Fig.* 75 we have the usual form of cross-section of cast-iron beams, in which the bottom flange *AB* contains four times as much metal as the top flange *CD*. It was customary, fifty years since, to make the top and bottom flanges equal. (See Tredgold on Cast-Iron, Vol. I., Art. 37, Plate I.)

A

FIG. 75.

Mr. Eaton Hodgkinson (who in 1842 edited a fourth edition of Tredgold's first volume, and in 1846 added a second volume to that valuable work) made many important experiments on cast-iron. Among the valuable deductions resulting from these experiments was this: that cast-

iron resists compression with about seven times the force that it resists tension (Vol. II., Art. 34); and that the form of section of a beam which will resist the greatest transverse strain, is that in which the bottom flange contains six times as much metal as the top flange (Vol. II., Art. 138, page 440). If beams of cast-iron for buildings were required to serve to the full extent of the power of the metal to resist *rupture*, the proportion between the areas of top and bottom flanges should be as 1 to 6. If, on the other hand, they be subjected only to very light strains, the areas of the two flanges ought to be nearly if not quite equal. In view of the fact that in practice it is usual to submit them to strains greater than the latter, and less than the former, therefore an average of the proportions required in these two cases is that which will give the best form for use. Guided by these considerations, it is found that when the flanges are as 1 to 4, we have a proportion which approximates very nearly the requirements of the case.

582.—Flanges and Web—Relative Proportion.—The web, or vertical part which unites the top and bottom flanges, needs only to be thick enough to resist the shearing strain upon the metal; a comparatively small requirement. Owing, however, to a tendency in castings to fracture in cooling, the thickness of the web should not be much less than that of the flanges, and the points of junction between the web and flanges should be graduated by a small bracket or easement in each angle. (Tredgold's Cast-Iron, Vol. II., Art. 124.) The thickness of the three parts—web, top flange and bottom flange—may with advantage be made in proportion as 5, 6 and 8. Made in these proportions, the width of the top flange will be equal to one third of that of the bottom flange; for if $w_{\prime}$ equal the width of the bottom flange and $w_{\prime\prime}$ that of the top flange, $t_{\prime}$ equal the thick-

ness of bottom flange and t_{u} that of the top, $a_{,}$ equal the area of the bottom flange and a_{u} the area of the top flange, then $a_{,} = w_{,}t_{,}$ and $w_{,} = \frac{a_{,}}{t_{,}}$; also, $a_{u} = w_{u}t_{u}$ and $w_{u} = \frac{a_{u}}{t_{u}}$; and from these, remembering that $a_{,} = 4a_{u}$, and that

$$t_{u} : t_{,} :: 6 : 8,$$

we have

$$w_{u} = \frac{a_{u}}{t_{u}} = \frac{\frac{1}{4}a_{,}}{\frac{6}{8}t_{,}} = \frac{1}{3}\frac{a_{,}}{t_{,}} = \frac{1}{3}w_{,}$$

or the width of the top flange equals one third of that of the bottom flange.

583.—Load at Middle.—Mr. Hodgkinson found, in his experiments, that the strength was nearly as the depth and as the area of the bottom flange. For the breaking weight, W, he gives

$$W = \frac{ca_{,}d}{l} \qquad (278.)$$

an expression for the relative values of the dimensions and weight; in which W is the breaking weight at the middle, l the length of the beam, d its depth, $a_{,}$ the area of the bottom flange, and c is a constant, to be derived from experiment. This constant, when the weight was in tons and the dimensions all in inches, he found to be 26. Taking the weight in pounds and the length in feet, we have $4853\frac{1}{3}$ for the constant, or say 4850, and therefore

$$W = \frac{4850a_{,}d}{l}$$

When a is the factor of safety,

$$W = \frac{4850a_{,}d}{al}$$

or

$$a_{,} = \frac{Wal}{4850d} \qquad (279.)$$

which is a rule for the area of the bottom flange of a cast-iron beam, required to sustain safely a load at the middle. The area of the top flange is to be made equal to $\frac{a_{\prime}}{4}$, and the thicknesses of the web and top and bottom flanges are to be in proportion as 5, 6 and 8.

584.—Example.—What should be the dimensions of the cross-section of a cast-iron beam 20 feet long between supports and 24 inches high at the middle, where it is to carry 30,000 pounds; with the factor of safety equal to 5?

Here $W = 30000$, $a = 5$, $l = 20$ and $d = 24$; and, by formula (*279.*),

$$a_{\prime} = \frac{30000 \times 5 \times 20}{4850 \times 24} = 25 \cdot 773$$

or the area of the bottom flange should be $25\frac{3}{4}$ inches. Now the thickness will depend upon the width, and this is usually fixed by some requirement of construction. If the width be 12 inches, then the thickness of the bottom flange will be $\frac{25 \cdot 77}{12} = 2 \cdot 15$, or $2\frac{1}{8}$ inches full. The width of the top flange will equal $\frac{w_{\prime}}{3} = \frac{12}{3} = 4$ (see *Art*, **582**), and its thickness will be $\frac{6}{8}t_{\prime} = \frac{6}{8} \times 2 \cdot 15 = 1 \cdot 61$, or $1\frac{5}{8}$ inches; while the thickness of the web will be $\frac{5}{8}t_{\prime} = \frac{5}{8} \times 2 \cdot 15 = 1 \cdot 34$, or $1\frac{3}{8}$ inches.

585.—Load Uniformly Distributed.—A load uniformly distributed will have an effect at any point in a beam equal to that which would be produced by half of the load if it were concentrated at that point (*Art.* **214**). Therefore, if

U equals the load uniformly distributed, $\frac{1}{2}U = W$ in formula (*279.*), or

$$a_{,} = \frac{\frac{1}{2}Ual}{4850d}$$

$$a_{,} = \frac{Ual}{9700d} \qquad (280.)$$

which is a rule for cast-iron beams to carry a uniformly distributed load.

This is precisely the same as the previous rule, except in the coefficient. The example given in *Art.* **584** will therefore serve to illustrate this rule, as well as the previous one.

586.—Load at Any Point—Rupture.—From formula (*278.*) we have

$$Wl = ca_{,}d$$

and, by a comparison of formulas (*21.*) and (*23.*),

$$Wl = 4W\frac{mn}{l}$$

therefore, in the above, substituting this value of Wl, we obtain

$$ca_{,}d = 4W\frac{mn}{l}$$

or

$$a_{,} = W\frac{4mn}{cdl} \qquad (281.)$$

which is a rule for the area of the bottom flange at any point in the length of the beam. The weight given by this rule is just sufficient to rupture the bottom flange.

587.—Safe Load at Any Point.—The value of c, for a concentrated load, is (*Art.* **583**) 4850, hence

$$\frac{4}{c} = \frac{4}{4850} = \frac{1}{1212\frac{1}{2}}$$

In formula (*281.*), substituting for $\frac{4}{c}$ this value, and inserting a, the factor of safety, then

$$a_{\prime} = \frac{Wamn}{1212\frac{1}{2}dl} \qquad (282.)$$

which is a rule for the area of the bottom flange at any point; W, the safe load, being concentrated at that point.

588.—Example.—In a cast-iron beam 20 feet long between bearings: What should be the area of the bottom flange at eight feet from one end, at which point the beam is 20 inches high and carries 25,000 pounds; the factor of safety being equal to 5?

Here $W = 25000$, $a = 5$, $m = 8$, $n = 12$, $d = 20$ and $l = 20$; and by formula (*282.*)

$$a_{\prime} = \frac{25000 \times 5 \times 8 \times 12}{1212\frac{1}{2} \times 20 \times 20} = 24 \cdot 74$$

or the area should be $24\frac{3}{4}$ inches.

589.—Safe Load Uniformly Distributed—Effect at Any Point.—This effect at any point is equal to that produced by half the load were it all concentrated at that point (*Art.* **214**); therefore, if U represent the uniformly distributed load, then by formula (*282.*)

$$a_{\prime} = \frac{\frac{1}{2}Uamn}{1212\frac{1}{2}dl}$$

$$a_{\prime} = \frac{Uamn}{2425dl} \qquad (283.)$$

which is a rule for the area of the bottom flange of a cast-iron beam at any point, to carry safely a uniformly distributed load. If the depth of the beam remain constant throughout the length, then $a_{,}$ will vary as the rectangle mn.

From formula (*283.*) we have

$$d = \frac{Uamn}{2425a_{,}l} \qquad (284.)$$

which is a rule for the depth of a beam at any point, to carry safely a uniformly distributed load. If the area of the bottom flange remain constant throughout the length, then d will vary as the rectangle mn.

590.—Form of Web.—By the last formula, (*284.*), it will be seen that when $a_{,}$, the area of the bottom flange, remains constant throughout the length of the beam, then the depths will vary in proportion to the rectangle of the two segments, m and n, of the length. The corresponding curve which may be drawn through the tops of the ordinates denoting the various depths, is that of a parabola (*Art.* **212**). Instead of computing the depths at frequent intervals, therefore, it will be sufficient to compute the depth at the centre only, and then give to the web the form of a parabola.

591.—Two Concentrated Weights—Safe Load.—Formula (*23.*) is appropriate for a concentrated load at any point in the length of a beam, and formula (*30.*) is for *two* concentrated loads at any given points.

A comparison of these formulas shows that

$$4Wa\frac{mn}{l} = 4a\frac{m}{l}(Wn + Vs)$$

In *Art.* **586** we have

$$ca_{,}d = 4W\frac{mn}{l}$$

which is an expression for the breaking load. Inserting a, the symbol of safety, in this expression, we have

$$ca_{,}d = 4Wa\frac{mn}{l}$$

an expression for the safe load for cast-iron beams. If for the first member of this equation there be substituted its value as above,

$$4a\frac{m}{l}(Wn + Vs)$$

we shall have

$$ca_{,}d = 4a\frac{m}{l}(Wn + Vs)$$

an expression for two concentrated safe loads. From this we have

$$a_{,}d = \frac{4}{c}a\frac{m}{l}(Wn + Vs)$$

In *Art.* **587** we have $\frac{4}{c} = \frac{1}{1212\frac{1}{2}}$, therefore

$$1212\tfrac{1}{2}a_{,}d = a\frac{m}{l}(Wn + Vs)$$

or

$$a_{,} = \frac{a\frac{m}{l}(Wn + Vs)}{1212\frac{1}{2}d} \qquad (285.)$$

which, in a beam carrying two concentrated loads, is a rule

for the area of the bottom flange at the location of W, one of the loads, as in *Fig.* 76; and (see *Art.* **153**)

$$a_{,} = \frac{a\frac{s}{l}(Vr + Wm)}{1212\frac{1}{2}d} \qquad (286.)$$

which, in a beam carrying two concentrated loads, is a rule for the area of the bottom flange at the location of V, one of the loads, as in *Fig.* 76.

592.—Examples.—As an application of rules (*285.*) and (*286.*), let it be required to ascertain the dimensions of a cast-iron girder to sustain a brick wall in which there are three windows, as in *Fig.* 76, so disposed as to concentrate the weight of the wall into two loads, as at W and V. Let l, the length in the clear of the supports, $= 20$, $m = 7$, $n = 13$, $s = 6$ and $r = 14$ feet, and the height of the girder at W and V equal 25 inches. Also, let the wall be 16 inches thick, and so much of it as is sustained at W measure 250 cubic feet, at 110 pounds per foot, or 27,500 pounds. Likewise, suppose the weight upon V to equal 27,000 pounds.

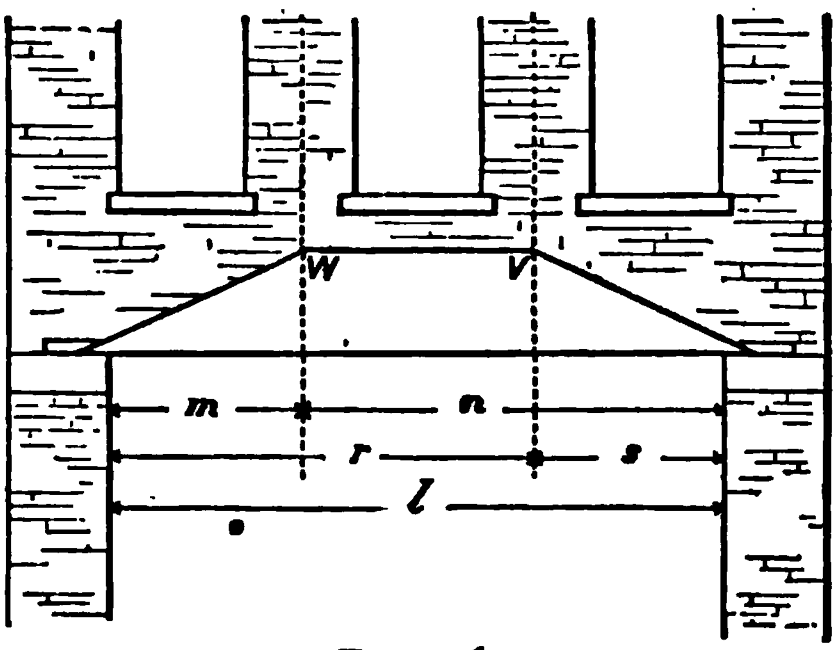

FIG. 76.

Taking the factor of safety at 5 we now have, by formula (*285.*),

$$a_{,} = \frac{5 \times \frac{7}{20}[(27500 \times 13) + (27000 \times 6)]}{1212\frac{1}{2} \times 25} = 29 \cdot 99$$

or the area of flange is required to be 30 inches at W; and, by formula (*286.*),

$$a_{,} = \frac{5 \times \frac{6}{20}\left[(27000 \times 14) + (27500 \times 7)\right]}{1212\frac{1}{2} \times 25} = 28 \cdot 23$$

or the area of flange is required to be $28\frac{1}{4}$ inches at V.

As the wall is 16 inches thick, the width of the bottom flange should be 16 inches, and its thickness therefore should be

$$\frac{30}{16} = 1 \cdot 875 \quad \text{inches at} \quad W$$

$$\frac{28 \cdot 23}{16} = 1 \cdot 764 \quad \text{inches at} \quad V$$

From W to V the thickness is to be graded regularly from $1 \cdot 875$ to $1 \cdot 764$; while from W to the end next W it is to be equal to that at W, $1\frac{7}{8}$ inches thick, and from V to the end next V it is to be $1\frac{3}{4}$ inches thick.

The width of the top flange is to be (*Art.* **582**) one third of the width of the bottom flange, or $\frac{16}{3} = 5\frac{1}{3}$ inches. Proportioning the three parts as 5, 6 and 8 (*Art.* **582**), the thickness of the top flange will be

$$\frac{6}{8} \times 1\frac{3}{4} = 1\frac{5}{16} \quad \text{inches at} \quad V$$

$$\frac{6}{8} \times 1\frac{7}{8} = 1\frac{13}{32} \quad \text{inches at} \quad W$$

The thickness is to be graded regularly between W and V, and thence to each end of the beam the thickness is to be that of W and V respectively. The web is to be of the shape shown in *Fig.* 76, and is to be (*Art.* **582**)

$$\frac{5}{8} \times 1\frac{3}{4} = 1\frac{3}{32} \quad \text{at} \quad V \qquad \text{and}$$

$$\frac{5}{8} \times 1\frac{7}{8} = 1\frac{11}{64} \quad \text{at} \quad W$$

or, say $1\frac{1}{8}$ inches, averaging it throughout.

593.—Arched Girder.—A beam such as shown in *Fig.* 77 is known as the "bow-string girder," in which the curved part is a cast-iron beam of the **T** form of cross-section, and the feet of the arch are held horizontally by a wrought-iron tie-rod. This beam, although very popular with builders, is by no means worthy of the confidence which is placed in it. With an appearance of strength, it is in reality one of the weakest beams used. Without the tie-rod its strength is very small, much smaller than if the **T** section were reversed so as to have the flange at the bottom, thus, **⊥** (Tredgold, Vol. II., pp. 414 and 415).

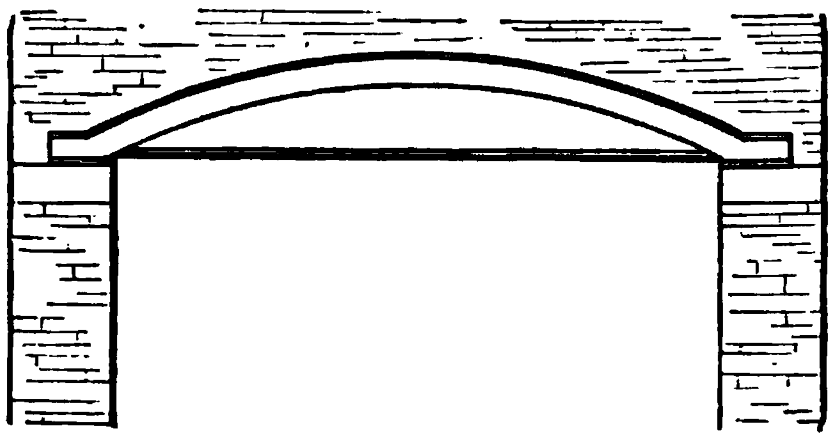

FIG. 77.

594.—Tie-Rod of Arched Girder.—The action of a concentrated weight at the middle of a tubular girder, in producing *tension* in the bottom flange, is explained in *Art.* **551.** The tension in the tie-rod of an arched girder is produced in precisely the same manner, and therefore the rule (*form.* *265.*) there given will be applicable to this case, when modified as required for a uniformly distributed load; or, for W substituting its value, $\frac{1}{2}U$ (*Art.* **585**). Then, upon the presumption that there is sufficient material in the cast arch to resist the thrust, we have

$$a_{\prime} = \frac{\frac{1}{2}Ul}{4dk}$$

in which d is in feet. If d be taken in inches, then

$$a_{\prime} = \frac{Ul}{\frac{2}{3}dk} \qquad (287.)$$

which is a rule for the area of the cross-section of the tie-rod in an arched girder; in which $a_{,}$ is the area of the cross-section of the rod, U is the weight in pounds equally distributed over the beam, l is the length in feet between the supports, d, in inches, is the depth or versed sine of the arc, or the vertical distance at the middle of the beam from the axis of the tie-rod to the centre of gravity of the cross-section of the cast-iron arch, and k is the weight in pounds which may safely be trusted when suspended from the end of a vertical rod of wrought-iron of one square inch section. If this latter be put at 9000 pounds, then

$$a_{,} = \frac{Ul}{6000d}$$

Now $a_{,}$ is the area of the tie-rod. The area of any circle is equal to the square of its diameter multiplied by ·7854, or

$$a_{,} = \cdot 7854 D^2$$

and since, by formula (*287.*),

$$a_{,} = \frac{Ul}{\frac{2}{3}dk} \qquad \text{therefore}$$

$$\cdot 7854 D^2 = \frac{Ul}{\frac{2}{3}dk} \qquad \text{and}$$

$$D = \sqrt{\frac{Ul}{\cdot 5236dk}} \qquad (288.)$$

If k, the safe resistance to tension per inch, be taken at 9000 pounds, the rule becomes

$$D = \sqrt{\frac{Ul}{4712d}} \qquad (289.)$$

which is a rule for the diameter of the tie-rod of an arched girder.

595.—Example.—What should be the diameter of the tie-rod of an arched girder, 20 feet long in the clear between supports, and 24 inches high from the axis of the tie-rod to the centre of gravity of the cross-section at the middle of the arched beam; the load being 40,000 pounds equally distributed over the length of the beam?

Here we have $U = 40000$, $l = 20$ and $d = 24$; and therefore, by formula (*289.*),

$$D = \sqrt{\frac{40000 \times 20}{4712 \times 24}} = 2 \cdot 66$$

or the diameter of the rod, with the safe resistance to tension taken at 9000 pounds, should be $2\frac{5}{8}$ inches.

596.—Substitute for Arched Girder.—The cast-iron arch of an arched girder serves to resist compression. Its place can as well be filled by an arch of brick, footed on a pair of cast-iron skew-backs, and these held in position by a pair of tie-rods, as in *Fig.* 78.

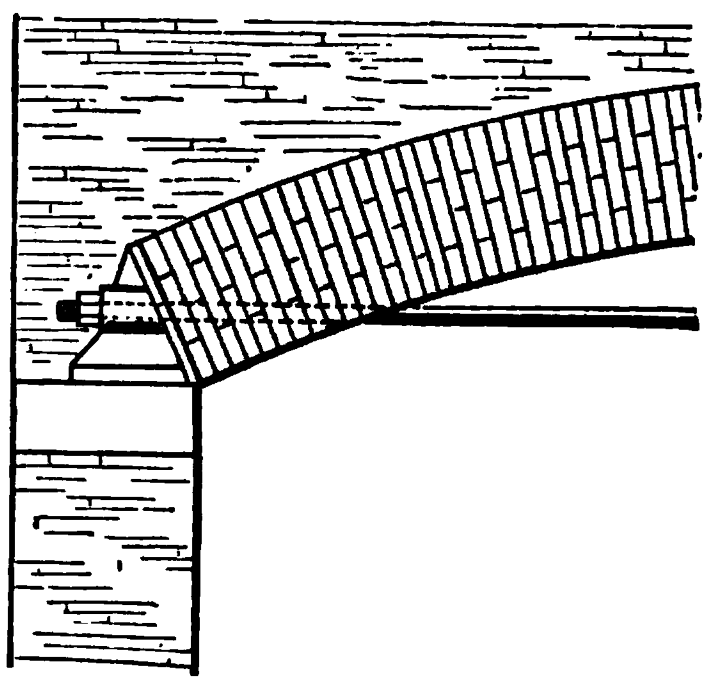

FIG. 78.

To obtain a rule for the diameter of each rod, we have as above, in *Art.* **594**,

$$a_{,} = \cdot 7854 D^2$$

This is for one rod. When $a_{\prime}$ is put for the joint area of two rods, we will have

$$a_{\prime} = \cdot 7854 D_{\prime}^{2} \times 2$$

Comparing this with formula (*287.*), we have

$$\cdot 7854 \times 2 \times D_{\prime}^{2} = \frac{Ul}{\frac{2}{3} dk} \qquad \text{or}$$

$$D_{\prime}^{2} = \frac{Ul}{\frac{2}{3} \times 2 \times \cdot 7854 dk}$$

and when k is taken at 9000 (*Art.* **594**)

$$D_{\prime}^{2} = \frac{Ul}{9425 d}$$

$$D_{\prime} = \sqrt{\frac{Ul}{9425 d}} \qquad (290.)$$

This is a rule in which $D_{\prime}$ represents the diameter of each of the two required rods.

For example, see *Art.* **595.**

An arch of brick, well laid and secured in this manner, will serve quite as well as the cast-iron arch, and may be had at less cost. The best supports, however, to carry brick walls are those made of rolled-iron beams, putting two or more of them side by side and bolting them together. (See *Art.* **489**, *form.* *228.*)

QUESTIONS FOR PRACTICE.

597.—What should be the dimensions of cross-section of a cast-iron girder, 23 feet long between supports, and 27 inches high at the middle, at which point it is to carry 40,000 pounds; with 5 as the factor of safety? The width of bottom flange is 16 inches.

598.—In a girder of the same length, height and width: What should be the cross-section if the weight be 60,000 pounds and be uniformly distributed; the factor of safety being 5?

599.—In a girder of the same length, and of the same height and width at 8 feet from one end, where it is required to carry 50,000 pounds, with a factor of safety of 5: What should be the dimensions of cross-section?

600.—In a girder 25 feet long between bearings, carrying a load of 40,000 pounds at 10 feet from one end, with 5 as the factor of safety, and having 30 inches area of cross-section in the bottom flange: What should be the depth of the girder?

601.—A girder, 25 feet long and 30 inches high, is required to carry, with 5 as a factor of safety, two weights, one of 25,000 pounds at 8 feet from one end, and the other of 30,000 pounds at 6 feet from the other end: What should

be the dimensions of cross-section at each weight, the bottom flange being 16 inches wide?

602.—In an arched girder, 24 feet long between bearings, with a versed sine or height of 30 inches from the axis of the rod to the centre of gravity of the arched beam at the middle, and with the load on the girder taken at 80,000 pounds uniformly distributed: What ought the diameter of the tie-rod to be?

CHAPTER XXII.

FRAMED GIRDERS.

ART. **603.—Transverse Strains in Framed Girders.**—This work, a treatise elucidating the Transverse Strain, would seem to have reached completion with the end of the discussion on simple beams; but when it is recognized that the formation of a deep girder, by a combination of various pieces of material, is but a continuation of the effort to gain strength in a beam, by concentrating its material far above and below the neutral axis, as is done in the tubular girder and rolled-iron beam, it is clear that the subject of framed girders is properly included within a treatise upon the transverse strain. The subject of framed girders, however, will here be discussed so far as to develop only the more important principles involved. For examples in greater variety, the reader is referred to other works (Merrill's Iron Truss Bridges, and Bow's Economics of Construction).

604.—Device for Increasing the Strength of a Beam.—The use of simple beams is limited to comparatively short spans; for beams cut from even the largest trees can have but comparatively small depth. The power of a beam to resist cross-strain can be considerably increased by a very simple device. Let *Fig.* 79 represent the side view of a long beam of wood, from which let *ACDB*, the upper part of the beam, be cut. With the pieces thus removed, and the addition of another small piece of timber, there may be con-

structed the frame shown in *Fig.* 80, which is capable of sustaining a greatly increased load. This increase will be in

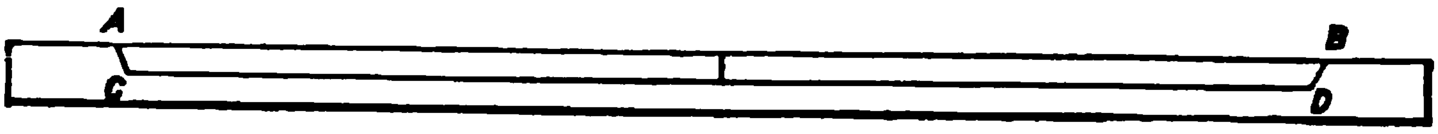

FIG. 79.

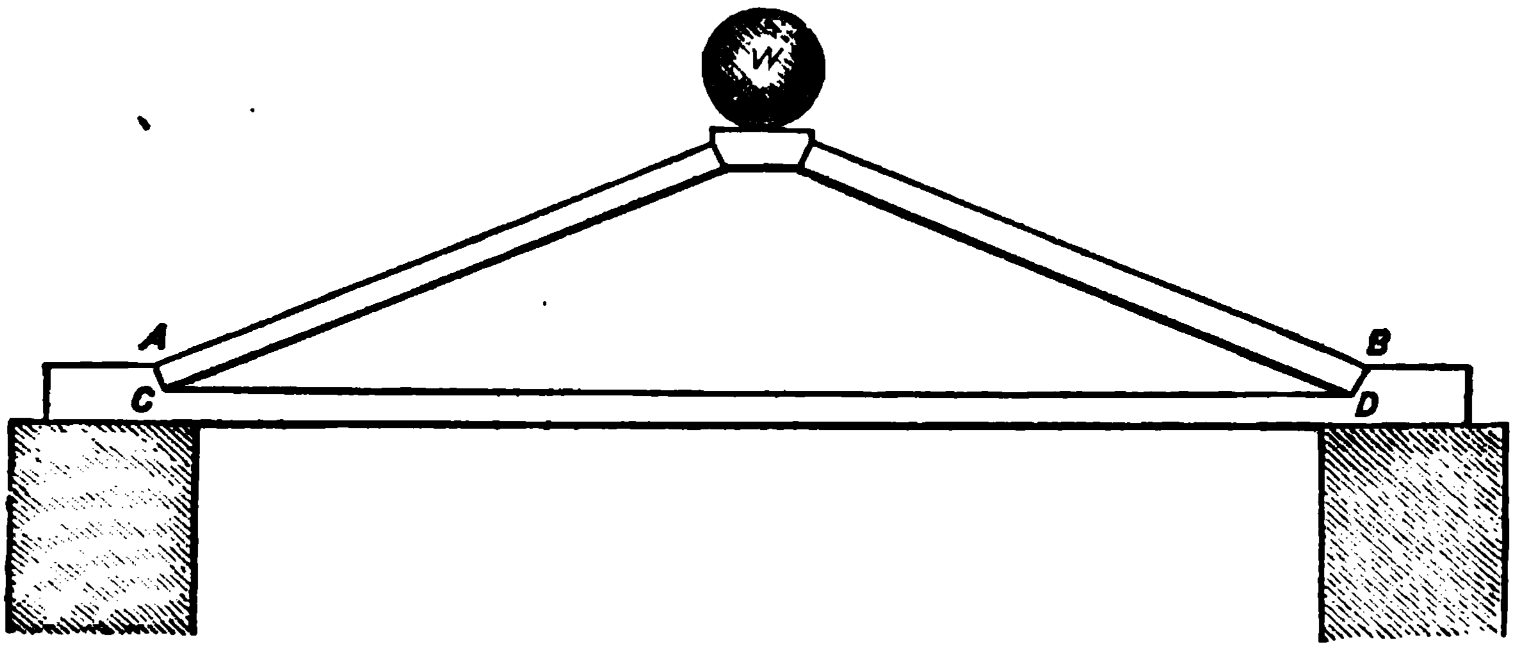

FIG. 80.

proportion to the depth of the frame (*Art.* **583**), and is obtained here by increasing the distance between the fibres which resist compression and those which resist tension. It is upon this principle that roof trusses and bridge girders, alike with common beams, all depend for their stability.

605.—Horizontal Thrust.—In a frame such as *Fig.* 80, the horizontal strains produced by the weight W are balanced; or, the tension caused in the tie CD is equal to the compression caused in the short timber on which the weight rests. If the tie CD were removed, it is obvious that the weight W, acting through the two struts AW and BW, would push the two abutments AC and BD from each other, and, descending, fall through between them; unless the abutments were held in place by resistance other than that contained in the frame—such, for instance, as outside buttresses.

From this we learn the importance of a tie-beam; or, in its absence, of sufficient buttresses. From this we may also learn why it is that roof trusses framed without a horizontal tie at foot so invariably push out the walls, when constructed without exterior buttresses.

606.—Parallelogram of Forces—Triangle of Forces.—A discussion of the subject of framed girders can only be intelligently understood by those who are familiar with some of the more simple and fundamental principles of statics. One of these principles is known as the *parallelogram of forces*, or the *triangle of forces*, and is useful to the architect in measuring oblique strains due to vertical and horizontal pressures. Proof of the truth of this principle may be found in most mathematical works. (See Cape's Math., Vol. II., p. 118; chap. on Mechs., Art. 20.) In this chapter its application to construction will be shown.

In *Fig.* 81, let the lines AW and BW represent the axes of two timber struts, which, meeting at the point W, sus-

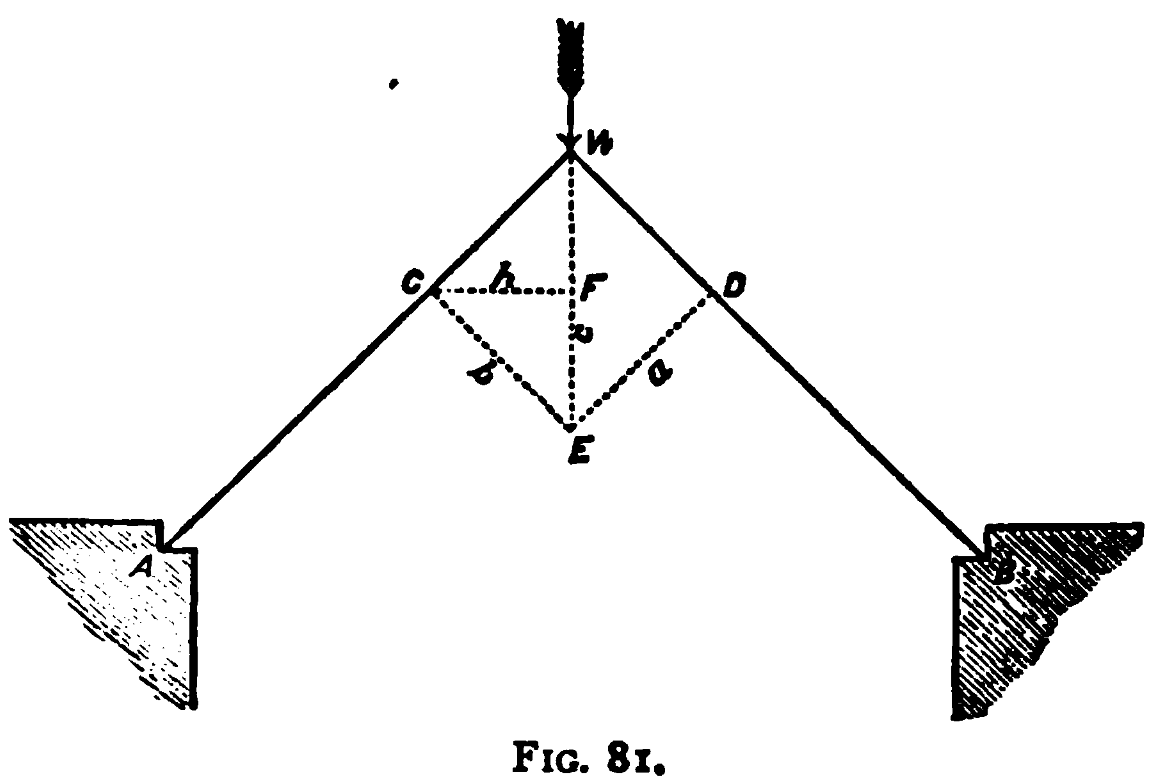

Fig. 81.

tain a weight, or vertical pressure, as indicated by the arrow at W. Then, let the vertical line WE, drawn by any

convenient scale, represent the number of pounds, or tons, contained in the vertical weight at W. From E, draw ED parallel with AW, and EC parallel with BW. $CWDE$ is the parallelogram of forces, and possesses this important property—namely, that the three lines WE, EC and CW, forming a triangle, are in proportion to three forces; the weight at W, the strain in WB, and the strain in WA.

The same is true of the other triangle WED; or, to designate more particularly, we have:—as the line WE is to the weight at W, so is the line CE, or WD, to the strain in WB; and also:—as the line WE is to the weight at W, so is the line DE, or WC, to the strain in AW. Indicating the lines by the letters a, b and c, as in the figure, we have

$$c : a :: W_{,} : A_{,}$$

$$A_{,} = W_{,}\frac{a}{c} \qquad (291.)$$

in which $A_{,}$ equals the strain caused by the weight $W_{,}$ through the line WA; and

$$c : b :: W_{,} : B_{,}$$

$$B_{,} = W_{,}\frac{b}{c} \qquad (292)$$

in which $B_{,}$ equals the strain caused by the weight $W_{,}$ through the line WB.

607.—Lines and Forces in Proportion.—The above proportions hold good when the two lines AW and BW are inclined at any angle, and whether they are of equal or of unequal lengths; indeed, the principle is general in its applica-

tion, for in all cases where the three sides of a triangle are respectively drawn parallel to the direction of three several forces which are in equilibrium, then the lengths of the three lines will be respectively in proportion to the three forces.

608.—Horizontal Strain Measured Graphically.—In *Fig.* 81, and in the triangle *WCE*, draw, from *C*, the horizontal line *CF*, or *h*; then we have the line *b*, in proportion

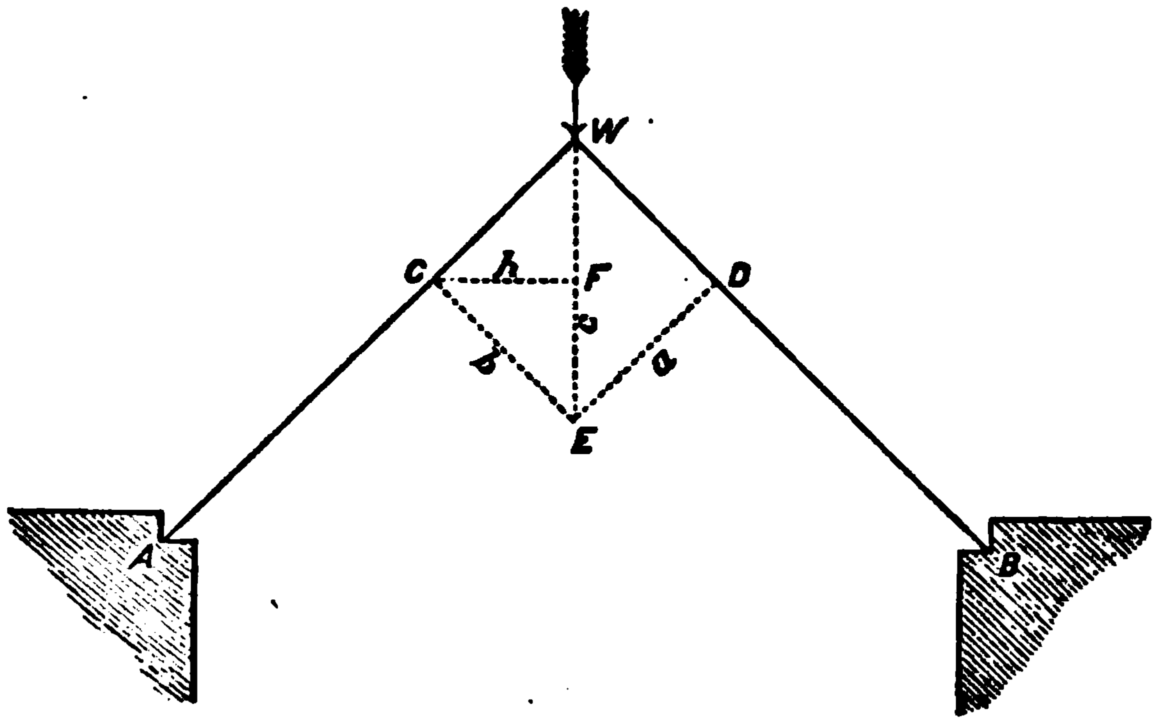

FIG. 81.

to the line *h*, as $B_{,}$, the strain in *WB*, is to $H_{,}$, the horizontal strain; or,

$$b : h :: B_{,} : H_{,} = B_{,}\frac{h}{b}$$

and by substituting the value of $B_{,}$ in formula (*292.*) have

$$H_{,} = B_{,}\frac{h}{b} = W_{,}\frac{bh}{cb} = W_{,}\frac{h}{c}$$

or the horizontal strain is measured by the quotient arising from a division by the line *c*, of the product of the weight

$W_{\prime}$ into the line h; or,

$$c : h :: W_{\prime} : H_{\prime}$$

$$H_{\prime} = W_{\prime}\frac{h}{c} \qquad (293.)$$

This measures the horizontal strain at AB, or at $W_{\prime}$ for it is the same at all points of such a frame, whatever the angle of inclination of the struts, or whether they are inclined at equal or unequal angles.

609.—Measure of Any Number of Forces in Equilibrium. —In *Fig.* 82, let AB be the axis of a horizontal timber, supported at A and B, and let AC, CD and DB be three iron rods, with two weights R and P suspended from the

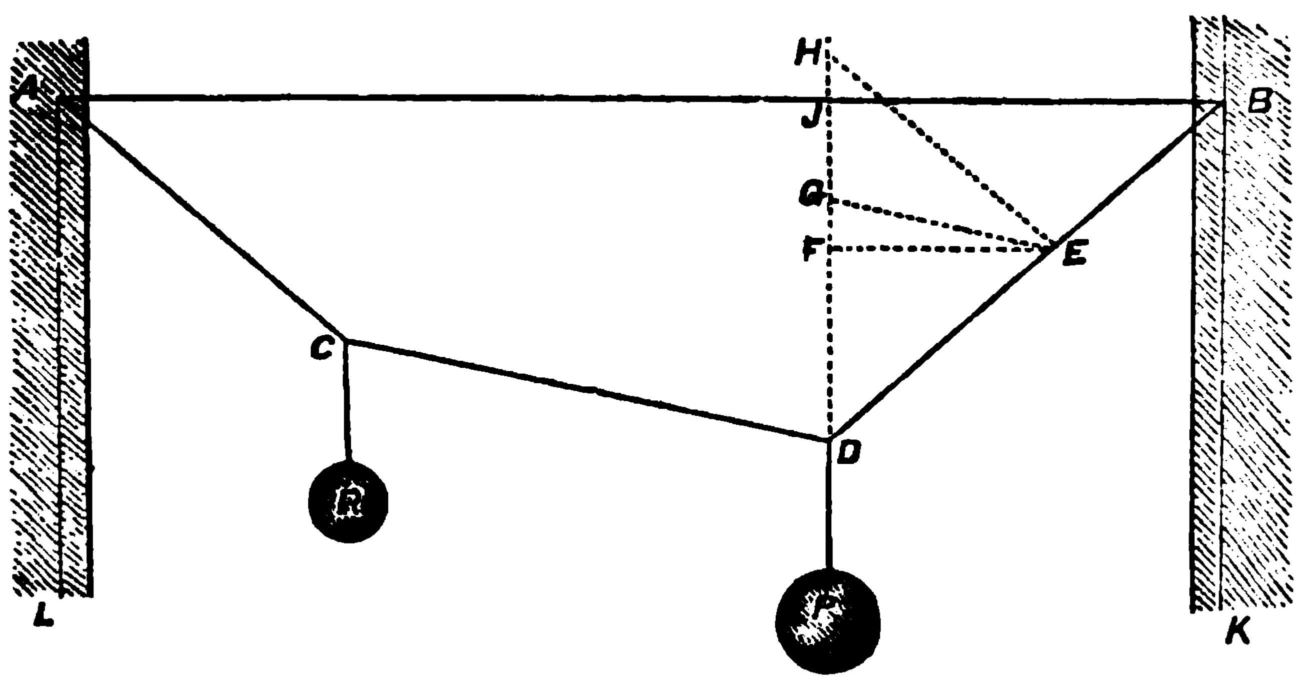

Fig. 82.

points C and D. The iron rods being jointed at A, C, D and B, so as to permit the weights to move freely, and thus to adjust themselves to an equilibrium, the whole frame $ABDC$ will be equilibrated.

From D erect a vertical, DH, and by any convenient scale make DG equal to the weight P, and GH equal to the weight R. From G, draw GE parallel with CD, and from H draw HE parallel with AC. The sides of the

triangle *GED* are parallel with the three lines *CD*, *DB* and *DP*, and consequently are in proportion as the strains in the three lines *CD*, *DB* and *DP*. Again; the sides of the triangle *HEG* are parallel with the lines *AC*, *CD* and *CR*, and consequently are in proportion as the strains in the lines *AC*, *CD* and *CR*. From *E* draw *EF* horizontal. Then the sides of the triangle *FED*, being parallel with the lines *BA*, *BD* and *BK*, are in proportion to the strains in these lines. Also, the sides of the triangle *HEF*, being parallel to the lines *AB*, *AC* and *AL*, are in proportion

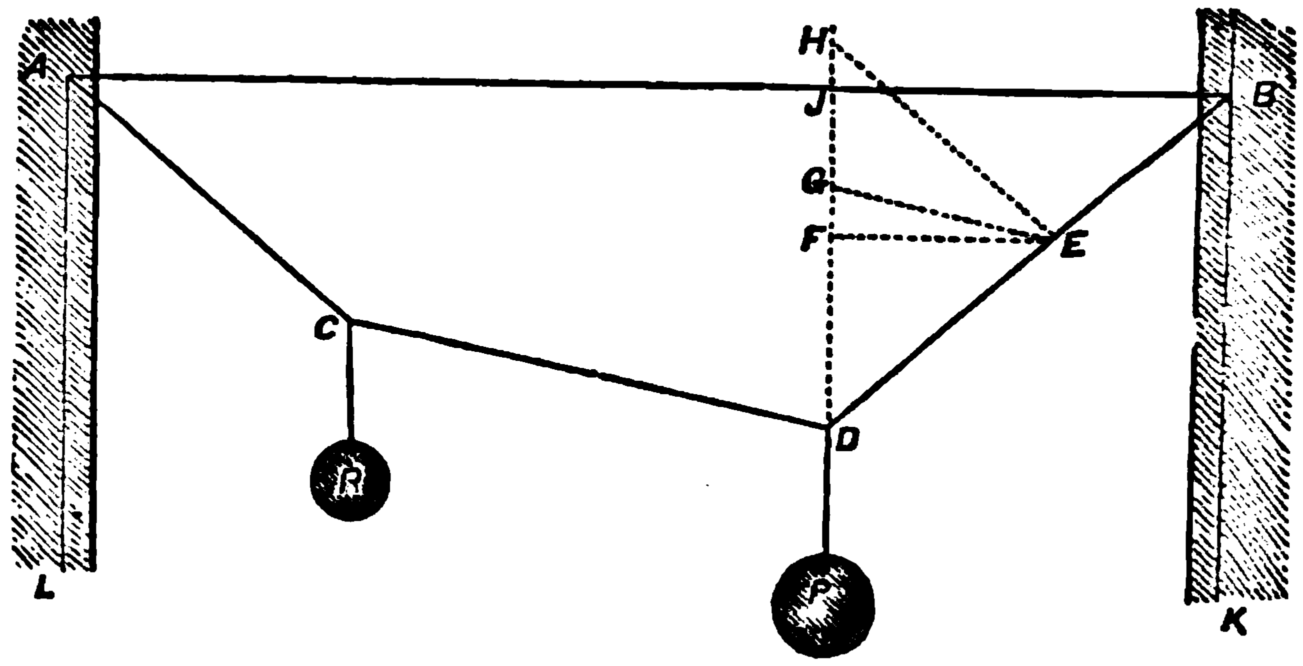

Fig. 82.

to the strains in these lines. Thus, in the triangles within *HDE*, we have the measures of all the strains of the *funicular* or string polygon *ABDCA*; *FE* being the horizontal strain, *FD* the vertical strain or load on *BK*, and *HF* the vertical strain or load on *AL*.

610.—Strains in an Equilibrated Truss.—In *Fig.* 82 the strains in the lines *AC*, *CD* and *DB* are tensile, while that in *AB* is compressive. If the lines *AC*, *CD* and *DB* were above the line *AB*, instead of below it, then these strains would all be reversed; those which are tensile in the figure would then be compressive, while that which is com-

pressive would then be tensile; but the *amount* of strain in each would be the same and be measured as in *Fig.* 82.

For example: Let *Fig.* 83 represent an equilibrated frame; the pieces *AC*, *CD*, *DE*, *EF* and *FB* suffering compression

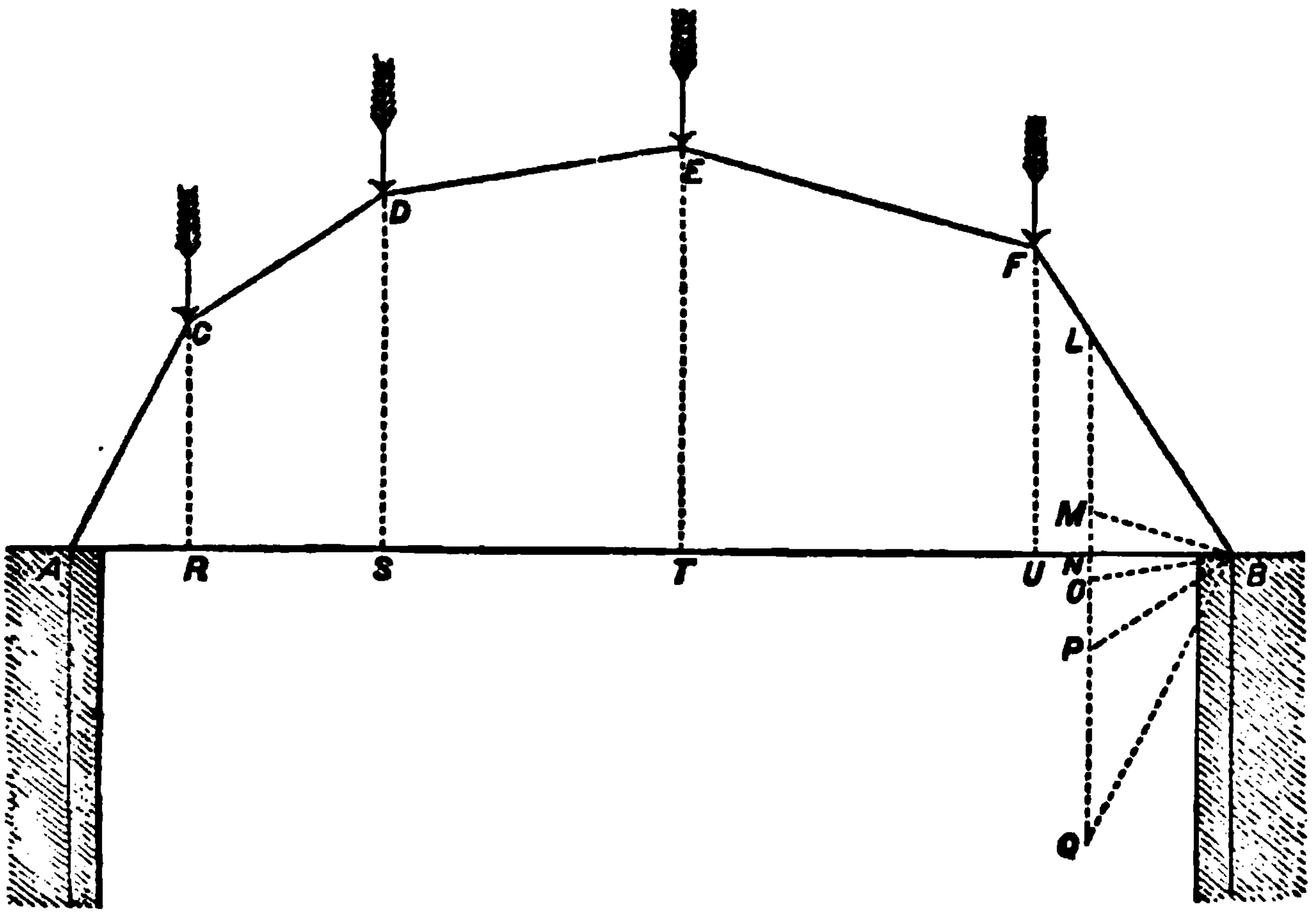

FIG. 83.

from the vertical pressures indicated by the arrows at *C*, *D*, *E* and *F*, while *AB*, a tie, prevents the frame from spreading. Draw the vertical line *LQ*, and from *B* draw radiating lines, parallel respectively with the several lines *AC*, *CD*, *DE* and *EF*, and cutting the line *LQ* at the points *Q*, *P*, *O* and *M*. Then the several lines *BQ*, *BP*, *BO*, *BM* and *BL* will be in proportion, respectively, to the strains in *AC*, *CD*, *DE*, *EF* and *FB*; and the lines *LM*, *MO*, *OP* and *PQ* will be in proportion, respectively, to the vertical pressures at *F*, *E*, *D* and *C*; while the line *LN* will represent the vertical pressure on *B*, and *NQ* that on *A*, and the line *NB* the horizontal thrust in *AB*.

611.—From Given Weights to Construct a Scale of Strains.—The construction of the scale of strains *LBQ*, as here given, is proper in a case where the points *C*, *D*, *E*

and F are fixed, and the weights and strains are required. When the weights at C, D, E and F, with their horizontal distances apart, and the two heights RC and UF, are given; then, to find the scale of strains, and incidentally the heights of the points D and E, proceed thus: From B

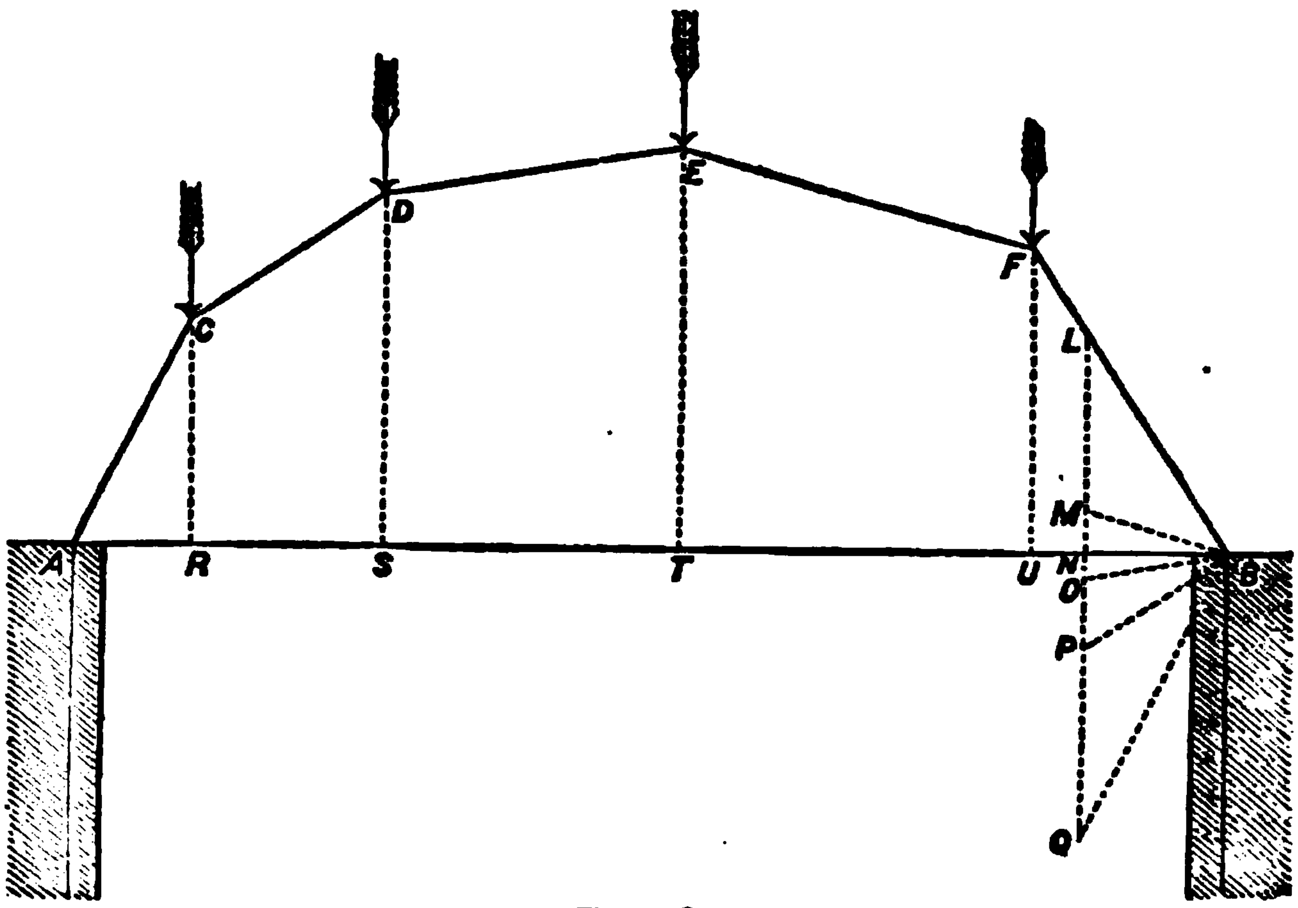

Fig. 83.

draw BQ parallel with AC, make the vertical QL equal, by any convenient scale, to the sum of the weights at C, D, E and F, and upon this vertical lay off in succession the distances LM, MO, OP and PQ, equal respectively to the weights at F, E, D and C. Then, the several lines BL, BM, BO, BP and BQ will, by the same scale, measure the several strains in BF, FE, ED, DC and CA, and BN will measure the horizontal strain.

612.—Example.—In constructing *Fig.* 83, the weights given are 11,899 pounds at F, 4253 pounds at E, 4464 at D and 11,384 at C; being a total of 32,000 pounds. The distances AR, RS, etc., are successively 8, 13, 20, 24 and 13; in all 78 feet. The height $RC = 15$, and $UF = 20\frac{1}{4}$.

With these dimensions all laid down as in *Fig.* 83, draw BQ parallel with AC. Draw the line LQ vertical, and at such a distance from B as that its length shall, by a scale of equal parts, be equal to the total load on the four points C, D, E and F; or to a multiple of the total load. For example: a scale of 100 parts to the inch will be convenient in this case, by appropriating 4 parts to the thousand pounds. The 32,000 pounds require $32 \times 4 = 128$ parts for the length of the line LQ, and the several other weights and distances require as follows:

$$
\begin{aligned}
LM &= 4 \times 11 \cdot 899 = 47 \cdot 596 \\
MO &= 4 \times 4 \cdot 253 = 17 \cdot 012 \\
OP &= 4 \times 4 \cdot 464 = 17 \cdot 856 \\
PQ &= 4 \times 11 \cdot 384 = 45 \cdot 536
\end{aligned}
$$

The sum of these,

$$LQ = 47 \cdot 596 + 17 \cdot 012 + 17 \cdot 856 + 45 \cdot 536 = 128$$

as before. Therefore, draw LQ at such a distance from B that it will, by the scale named, equal 128 parts. On this line lay off the distances $LM = 47 \cdot 596$, $MO = 17 \cdot 012$, etc., as above given. Join B with each of the points P, O and M. These lines give the directions of the lines CD, DE and EF; therefore, draw FE parallel with BM, ED parallel with BO, and DC parallel with BP.

By applying the scale to the lines radiating from B, the strains in the several lines AC, CD, etc., will be shown.

BQ, by the scale, measures 80 parts, therefore $\frac{80}{4} = 20$; or the strain in AC is 20,000 pounds.

BP measures 45 parts, and $\frac{45}{4} = 11\frac{1}{4}$; or the strain in CD is 11,250 pounds.

BO measures 38, and $\frac{38}{4} = 9\frac{1}{2}$; or the strain in DE is 9500.

BM measures 39, and $\frac{39}{4} = 9\frac{3}{4}$; or the strain in EF is 9750.

BL measures 69·5, and $\frac{69 \cdot 5}{4} = 17 \cdot 375$; or the strain in FB is 17,375.

BN measures 37·5, and $\frac{37 \cdot 5}{4} = 9 \cdot 375$; or the horizontal strain is 9375 pounds.

Also, as LN measures 58, therefore $\frac{58}{4} = 14{,}500$, equals the load on B; and as NQ measures 70, therefore $\frac{70}{4} = 17{,}500$, equals the load on A; and the two loads A, and B, together equal $17500 + 14500 = 32000$, equals the total load.

In practice the diagram should be large, for the accuracy of the results will be in proportion to the size of the scale, as well as to the care with which it is drawn and measured. The size above taken is large enough for the purposes of illustration merely, but in practice the diagram should be drawn at a scale of 12 feet to the inch; or, still better, at 8 feet. (See *Art.* **615**.)

613.—Horizontal Strain Measured Arithmetically.—In the last article, directions were given for locating the line LQ, *Fig.* 83. This line may be located more precisely by arithmetical computation, and the horizontal thrust be thus defined more accurately than is there done. In *Fig.* 84, showing parts of *Fig.* 83 enlarged, we have the triangles ACR and BFU, the same as in *Fig.* 83.

The triangle ACR, as stated (*Art.* **612**), has a base of 8 and a height of 15. Make AY equal 10, and draw YZ vertical. We now have this proportion,

$$AR : RC :: AY : YZ$$

or

$$8 : 15 :: 10 : YZ = \frac{10 \times 15}{8} = 18 \cdot 75$$

Again; triangle BFU, as stated (*Art.* **612**), has a base of 13 and a height of $20\frac{1}{4}$. Make BX equal 10, and draw XV

vertical. We have now this proportion,

$$BU : UF :: BX : XV \qquad \text{or}$$

$$13 : 20\tfrac{1}{4} :: 10 : XV = \frac{10 \times 20\tfrac{1}{4}}{13} = 15 \cdot 577$$

Thus we have the two angles at A and B comparable, for, with a common base of 10, the one at A has a height of 18·75, while the one at B is 15·577. The two triangles may now be put together at the line BU. Extend the vertical line VX to W, make XW equal to $YZ = 18 \cdot 75$, and join W and B. Then the triangle BXW equals the triangle AYZ, and BW is parallel with AZ.

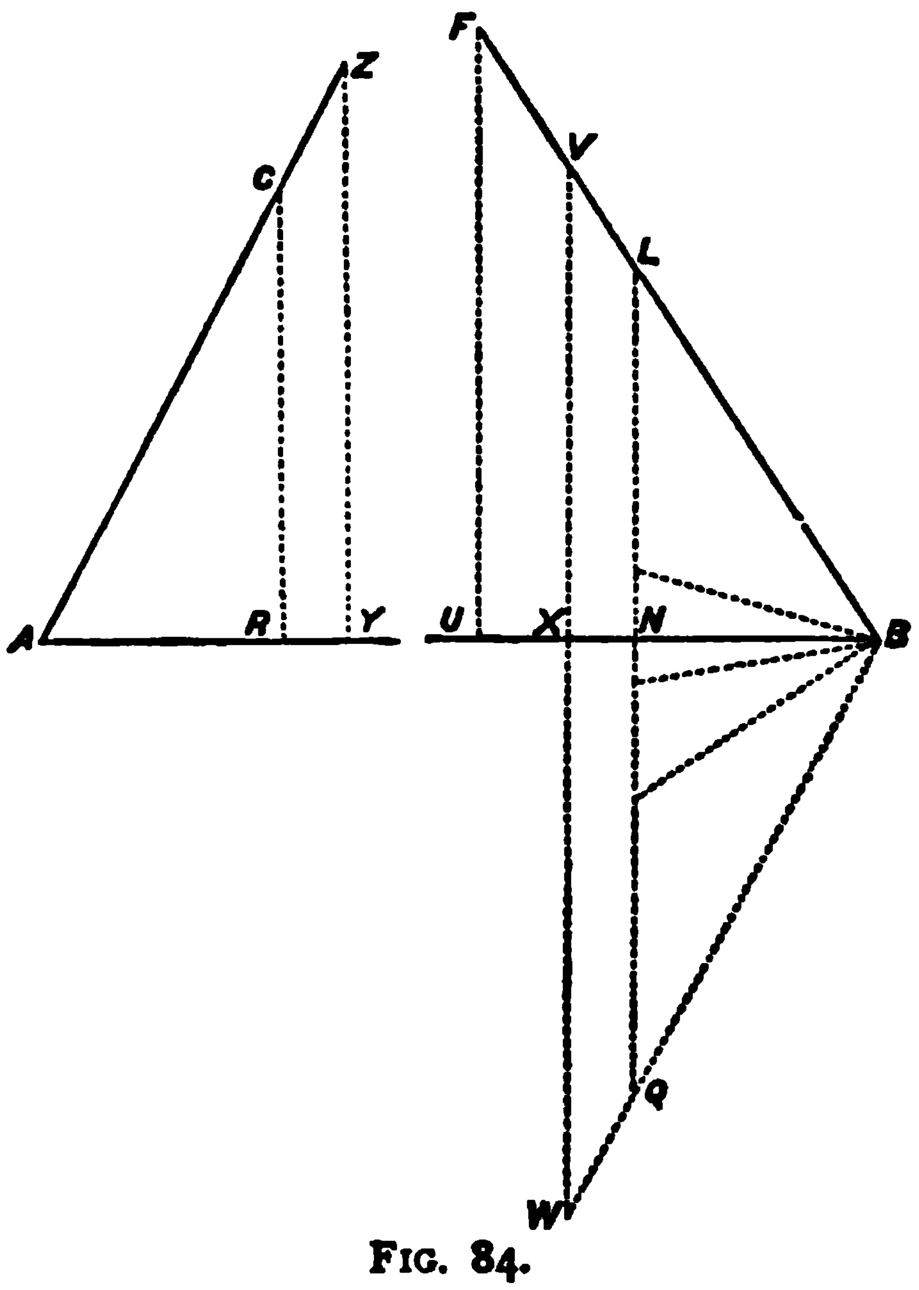

FIG. 84.

The problem now is to locate the point N, so that the vertical LQ drawn through N shall be equal, by any given scale, to the total of the loads at C, D, E and F. To do this we have

$$BN : NL :: BX : XV = \frac{BX \times NL}{BN}$$

$$BN : NQ :: BX : XW = \frac{BX \times NQ}{BN}$$

By addition we have

$$XV + XW = \frac{BX \times NL}{BN} + \frac{BX \times NQ}{BN} = \frac{BX(NL + NQ)}{BN}$$

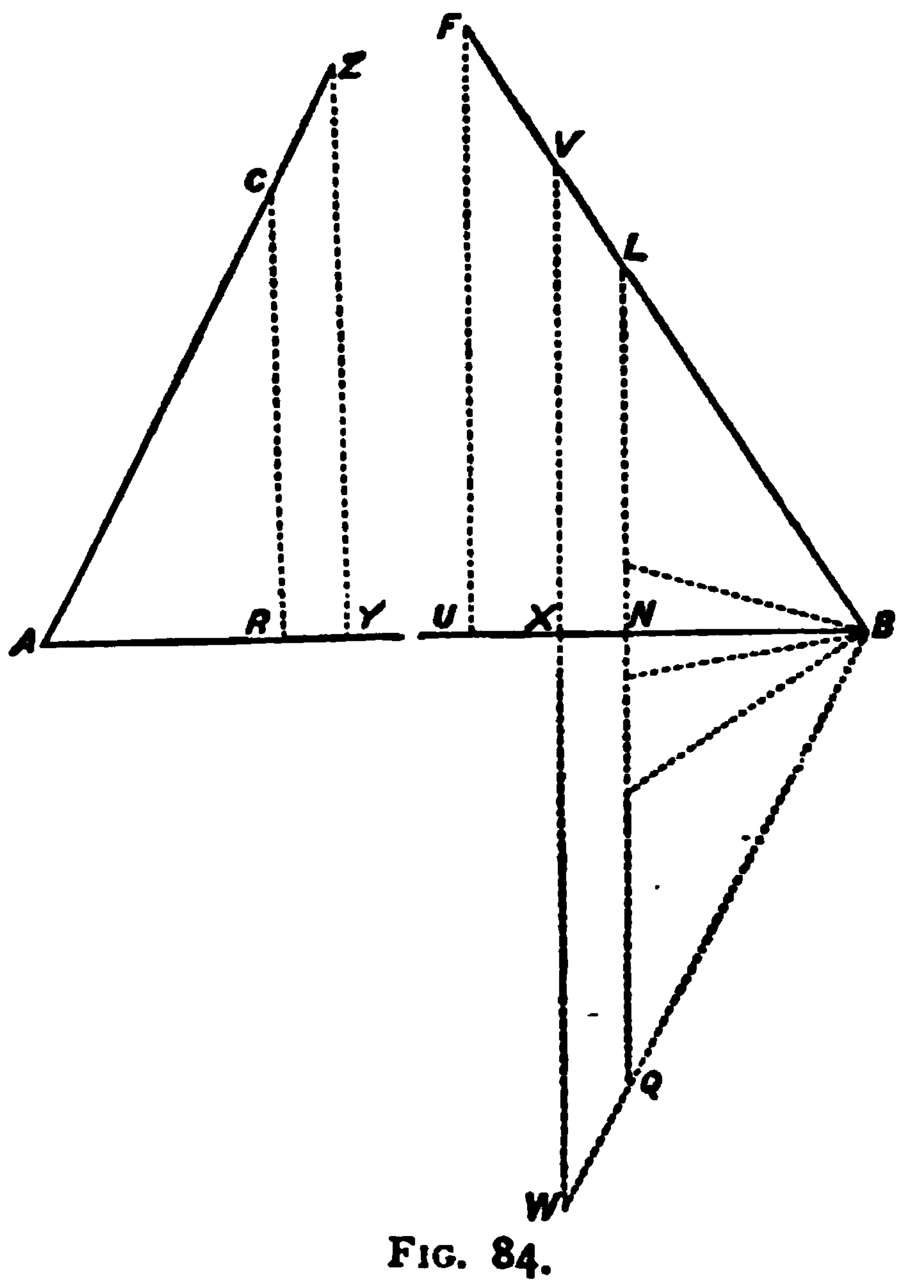

FIG. 84.

By the diagram, $XV + XW = VW$, and $NL + NQ = LQ$, and therefore

$$VW = \frac{BX \times LQ}{BN}$$ or

$$VW : BX :: LQ : BN = \frac{BX \times LQ}{VW}$$

The total load is 32,000, for which we may, putting one for a thousand, make LQ equal to 32; and, since VW equals $YZ + VX = 18 \cdot 75 + 15 \cdot 577 = 34 \cdot 327$, and $BX = 10$, we have

$$BN = \frac{10 \times 32}{34 \cdot 327} = 9 \cdot 322$$

defining accurately the horizontal thrust BN as $9 \cdot 322$, or 9322 pounds.

614.—Vertical Pressure upon the Two Points of Support.—This pressure was shown in *Art.* **612** by the diagram, but may be more accurately determined by arithmetical computation, as follows: In the last article the horizontal thrust BN was shown to be 9322 pounds. We have the proportion

$$BX : XV :: BN : NL$$

$$10 : 15 \cdot 577 :: 9 \cdot 322 : NL = \frac{15 \cdot 577 \times 9 \cdot 322}{10} = 14 \cdot 521$$

or the vertical strain upon the support B is 14,521 pounds. To find that upon the support A we have

$$BX : XW :: BN : NQ$$

$$10 : 18 \cdot 75 :: 9 \cdot 322 : NQ = \frac{18 \cdot 75 \times 9 \cdot 322}{10} = 17 \cdot 479$$

or the vertical strain upon the support A is 17,479 pounds and the two, $17479 + 14521 = 32000$, equals the total load.

615.—Strains Measured Arithmetically.—The resulting strains in *Fig.* 83, as obtained by scale in *Art.* **612**, are close approximations, and are near enough for general purposes. The exact results can be had arithmetically, as in *Arts.* **613** and **614**. For example: In *Art.* **612** the horizontal thrust was found by scale to be 9375, while in *Art.* **613** it was more accurately defined by arithmetical process to be 9322. So, also, the portions of the total load borne by the two

supports, A and B, were found by scale to be 17,500 and 14,500, respectively, while in *Art.* **614** they were accurately fixed at 17,479 on A and 14,521 on B. A carefully drawn diagram at a large scale will generally be sufficient for use, but it is well, in important cases, to compute the dimensions also. When both are done, the scale measurements serve as a check against any gross errors in the computations.

In *Art.* **612** the strains in the several timbers are given, as ascertained by scale. By the rules for computing the sides of a right-angled triangle (the 47th of first book of Euclid), the several strains, as represented by BL, BM, BO, etc. (*Fig.* 83), may be found arithmetically. The following list shows the results by this method, side by side with those by scale:

	By Scale.		By Computation.
$BL =$	17,375	and	17,256
$BM =$	9,750	"	9,684
$BO =$	9,500	"	9,464
$BP =$	11,250	"	11,138
$BQ =$	20,000	"	19,810

616.—Curve of Equilibrium—Stable and Unstable.—When the positions of the supporting timbers AC, CD, DE, etc. (*Fig.* 83), are regulated in accordance with the weights upon the points C, D, E, etc., and, as shown in *Art.* **611**, the frame is in a state of equilibrium; and a curve drawn through the points A, B, C, D, etc., is called the *curve of equilibrium*. When the several weights are numerous, are equal, and are located at equal distances apart; or, when the load is uniformly distributed over the length of the frame, this curve is a parabola. In these cases, if the rise

be small in comparison with the base, the curve is nearly the same as a segment of a circle, and the latter may be used without serious error. (Tredgold's Carpentry, Arts. 57 and 171.)

The pressures in an equilibrated frame act only in the axes of the timbers composing the frame, and these carry the effects of the several weights on, from point to point, until they successively arrive at A and B, the points of final support. A frame thus balanced is not, however, stable, for if subjected to additional pressure, however small, at any one of the points of support, it is liable to derangement; and if so deranged it has no inherent tendency to recover itself, but the distortion will increase until total failure ensues. A frame thus conditioned is therefore said to be in a state of *unstable equilibrium;* while a frame of suspended pieces, as in *Fig.* 82, is said to be in a state of *stable equilibrium*, since, if disturbed by temporary pressures, it will recover its original position when they are removed.

617.—Trussing a Frame.—The tendency to derangement and consequent failure, in a frame such as *Fig.* 83, can be counteracted by additional pieces termed braces, located in any manner so as to divide the inclosed spaces into triangles. For example: it may be divided into the triangles ACS, CSD, DST, DTE, ETF, TFU and UFB. If these additional pieces be adequate to resist such pressures as they may be subjected to, and be firmly connected at the joints, the frame will thereby be rendered completely stable. Treated in this way the frame becomes a *truss*, from the fact that it has been trussed or braced.

618.—Forces in a Truss Graphically Measured.—When a frame is divided into triangles, as proposed in the last article, sometimes three or more pieces meet at the same

point. In such a case, owing to the complexity of the forces, it becomes difficult to trace, and, by the parallelogram of forces, to measure them all. Professor Rankine, in his "Applied Mechanics," somewhat extended the use of the triangle of forces in its application to such cases. It was afterward more fully developed and generalized by Professor J. Clerk Maxwell in a paper read before the British Association in 1867, and by him termed "Reciprocal Figures, Frames and Diagrams of Forces;" and Mr. R. H. Bow, C.E., F.R.S.E., in his "Economics of Construction," has simplified the method in its use, by a system of reciprocal lettering of lines and angles. By Professor Maxwell's method, the forces in any number of pieces converging at one point are readily determined. The principle involved is very simple, and is this: Construct a closed polygon, with lines parallel to the direction of, and equal in length to, the amount of the forces which in the framed truss meet at any point. A system of such polygons, one for each point of meeting of the forces, so constructed that in it no line representing any one force shall be repeated, is termed a *diagram of forces.*

619.—Example.—Let *Fig.* 85 represent a point of convergence of parts of a framed truss, and *Fig.* 86 be its corresponding diagram of forces, in which latter the lines are drawn parallel to the lines in *Fig.* 85. Designate a line in the diagram in the usual manner, by two letters, one at each end of the line, and indicate the corresponding line in *Fig.* 85 by placing the same two letters one on each side of the line. For instance, the line AB of 86 is parallel with that line of 85 which lies between the spaces A and B; and so of each of the other corresponding lines. In *Fig.* 86 the lines are in proportion to each other, respectively, as the forces in the corresponding lines of *Fig.* 85. Thus if AD (86), by any scale,

represents the force in the line between A and D (85), then will the line AB equal the force in the line between A and B; and in like manner for the other lines and strains.

620.—Another Example.—Let *Fig.* 87 represent the axial lines of the timbers of a roof truss, and its two sustaining piers, and let *Fig.* 88 be its corresponding diagram of forces.

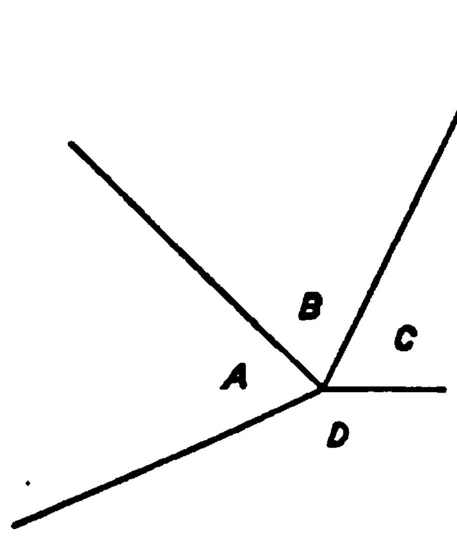

FIG. 85.

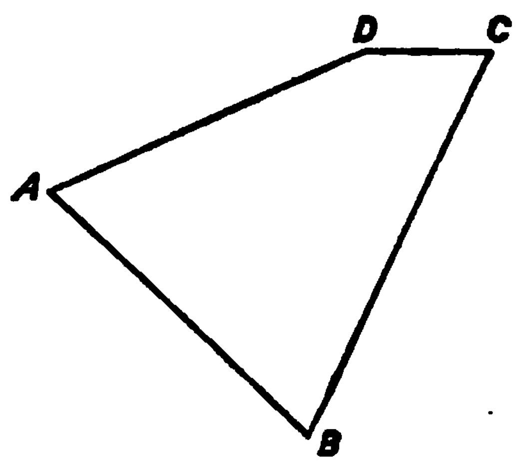

FIG. 86.

The truss being loaded uniformly, the three arrows (87)—one at the ridge and one at the apex of each brace—represent equal loads. Let these three loads be laid down to a suitable scale on the line FJ (88), one extending from F to G, another from G to H, and the third from H to J. The half of these, or FE, is the load sustained on one of the supports of *Fig.* 87, and the other half, EJ, is the load upon the other support. In *Fig.* 87 a letter is placed in each triangle, and one in each partly-enclosed space outside of the truss. Each line of the figure is to be designated by the two letters which it separates; thus, the line between A and E is called line AE, the line between A and B is called line AB, etc. In *Fig.* 88 the corresponding lines are designated by the same letters; the letters here being, as usual, at the ends of the lines. Also, it will be observed that while in the diagram any point is designated in the usual way by the letter standing at it, it is the practice in

the frame to designate a point by the several letters which cluster around it; for example, the point of support *FAE* (*Fig.* 87). The diagram, *Fig.* 88, is constructed by drawing a line parallel with each of the lines in *Fig.* 87. Thus the three

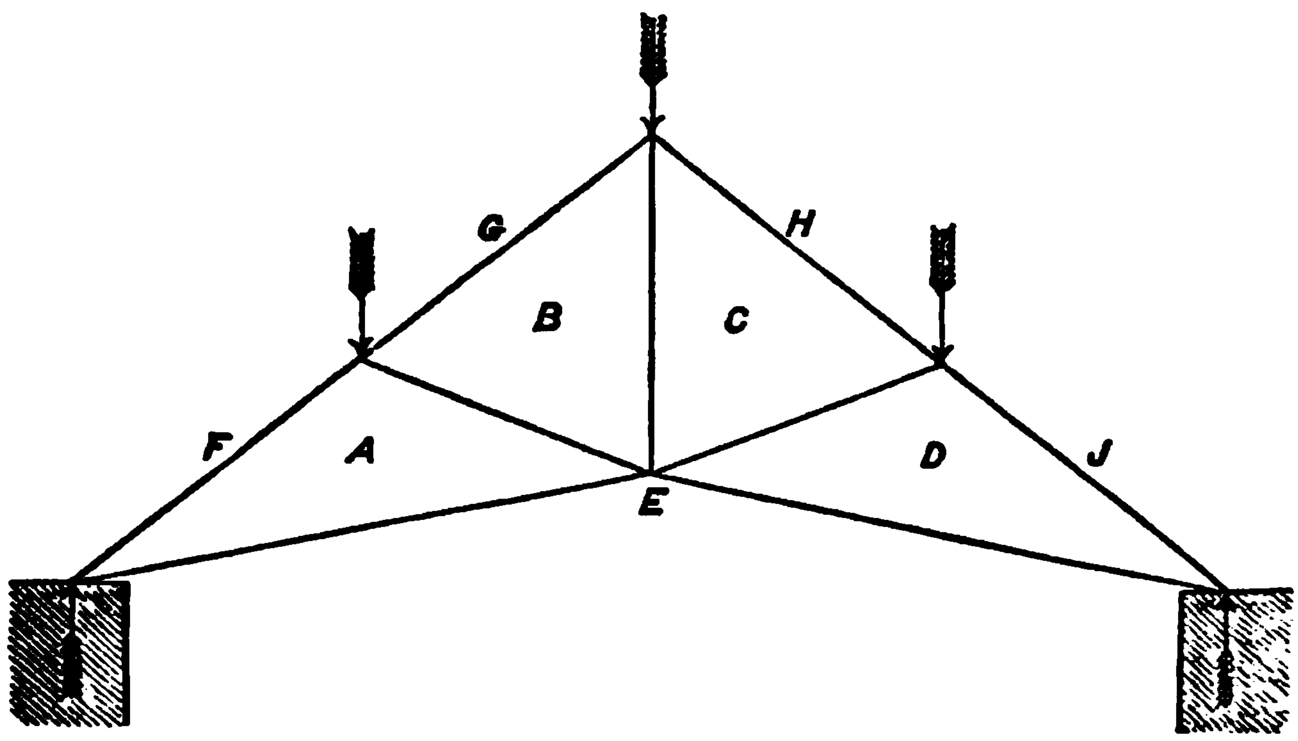

FIG. 87.

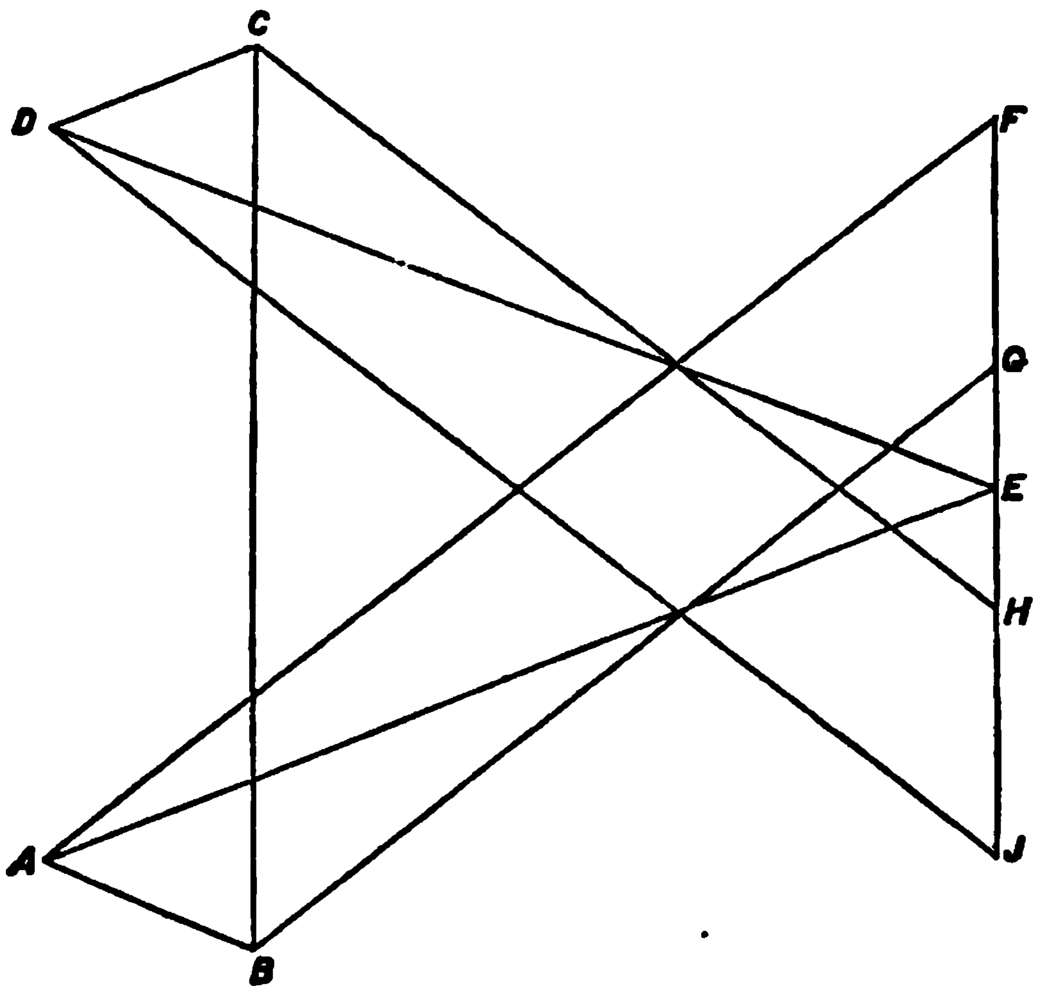

FIG. 88.

forces converging in *Fig.* 87 at *FAE*, the left-hand point of support, are *EF*, *FA* and *AE*; and in *Fig.* 88 the lines *EF*, *FA* and *AE*, drawn parallel with the corresponding lines of *Fig.* 87, form the triangle *EFA*. Taking the forces at

point *GBAF*, 87, we find them to be *AF*, *FG*, *GB* and *BA*, and drawing, in 88, lines parallel with these, we obtain the quadrangle *AFGBA*. Again, in 87 the forces at point *GBCH* are *BG*, *GH*, *HC* and *CB*, and drawing, in 88, lines parallel with these, we obtain the closed polygon *BGHCB*. At point *HCDJ* (87) the forces are *CH*, *HJ*, *JD* and *DC*, and, in 88, drawing the corresponding lines produces the quadrangle *CHJDC*. In 87 the forces at point *JED*, the right-hand support, are *DJ*, *JE* and *ED*, and in 88, the corresponding lines produce the triangle *DJE*. In 87, at point *ABCDE*, we have the five forces *EA*, *AB*, *BC*, *CD* and *DE*, and, in 88, the corresponding lines give the closed polygon *EABCDE*. This completes the diagram of forces, *Fig.* 88, in which the several lines, measured by the same scale with which the three loads were laid off on *FJ*, are equal to the corresponding forces in the similar parts of *Fig.* 87.

621.—Diagram of Forces.—In this manner the diagram of forces may be drawn to represent the strains in a framed truss, by carefully following the directions of *Art.* **618**; commencing by first laying down the forces which are known; from which the ones to be found will gradually be determined until the whole are ascertained.

622.—Diagram of Forces—Order of Development.—When more than two of the forces converging at any one point are undefined in amount, the diagram can not be completed. Thus, where three forces converge it is requisite to know one of them. Of four forces, two must be known. Of five forces, three must be known.

In constructing a diagram, the first thing necessary is to establish the line of loads, as *FJ* in *Fig.* 88, then to ascertain the portion of the total load which bears upon each of the

points of support, *AEF* and *JED* (*Art.* **56**) (one half on each when the load is disposed symmetrically), and, with this, to obtain the first triangle *FEA*. From this proceed up the rafters, or to where the points of convergence have the fewest strains, leaving the more complex points to be treated later. In this way the most of the forces which affect the crowded points will be developed before reaching those points.

623.—Reciprocal Figures.—By comparing *Figs.* 87 and 88, we see that the lines enclosing any one of the lettered spaces in the former are, in the latter, found to radiate from the same letter. The space *A* (87) has for its boundaries the lines *AF*, *AB* and *AE*, and these same lines in 88 radiate from the letter *A*. The space *E* (87) has for its enclosing lines *EF*, *EA*, *ED* and *EJ*, and these same lines are found to radiate from the point *E* (88). Thus the diagram (88) is a reciprocal of the frame (87).

624.—Proportions in a Framed Girder.—In order to treat of the method of measuring strains in trusses, we have digressed from the main subject. Returning now, and referring to the relations existing between a girder and a simple beam, as in *Arts.* **603** to **605**, we proceed to develop the proportion in a girder, between the length and depth.

A girder, as generally used, serves to support a tier of floor beams at a line intermediate between the walls of the building, and when sustained by posts at points not over 12 to 15 feet apart, may be made of timber in one single piece. But when a girder is required to span greater distances than these, it becomes requisite, by some contrivance, to increase its depth, in order to obtain the requisite strength. An increase of depth, however, may interfere with the demand for clear, unobstructed space in rooms so

large as those in which girders are required. To prevent this interference, the depth of the girder should be the least possible; although diminishing the depth will increase the cost; for the cost will be in proportion to the amount of material in the girder, and this will be in proportion to the strains in its several parts, and the strains will be inversely as the height. For economy's sake, therefore, as well as for strength, the girder should have a fair depth; modified, however, by the demand for unobstructed space.

Where other considerations do not interfere to prevent it, the depth of a framed girder should be from $\frac{1}{12}$ to $\frac{1}{8}$ of the length; the former proportion being for girders 25 feet long, and the latter for those 125 feet long. If these two rates be taken as the standard rates, respectively, of two girders thus differing in length 100 feet, and all other girders be required to have their depths proportioned to their lengths in harmony with these standards, their rates will be regularly graduated. In order to develop a rule for this, let the two standards be reduced to a common denominator, or to $\frac{2}{24}$ and $\frac{3}{24}$. If their difference, $\frac{1}{24}$, be divided into 100 parts, each part will equal

$$\frac{1}{24 \times 100} = \frac{1}{2400}$$

and will equal the difference in rate for every foot increase in length of girder; for the two standards are 100 feet apart. The scale of rates thus established is for lengths of girder from 25 to 125 feet, but it is desirable to extend the scale back over the 25 feet to the origin of lengths. To do this, we have for the difference in rates for this 25 feet, $25 \times \frac{1}{2400} = \frac{25}{2400} = \frac{1}{96}$. Deducting this from $\frac{1}{12}$ $(= \frac{8}{96})$, the rate at 25 feet, we have $\frac{8}{96} - \frac{1}{96} = \frac{7}{96}$, the rate between depth and length at the origin of lengths (if such a thing were there possible). Now if to this base of rates we

add the increase ($\frac{1}{2400}$ of the length) the sum will be the rate at any given length. As an example: What should be the rate, by this rule, for a girder 125 feet long? For this the difference in rate is $125 \times \frac{1}{2400} = \frac{125}{2400} = \frac{5}{96}$. Adding this to the base of rates, or to $\frac{7}{96}$, as above, and the sum $\frac{5}{96} + \frac{7}{96} = \frac{12}{96} = \frac{1}{8}$, the required rate. This is one of the standard rates. The other standard may be found by the rule thus, $25 \times \frac{1}{2400} = \frac{25}{2400} = \frac{1}{96}$. Adding this to the base $\frac{7}{96}$, gives $\frac{8}{96} = \frac{1}{12}$, the standard rate. We have therefore for the rate at any length

$$r = \frac{7}{96} + \frac{1}{2400}\,l = \frac{175}{2400} + \frac{l}{2400} = \frac{175 + l}{2400}$$

$$r = \frac{175 + l}{2400}$$

This gives the rate of depth to length, and since the depth is equal to the rate multiplied by the length, therefore

$$rl = \frac{(175 + l)\,l}{2400} \qquad \text{or}$$

$$d = \frac{(175 + l)\,l}{2400} \tag{294.}$$

in which d is the depth between the axes of the top and bottom chords, and l is the length (between supports), both being in feet.

This rule will give the proper depth of a girder, and may be used when the depth is not fixed arbitrarily by the circumstances of the case. (See *Art.* **572**.)

625.—Example.—What should be the depth of a girder which is 40 feet long between supports?

By formula (*294.*),

$$d = \frac{(175 + 40) \times 40}{2400} = 3 \cdot 58\tfrac{1}{3}$$

or the economical depth is 3 feet and 7 inches, measured from the middle of the depths of the top and bottom chords.

Again: What should be the depth of a girder which is 100 feet long in the clear between supports? By (*294.*),

$$d = \frac{(175 + 100) \times 100}{2400} = 11 \cdot 458\tfrac{1}{3}$$

or the depth between the axes of the chords should be 11 feet $5\frac{1}{2}$ inches.

A girder 125 feet long would by this rule be 15 feet $7\frac{1}{2}$ inches, or $\frac{1}{8}$ of its length, in depth; while a girder 25 feet long would be 2 feet and 1 inch deep between the axes, or $\frac{1}{12}$ of the span.

626.—Trussing, in a Framed Girder.—One object to be obtained by the trussing pieces—the braces and rods—is to transmit the load from the girder to the abutments. The braces and rods forming the trussing may be arranged in a great variety of ways (see Bow's Economics of Construction), but that system is to be preferred which will take up the load of the girder at proper intervals, and transmit it to its two supports in the most direct and economical manner.

Just which of the great number of systems proposed will the more nearly perform these requirements it will perhaps be somewhat difficult to determine, but the one in which the struts and ties are arranged in a chain of isosceles triangles is quite simple, and offers advantages over many others. It is therefore one which may be adopted with good results.

627.—Planning a Framed Girder.—After fixing upon the height (*Art.* **624**), the next point is as to the number of *panels* or *bays*. These should be of such length as to afford points of support at suitable intervals along the girder, and the rods and struts should be placed at such an angle as will

secure a minimum for the strains in the truss. To set the braces and ties always at the same angle, would result in furnishing points of support at intervals too short in the girders of short span, and too long in those of long span. So also, if the width of a bay be a constant quantity, there would be too great a difference in the angles at which the rods and struts would be placed. To determine the number of bays, so as to avoid as far as practicable these two objections—*first*, we have the number of the bays directly as the length of the truss and inversely as the depth, and, *second* (to vary this proportion as above suggested), we may deduct from this result a quantity inversely proportioned to the length of the girder. Combining these, we have, n being the number of bays,

$$n = \frac{l}{d} - \frac{120-l}{c}$$

and by substituting for d its value as in formula (*294.*),

$$n = \frac{l}{\frac{(175+l)l}{2400}} - \frac{120-l}{c}$$

$$n = \frac{2400}{175+l} - \frac{120-l}{c}$$

in which l is the length of the girder in feet, and c is a constant, to be developed by an application to a given case.

To this end we have, from the last formula,

$$\frac{120-l}{c} = \frac{2400}{175+l} - n$$

$$c = \frac{120-l}{\frac{2400}{175+l} - n}$$

With $n = 4 \cdot 5$ and $l = 20$, we have

$$c = \frac{120 - 20}{\frac{2400}{175 + 20} - 4 \cdot 5} = \frac{100}{12 \cdot 308 - 4 \cdot 5} = 12 \cdot 807$$

or, say $c = 12 \cdot 8$; and with this value

$$n = \frac{2400}{175 + l} - \frac{120 - l}{12 \cdot 8} \qquad (295.)$$

which is a rule for determining the number of bays in a truss, when not determined arbitrarily by the circumstances of the case, and when the height of the girder is obtained as in formula (*294.*). In the resulting value of n, the fraction over a whole number is to be disregarded, unless greater than $\frac{1}{2}$, in which latter case unity should be added to the whole number.

628.—Example.—What should be the number of bays in a truss 80 feet long?

Here $l = 80$, and, by the formula, (*295.*),

$$n = \frac{2400}{175 + 80} - \frac{120 - 80}{12 \cdot 8} = \frac{2400}{255} - \frac{40}{12 \cdot 8} = 6 \cdot 287$$

or the required number of bays is six; disregarding the decimal $\cdot 287$ because it is less than $\frac{1}{2}$.

629.—Example.—How many bays are required in a girder 110 feet long?

Here $l = 110$, and, by formula (*295.*),

$$n = \frac{2400}{175 + 110} - \frac{120 - 110}{12 \cdot 8} = 7 \cdot 64$$

or, adding unity for the decimal $\cdot 64$, the number required is 8.

630.—Number of Bays in a Framed Girder.—According to the above rule, the number of bays or panels required in framed girders of different lengths is as follows:

Girders	from	20	to	59	feet	long	should	have	5	bays.
"	"	59	"	85	"	"	"	"	6	"
"	"	85	"	107	"	"	"	"	7	"
"	"	107	"	127	"	"	"	"	8	"
"	"	127	"	146	"	"	"	"	9	"

In cases where the length exceeds 120 feet, the quantity of the formula to be deducted $\left(\frac{120-l}{12 \cdot 8}\right)$ becomes a negative quantity, and, since deducting a negative quantity is equivalent to adding a positive one, the result may be added, thus:

$$-\frac{120-144}{12 \cdot 8} = -\frac{-24}{12 \cdot 8} = -(-1 \cdot 875) = +1 \cdot 875$$

631.—Forces in a Framed Girder.—Let *Fig.* 89 represent the *axial* lines of a framed girder, or the imaginary lines passing through the axes of the several pieces composing the frame. Let the load, equally distributed, be divided into six parts, one of which acts at the apex of each lower triangle. We may notice here that in a truss with an even number of lower triangles, as in *Fig.* 89, there is an even number of loads, one half of which are carried by the struts and rods to one point of support, and the other half to the other support. Thus the load *PQ*, at point *PEFGQ*, is sustained by the top chord, and by the strut *EF*. The portion passing down this strut is carried by the rod *DE* up to the top chord, and thence, together with the load *OP*, at point *OCDEP*, down by the strut *CD* to the bottom chord. This accumulated load is carried by the rod *BC* up to the top chord,

and thence, with the addition of the last load *AO*, at *ABCO*, finally reaches, through the strut *AB*, the point of support for that end of the truss. The three weights on the other side are in like manner conveyed to *MNT*, the other point of support. We here see the manner in which, in a framed girder upon which the load is uniformly distributed, one half is carried by the trussing pieces to each point of support.

632.—Diagram for the above Framed Girder.—*Fig.* 90 is a diagram constructed as per *Arts.* **619** and **620**, and represents the strains or forces in the framed girder of *Fig.* 89.

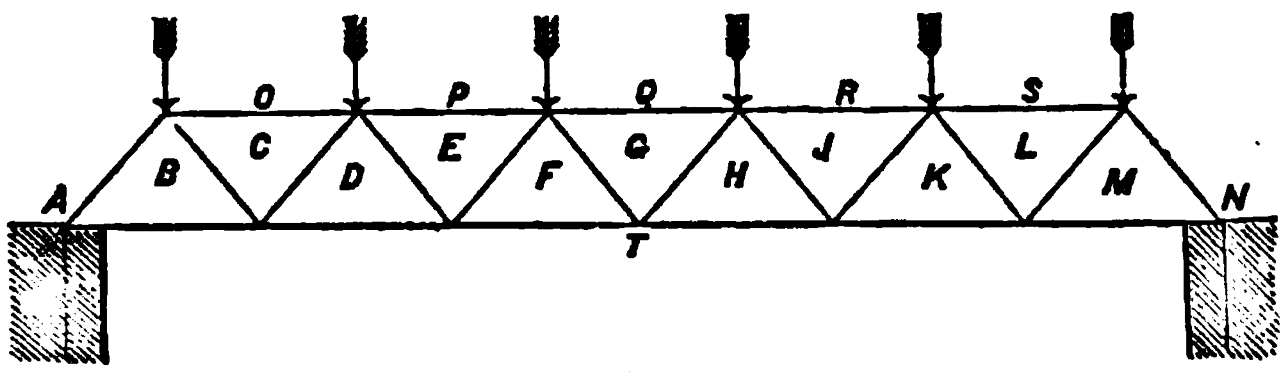

FIG. 89.

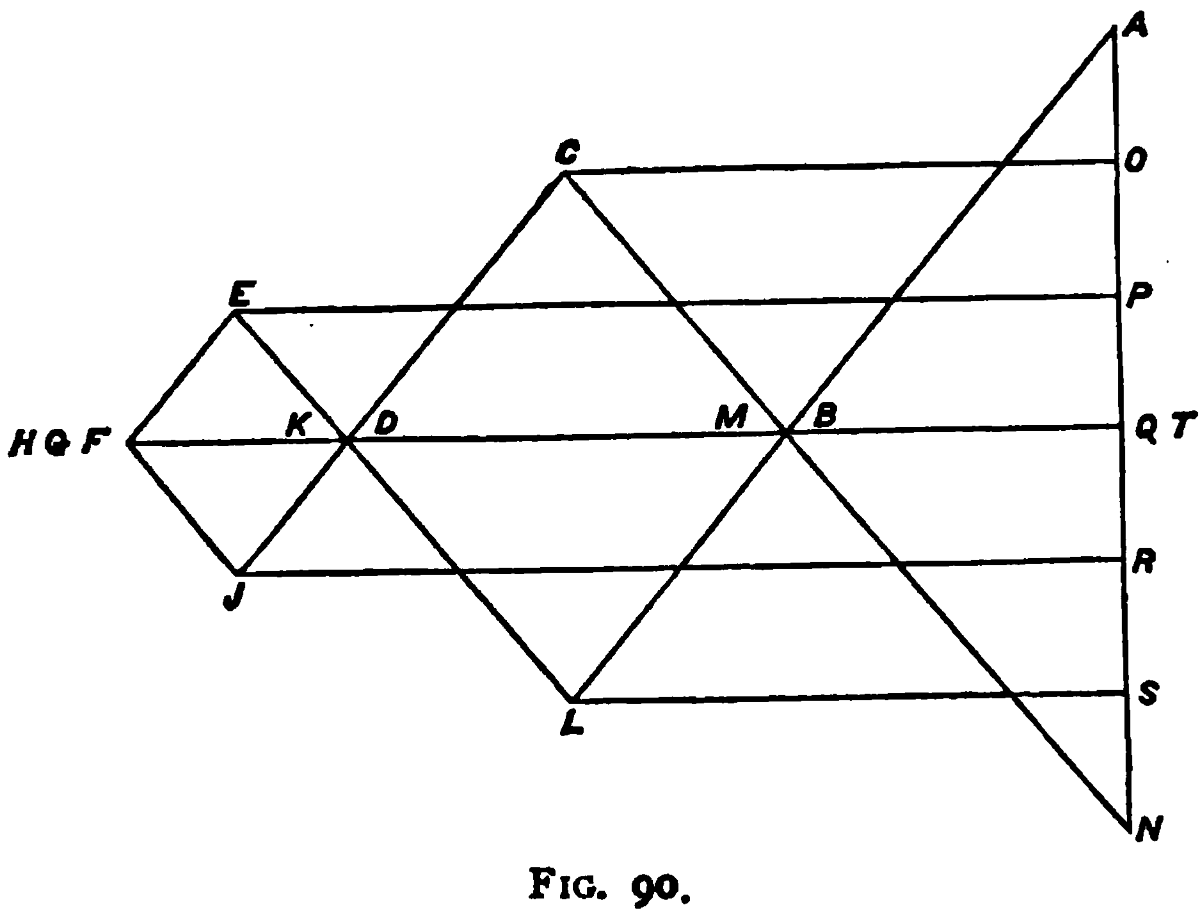

FIG. 90.

To construct this diagram, we proceed as follows: Upon the vertical line *AN* lay off the several distances *AO*, *OP*, *PQ*, *QR*, *RS* and *SN*; each equal by any convenient

scale to one of the six equal loads resting upon the top of 89. The load at the apex of the triangle B, or point $ABCO$ (89), is placed from A to O in 90. The load OP, at point $OCDEP$ (89), is placed from O to P in 90; and so on with the other loads. The other lines of *Fig.* 90 are obtained by drawing them parallel with the corresponding lines of *Fig.* 89, as per directions in *Art.* **618**. Commencing at the point ABT (89), we draw (in 90) three lines parallel with the direction of the forces at that point. The first of these is the vertical pressure upon the point of support ABT, which in this case equals one half the total load, or AQ, or AT of

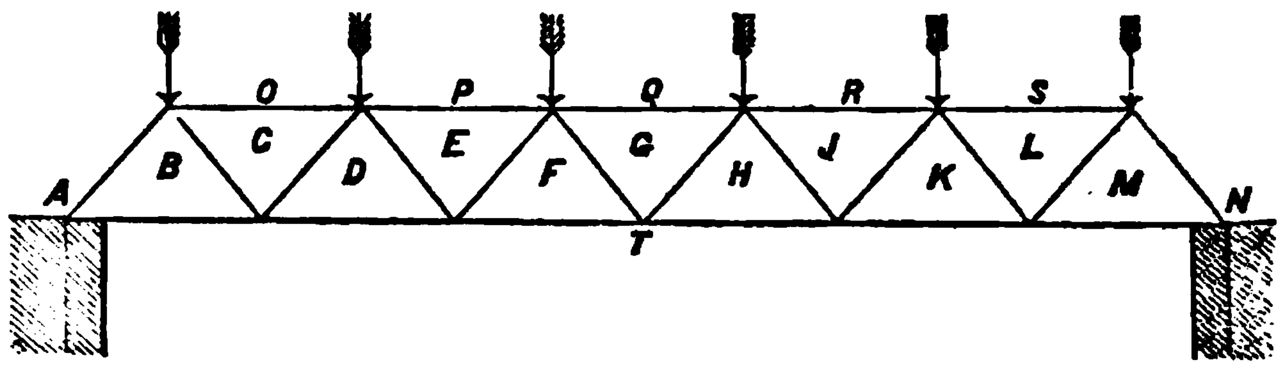

FIG. 89.

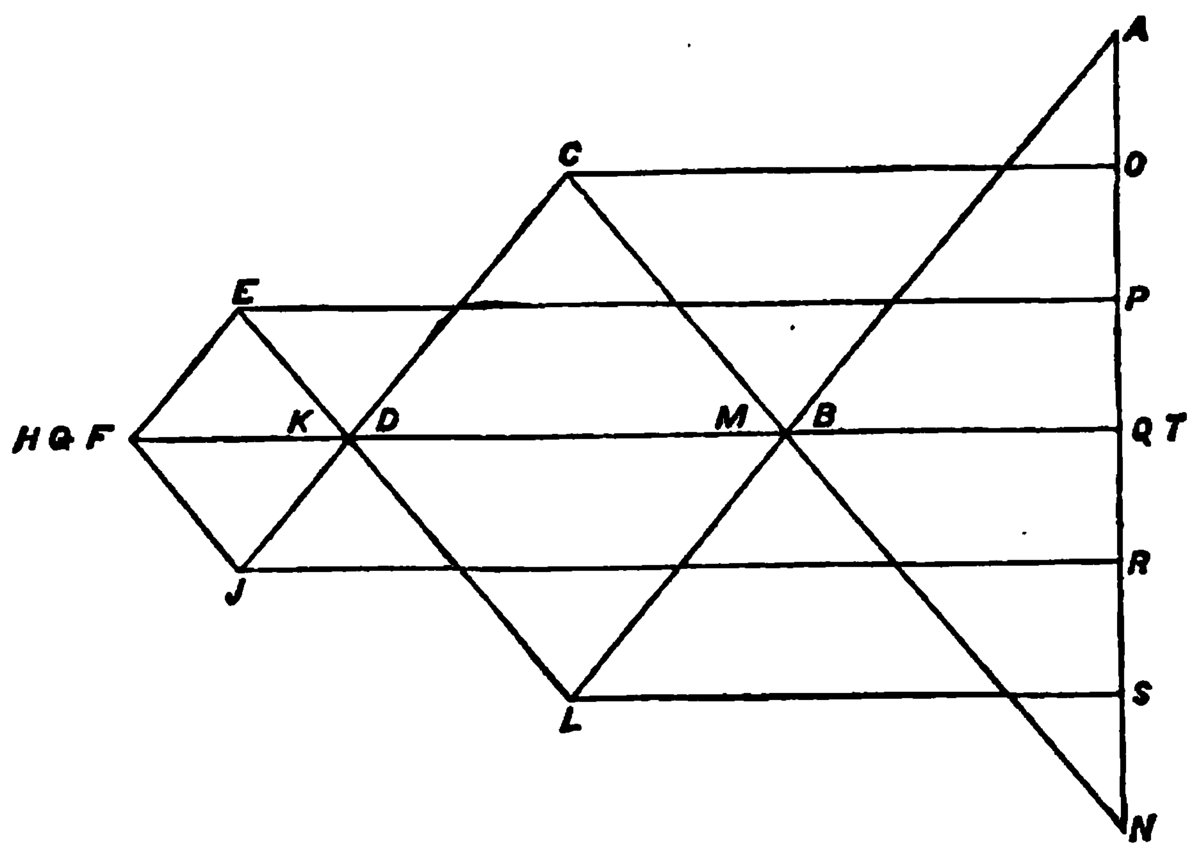

FIG. 90.

Fig. 90. Next, from T, draw the horizontal line TB, and from A, the inclined line AB, parallel with the brace AB of 89. These two lines meet at B, and we have the triangle ABT, representing the three forces converging at the point of support ABT. For the four forces at the point

ABCO in 89, we proceed as follows: We already have the forces *AO* and *AB*. From *B* in 90, draw *BC* parallel with the rod *BC* of 89; and from *O* in 90, draw *OC* parallel with *OC* of 89. These two lines intersect at *C*, completing the polygon *ABCOA*, the sides of which are in proportion as the forces in the several lines converging at the point *ABCO* of *Fig.* 89. Proceeding to the point *BCDT* (*Fig.* 89), we find, of the four forces converging there, that two are already drawn, *TB* and *BC*. From *C* draw *CD* parallel with the brace *CD* of *Fig.* 89; and from *T* draw *TD*. These two lines will meet at *D* and complete the polygon *TBCDT*, the sides of which measure the forces in the lines converging at the point *BCDT* of 89. The next in order is the point *OCDEP*. Of the five forces concentrating here, we already have, in *Fig.* 90, three, *PO*, *OC* and *CD*. To find the other two, draw from *D* the line *DE* parallel with the line *DE* of 89, and from *P* draw *PE* parallel with line *PE* of 89. These two lines will meet at *E* and complete the polygon *POCDEP*, which measures the forces in the lines concentrating at point *OCDEP*. Proceeding now to the point *DEFT* of *Fig.* 89, we find four forces, two of which, *TD* and *DE*, are already determined. For the other two, draw from *E* the line *EF* parallel with *EF* in 89, and from *T*, *TF* parallel with the line *TF* of 89. These two lines meet in *F* and complete the polygon *TDEFT*, which measures the forces in the lines converging at the point *DEFT* of *Fig.* 89. The next in order is the point *PEFGQ* in 89, where five lines converge. The forces in three of these we have already—namely, *QP*, *PE* and *EF*. Draw from *F* a line parallel with the line *FG* of 89, and from *Q* a line parallel with *QG* of 89. These two intersect at *G* and complete the polygon *QPEFGQ*, which gives the forces in the lines around the point *PEFGQ* of *Fig.* 89.

In this last proceeding we meet with a peculiarity. The line *FG* has no length in *Fig.* 90. It commences and ends at the same point, since *G* is identical with *F*. This seems to be an error, but it is not. It is correct, for an examination of *Fig.* 89 will show that the two inclined lines meeting at the foot of the triangle *G* do not assist in carrying the weights upon the top chord, and may therefore, in so far as those weights are concerned, be dispensed with, so that the space occupied by the three triangles *F*, *G* and *H* may be left free, and be designated by one letter only instead of three. In place of five, there are in fact only four forces meeting at the point *PEFGQ*, and these four are represented in *Fig.* 90 by the polygon *QPEFQ*.

The above analysis is in theory strictly correct, and yet in practice it is not so, for in such cases as this there is always more or less weight on the lower chord at the middle point. If nothing more, there is the weight of the timber chord itself, and this should be considered.

In *Art.* **634** a truss with weights at the points of each chord will be found discussed, and the facts as found in practice there developed.

The construction of one half of the diagram (*Fig.* 90) has now been completed. The other half is but a repetition of it in reversed order, and need not here be shown in detail. In drawing the lines for the latter half, it will be seen that the point *H* is identical with the point *F*, and that *K* and *M* coincide respectively with *D* and *B*.

633.—Gradation of Strains in Chords and Diagonals.—In considering the forces shown in *Fig.* 90, we find that those in the chords *increase* towards the middle of the girder, while the forces in the diagonals *decrease* towards the middle. Thus, in *Fig.* 90, of the lines representing the upper chord,

PE is longer than *OC*, and *QG* is longer than *PE*, indicating a corresponding increase in the lines *OC*, *PE* and *QG* of 89. So in the lower chord, we have a successive increase of forces, as seen in a comparison of the lengths of the lines *TB*, *TD* and *TF* of *Fig.* 90, representing the chord at the several bays *B*, *D* and *F* of *Fig.* 89. The diagonal lines *AB*, *CD* and *EF* in 90 show decreasing forces in the diagonals *AB*, *CD* and *EF* in 89—decreasing towards the middle of the girder. These facts are useful to remember when constructing a diagram of forces, as a knowledge of this law of increase in the chords and decrease in the diagonals will assist in more readily laying out the lines of the diagram correctly.

634.—Framed Girder with Loads on Each Chord.—Let *Fig.* 91 represent such a girder, and let *Fig.* 92 be the corresponding diagram of forces. In constructing this diagram, we lay off upon a vertical line, by any convenient scale, the several distances *KL*, *LM*, *MN*, *NO* and *OP*, respectively equal to the several loads upon the upper chord of *Fig.* 91. Make *PV* equal, by the same scale, to the *sum* of the several weights suspended from the lower chord. Divide *KV* at *U* into two parts, in proportion to the two parts into which the total load is divided and borne by the two points of support, *AUK* and *JQP*, of 91 (*Art.* 56). In this case, the load being symmetrically disposed, the two parts are equal, or $KU = UV$. From *U* and *K* draw lines parallel to the corresponding lines *UA* and *KA* (91). These will meet at *A* and complete the triangle of forces for the point *AUK* of *Fig.* 91. From *A* in 92 draw the line *AB*, and from *L* the line *LB*. These meet at *B* and complete the polygon *KLBAK* for the forces at the point *KABL* of 91. Starting from *U*, set off upon the vertical

line *KV* the several distances *UT*, *TS*, *SR* and *RQ*, respectively equal to the several loads *UT*, *TS*, *SR* and *RQ* as found in 91. For the forces at the point *ABCTU*, draw the line *BC* from *B*, and the line *TC* from *T*, each parallel with its corresponding line in 91. These lines meet at *C* and complete the polygon *ABCTUA*, which gives the forces converging in the point *UABCT*.

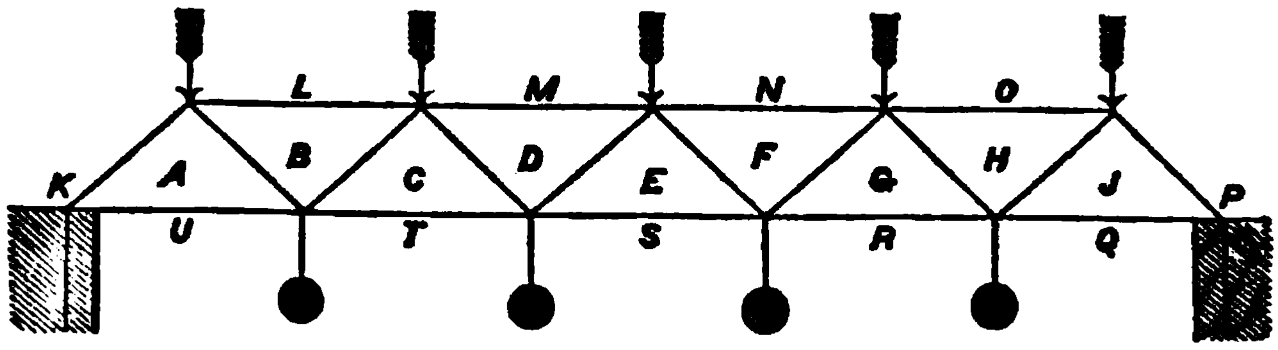

FIG. 91.

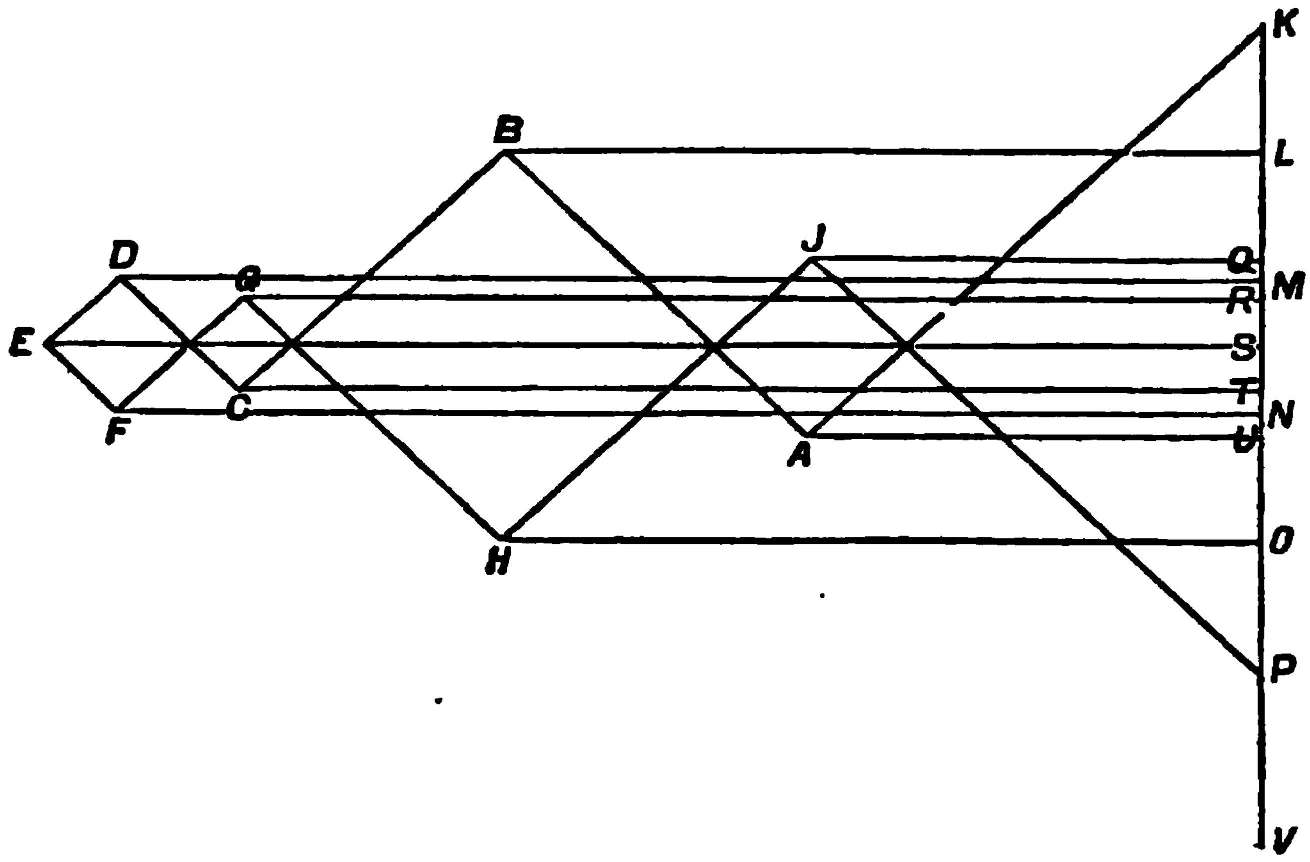

FIG. 92.

For the point *LBCDM*, draw from *C* the line *CD*, and from *M* the line *MD*, each parallel with its corresponding line in 91. These lines, meeting in *D*, complete the polygon *MLBCDM*, which gives the forces surrounding the point *LBCDM*. For the point *TCDES*, draw from *D* the line *DE*, and from *S* the line *SE*, respectively

parallel with the corresponding lines of *Fig.* 91. They will meet at E and complete the polygon $TCDEST$, which measures the forces around the point $TCDES$. For the point $MDEFN$, draw from E the line EF, and from N the line NF, parallel with EF and NF of 91; and they, meeting at F, will complete the polygon $MDEFNM$, thus giving the forces converging at the point $MDEFN$.

The correspondence of lines in the two figures has now been traced to a point beyond the middle of the framed girder. The remainder of *Fig.* 92 may be traced for the other half of 91, by a continuance of the process used in tracing the first half. Since, in this instance, the loading and plan of the girder are symmetrical, and hence the several forces in the lines of one half of the girder respectively equal to those in the other half, the lines of the diagram as laid down for the one may be used for the other half.

635.—Gradation of Strains in Chords and Diagonals.— The gradation of the forces in *Fig.* 92 may (as was remarked in *Art.* **633**) be observed in the diagonals representing the lines KA, AB, BC, CD and DE, which diagonals *decrease* from the end towards the middle of the girder; and also in the lines representing the chords AU, BL, CT, DM and ES, which gradually *increase* from the end towards the middle.

636.—Strains Measured Arithmetically.—Let *Fig.* 93 represent a framed girder, in which the loads are symmetrically placed, and where L is put for the load on each point of bearing of the upper chord, and N for that suspended at each bearing point of the lower chord. Let a represent the vertical height of the girder, c the length of a diagonal, and b the base of the triangle formed with c and a.

637.—Strains in the Diagonals.—To analyze these, we commence at the middle of the girder. There being an odd number of loads upon the upper chord, one half of the

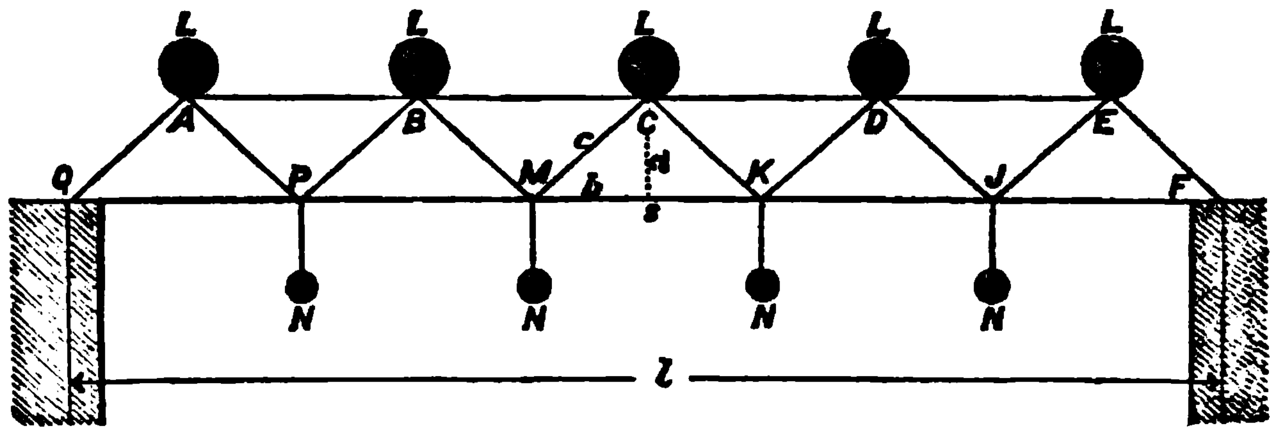

FIG. 93.

central one, L, is carried at Q, one of the points of support, and the other half at F, the other point. The effect of this upon the brace MC may be had from the relation of the sides of the triangle abc, for

$$a : c :: \tfrac{1}{2}L : \frac{c}{2a}L$$

$$2a : c :: L : \frac{c}{2a}L = D$$

equals the strain in the diagonal; or, when W equals the vertical load, equals $\frac{1}{2}L$,

$$D = W\frac{c}{a} \qquad (296.)$$

The vertical effect of this at M is $\frac{1}{2}L$, the same as it is at C. This amount, added to the suspended load N at M, equals $\frac{1}{2}L+N$, equals the total vertical force acting at M. This is sustained by the lines MK and BM, the latter standing at the same angle with MK as did MC. Hence the effect upon the diagonal is

$$a : c :: \tfrac{1}{2}L+N : (\tfrac{1}{2}L+N)\frac{c}{a}$$

equals the strain on the diagonal BM; and the vertical effect at M is equal to $\frac{1}{2}L + N$. Adding this to the load on the top chord at B, the sum, $\frac{3}{2}L + N$, is the total load at B, and it is supported by the forces in the lines PB and BC, constituting, with the weight, three forces, acting in the directions of the three sides of the triangle abc. The effect in the diagonal BP is therefore, as before, the load into the ratio $\frac{c}{a}$, or $(\frac{3}{2}L + N)\frac{c}{a}$. The vertical effect of this at P is equal to the vertical effect at B, or $\frac{3}{2}L + N$. Adding to it the load N at P, their sum, $\frac{3}{2}L + 2N$, is the total vertical effect at P; and, as before, the effect of this on the diagonal AP which carries it is $(\frac{3}{2}L + 2N)\frac{c}{a}$, with a vertical effect at A of $\frac{3}{2}L + 2N$, the same as at P. Adding the load L, at A, the sum, $\frac{5}{2}L + 2N$, equals the total vertical pressure at A. This is sustained by the forces in the lines QA and AB, which, with the weight, act in the direction of the sides of the triangle abc, and therefore the effect in the diagonal, as before, is $(\frac{5}{2}L + 2N)\frac{c}{a}$, while the vertical effect of this at Q is equal to the same effect as at A, or $\frac{5}{2}L + 2N$.

Thus, the loads on half the girder have, one by one, been picked up and brought along, step by step, until they are finally received upon Q, their point of support at one end of the girder.

It will be observed that this accumulated load, $\frac{5}{2}L + 2N$, coincides with the sum of the loads as seen upon one half of the figure, that is, to the $2\frac{1}{2}$ loads on the top chord and the two loads suspended from the bottom chord.

638.—Example.—Let it be required to show the strains in the diagonals of a framed girder 50 feet long, of five bays and $4\frac{1}{2}$ feet high.

Here b, the base of the measuring triangle, is equal to $\frac{50}{10} = 5$, and a, its height, equals the height of the girder, equals $4 \cdot 5$; and c, the hypothenuse of the right-angled triangle, is therefore

$$c = \sqrt{a^2 + b^2} = \sqrt{20 \cdot 25 + 25} = 6 \cdot 7268$$

The load L upon each point of the upper chord is 10,000 pounds, while N, the suspended load at each point of the bottom chord, is 2500 pounds.

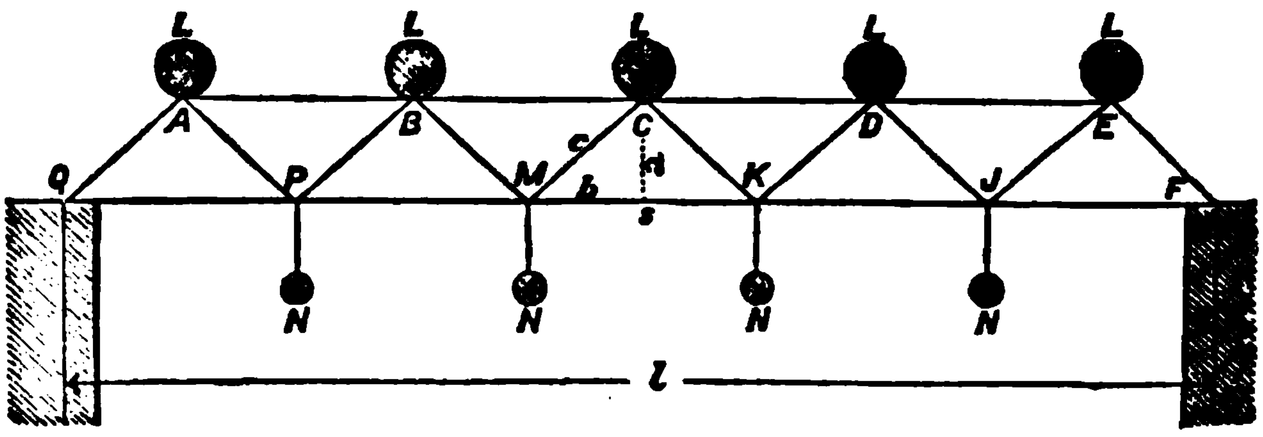

FIG. 93.

The strain upon the diagonals is, by formula (*296.*),

$$D = W \frac{c}{a}$$

The load on CM is $\frac{1}{2} L$, and therefore the strain in the diagonal CM is

$$D_{,} = L \frac{c}{2a} = 10000 \times \frac{6 \cdot 7268}{9} = 7474\tfrac{2}{9} \text{ pounds.}$$

The strain in the diagonal MB is

$$D_{,} = (\tfrac{1}{2} L + M) \frac{c}{a} = (5000 + 2500) \frac{6 \cdot 7268}{4 \cdot 5} = 11211\tfrac{1}{3} \text{ pounds.}$$

The strain in the diagonal BP is

$$D_2 = (\tfrac{3}{2}L + N)\frac{c}{a} = (15000 + 2500)\frac{6 \cdot 7268}{4 \cdot 5} = 26159\tfrac{7}{9} \text{ pounds.}$$

The strain in the diagonal PA is

$$D_4 = (\tfrac{3}{2}L + 2N)\frac{c}{a} = (15000 + 5000)\frac{6 \cdot 7268}{4 \cdot 5} = 29896\tfrac{8}{9} \text{ pounds;}$$

and the strain in the diagonal AQ is

$$D_6 = (\tfrac{5}{2}L + 2N)\frac{c}{a} = (25000 + 5000)\frac{6 \cdot 7268}{4 \cdot 5} = 44845\tfrac{1}{3} \text{ pounds.}$$

639.—Strains in the Lower Chord.—From the measuring triangle abc of *Fig.* 93 we have

$$a : b :: W : H$$

$$H = W\frac{b}{a} \qquad (297.)$$

in which H is the strain in the horizontal lines due to W the weight; and with this formula we may ascertain the horizontal forces in the chords of the girder.

First. In the lower chord. At the point Q we have, for W in the formula, one half the total load, or $(\tfrac{5}{2}L + 2N)$, and therefore

$$H_1 = (\tfrac{5}{2}L + 2N)\frac{b}{a}$$

equals the horizontal strain in QP.

For the next bay, PM, we have, for W, the same amount, plus that caused by the thrust in the strut BP, plus that due to the tension in the rod AP. These three amounts

are respectively $\frac{5}{2}L + 2N$, $\frac{3}{2}L + N$ and $\frac{3}{2}L + 2N$, and their sum is

$$(\tfrac{5}{2}L + 2N) + (\tfrac{3}{2}L + N) + (\tfrac{3}{2}L + 2N) = W = \tfrac{11}{2}L + 5N$$

equals the total weight causing horizontal strain in PM.

From this, the horizontal strain in PM is

$$H_2 = (\tfrac{11}{2}L + 5N)\frac{b}{a}$$

For the third, or middle bay, MK, we have the weight the same as for PM, together with that coming from the

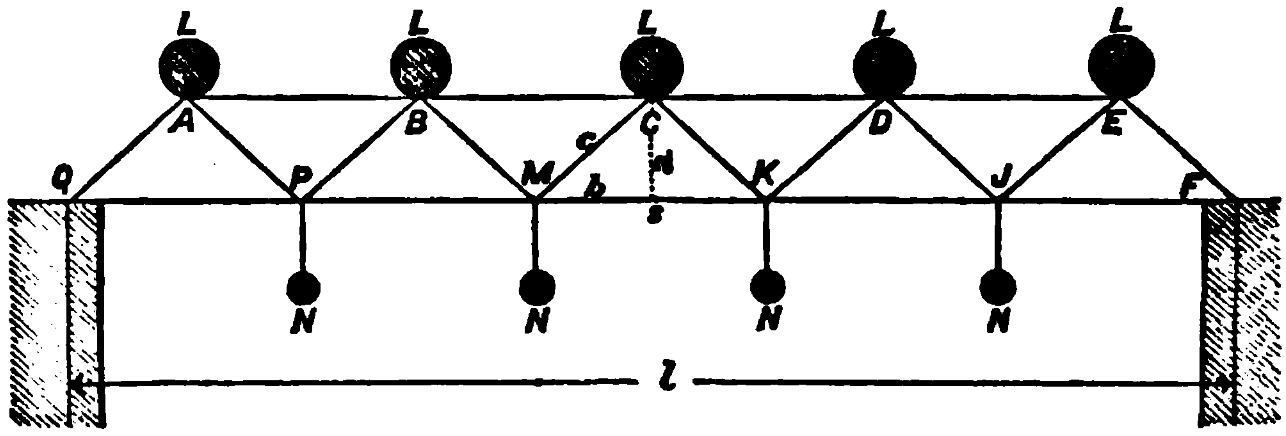

FIG. 93.

thrust of the strut CM, and from the tension of the rod BM. These three weights are $\frac{11}{2}L + 5N$, $\frac{1}{2}L$ and $\frac{1}{2}L + N$, or together,

$$(\tfrac{11}{2}L + 5N) + \tfrac{1}{2}L + (\tfrac{1}{2}L + N) = W = \tfrac{13}{2}L + 6N$$

and for the horizontal strain in MK we have

$$H_3 = (\tfrac{13}{2}L + 6N)\frac{b}{a}$$

This completes the strains in the lower chord, for those of the other end are the same as these.

640.—Strains in the Upper Chord.—For the first bay, AB, there are two compressions, namely: that due to the reaction from the strut AQ, and that from the tension in

the rod AP. The weight causing thrust in the strut is equal to half the total load, or $\frac{5}{2}L + 2N$, and the weight causing tension in the rod is $\frac{3}{2}L + 2N$; or, together, we have for the weight $4L + 4N$; and for the compression in AB,

$$H' = (4L + 4N)\frac{b}{a}$$

For the second bay, BC, we have this same thrust, plus that due to the reaction of the strut PB, plus that due to the tension in the rod BM. The three weights are $4L + 4N$, $\frac{3}{2}L + N$ and $\frac{1}{2}L + N$, and their sum is

$$(4L + 4N) + (\tfrac{3}{2}L + N) + (\tfrac{1}{2}L + N) = 6L + 6N$$

and the horizontal compression in BC is

$$H'' = (6L + 6N)\frac{b}{a}$$

641.—Example.—What are the horizontal strains in a girder of five bays, it being 50 feet long and $4\frac{1}{2}$ feet high, and having 10,000 pounds resting upon each bearing point of the upper chord, and 2500 pounds at each point of suspension in the lower chord?

Here, in the measuring triangle abc, $b = \frac{50}{10} = 5$ and $a = 4 \cdot 5$; from which $\frac{b}{a} = \frac{5}{4 \cdot 5} = 1\frac{1}{9}$. Hence, for each horizontal strain, we have

$$H = W\frac{b}{a} = 1\tfrac{1}{9}W = \tfrac{10}{9}W$$

Now, in the lower chord, we have, as in *Art.* **639**, for the bay QP, the weight

$$W = \tfrac{5}{2}L + 2N \qquad \text{and, therefore,}$$

$$H_{,} = \tfrac{10}{9}[(2\tfrac{1}{2} \times 10000) + (2 \times 2500)] = 33333\tfrac{1}{3} \text{ pounds;}$$

equals the horizontal tension in QP.

For the next bay, PM, we have for the weight, as per *Art.* **639**,

$$W = \tfrac{11}{2}L + 5N \qquad \text{and, therefore,}$$

$$H_2 = \tfrac{10}{9}[(5\tfrac{1}{2} \times 10000) + (5 \times 2500)] = 75000 \text{ pounds;}$$

equals the horizontal tension in PM.

For the third, or middle bay, MK, for the weight, as per *Art.* **639**, we have

$$W = \tfrac{13}{2}L + 6N \qquad \text{and, therefore,}$$

$$H_3 = \tfrac{10}{9}[(6\tfrac{1}{2} \times 10000) + (6 \times 2500)] = 88888\tfrac{8}{9} \text{ pounds;}$$

equals the horizontal tension in MK.

This completes the work for the lower chord, as the tensions in the other half are the same as those here found for this.

In the upper chord the weight causing compression in the first bay, AB, is, as per the last article,

$$W = 4L + 4N \qquad \text{and, therefore,}$$

$$H' = \tfrac{10}{9}[(4 \times 10000) + (4 \times 2500)] = 55555\tfrac{5}{9} \text{ pounds;}$$

equals the horizontal compression in AB.

For the next bay, BC, for the weight causing compression we have, as per last article,

$$W = 6L + 6N \qquad \text{and, therefore,}$$

$$H'' = \tfrac{10}{9}[(6 \times 10000) + (6 \times 2500)] = 83333\tfrac{1}{3} \text{ pounds;}$$

equals the horizontal compression in BC.

This completes the strains for the upper chord. Tabulated, these several horizontal strains stand thus:

For the lower chord:

In	*QP*	and	*JF*	the strains are	33,333⅓	pounds.
"	*PM*	"	*KJ*	" " "	75,000	"
"	*MK*			" strain is	88,888⅞	"

For the upper chord:

In	*AB*	and	*DE*	the strains are	55,555⅝	pounds.
"	*BC*	"	*CD*	" " "	83,333⅓	"

To test the correspondence of these results with those shown by the graphic method in *Figs.* 91 and 92, the student may make diagrams with the given figures at a scale as large as convenient, giving to L and N the proportions above assigned them, namely, $L = 4N$, and making the bays with a base of 10 and a height equal to 4·5. The results obtained should approximate those above given, in proportion to the accuracy with which the diagrams are made.

642.—Resistance to Tension.—Only in so far as tension is incidental to the transverse strain would it be proper to speak of the former in a work on the latter. In a framed girder, the lower chord and those diagonals which tend downwards towards the middle of the girder are subject to tension. The better material to resist this strain is wrought-iron, and this, in the diagonals at least, is usually employed. The weight with which this material may be safely trusted per square inch of sectional area varies according to the quality of the metal, from 7000 to 15,000 pounds. Ordinarily, it may be taken at 9000 pounds, but when the metal

and the work upon it are of superior quality, it is taken at as much as 12,000, or even higher in some special cases. This is the safe power of the metal per square inch of the sectional area. Let k equal this power, W equal the load to be carried, and A the sectional area of the bar, then

$$Ak = W$$

$$A = \frac{W}{k} \qquad (298.)$$

As an application of this formula, take the case of the diagonal AP, *Fig.* 93; the strain in which is $29,896\frac{3}{8}$ pounds. Putting $k = 9000$, we have

$$A = \frac{29896\frac{3}{8}}{9000} = 3 \cdot 3218$$

or the rod should contain $3\frac{1}{8}$ inches in its sectional area. Referring to a table of areas of circles, we find that the rod, if round, should be a trifle over 2 inches in diameter, or, if a flat bar 4 inches wide, it would need to be $\frac{5}{6}$ of an inch thick, since $4 \times \frac{5}{6} = 3 \cdot 333$.

The above is for the diagonals. The chords are usually of wood. When so made, the value per square inch sectional area may be taken at one tenth of the ultimate tensional power of the materials as given in Table XX. Since a chord is usually compounded of three or more pieces in width, and of lengths less than the length of the chord, it is necessary to see that the area of material determined by the use of formula (*298.*) is that of the *uncut* material, or of the uncut sectional area at all points in the length. Thus, were the pieces so assembled as to have no two heading joints occur at the same point in the length, or so near each other that the requisite bolts for binding the pieces together could not be introduced between the two joints, then the uncut sectional

area would be equal to that of all the pieces in the width except one. Should two joints occur at or near one point in the length, then the sectional area of all but two pieces in width must be taken; and so on for other cases.

Where care is exercised in locating the joints, the allowance for joints, bolt holes, and other damaging contingencies may be taken as amounting to as much as the net size; or, ordinarily, the net size should be doubled. Then for the total sectional area we have

$$k = \frac{T}{10 \times 2} = \frac{T}{20} \qquad \text{or}$$

$$A = \frac{W}{\frac{T}{20}}$$

$$A = \frac{20W}{T} \qquad (299.)$$

in which T is the ultimate resistance to tension, as found in Table XX.

As an illustration, take the case of the lower chord of *Fig.* 93, which, at the middle bay, has a horizontal strain of 88,889 pounds. From Table XX. we have the resistance to tension of Georgia pine equal to 16,000 pounds. By formula (*299.*)

$$A = \frac{20W}{T} = \frac{20 \times 88889}{16000} = 111 \text{ inches}$$

or the area should be not less than 111 inches. The chord may be $10 \times 12 = 120$ inches, and may be compounded of three pieces in width—a centre one of 4×12 and two outside pieces of 3×12 each.

643.—Resistance to Compression.—The top chord of a framed girder, and the struts or diagonals directed down-

ward towards the points of support, are in a state of compression.

The rules for determining the resistance to compression in posts or struts are numerous, and their discussion has occupied many minds. The theory of the subject will not be rehearsed here. For this the reader is referred to authors who have made it a special point, such as Tredgold, Hodgkinson, Rankine, Baker, Francis and others.

For short columns, the resistance is, approximately, in proportion to the area of cross-section of the post. As the post increases in length, the resistance per square inch of cross-section gradually diminishes.

In framed girders, the struts, and also the chords, when properly braced against lateral motion, are in lengths comparatively short, and hence the resistance which the material in them offers is not much less than when in short blocks. Baker* gives as the strain upon posts

$$t = t_{,}\left(1 + \frac{L^2 ma}{8I}\right)$$

Reducing this expression and changing the symbols to agree with those of this work, we have

$$f = \frac{C}{1 + \frac{l^2 ebd}{8 \times \frac{1}{12} bd^3}}$$

$$f = \frac{C}{1 + \frac{3}{2} er^2}$$

in which f equals the ultimate resistance of the post per inch of sectional area, C equals the ultimate resistance to compression of the material when in a short block, e the extension of the material per foot due to flexure, within

* Strength of Beams, Columns and Arches, by B. Baker, London, 1870, p. 182.

the limits of elasticity, as found in Table XX., and d is the dimension in the direction of the bending. This in a post will be the smaller of the two, or the thickness. Let h represent this thickness and be substituted in the above for d; then $r = \frac{l}{h}$ is the ratio of the length to the thickness or smallest dimension of the cross-section; l and h both being taken of the same denomination, either inches or feet.

The safe limit of load for posts is variously estimated at from 6 to 10. Putting a to represent this, and taking C for the ultimate resistance, as in Table XX., we have for the safe resistance

$$f = \frac{C}{a(1 + \frac{5}{8}cr^2)} \qquad (300.)$$

and when W equals the load to be carried, and A equals the sectional area, we have

$$Af = W \quad \text{or} \quad A = \frac{W}{f}$$

and, by substituting for f, its value, as in (300.),

$$A = \frac{W}{\frac{C}{a(1 + \frac{5}{8}cr^2)}}$$

$$A = \frac{a(1 + \frac{5}{8}cr^2)\,W}{C} \qquad (301.)$$

As an application, let it be required to find the area of the Georgia pine strut AQ in *Fig.* 93, the strain in which is (*Art.* **638**), say 45,000, and the length of which is 6·73.

The ratio r can not be assigned definitely in the formula, as h is unknown. From experience, however, a value may be assigned it approximating its true value, and after computation, if the result shows that the assigned value deviates materially from the true value, then a nearer ap-

proximation may be made for a second computation. The ratio in the case now considered is probably about equal to 12. We will take it at this amount for a trial. Take, from Table XX., the values of $C = 9500$ and $e = 0{\cdot}00109$, for Georgia pine. Make a, the factor of safety, equal to 10. The value of W is 45,000. Then, by formula (*301.*),

$$A = \frac{10\,[1 + (\frac{3}{2} \times 0{\cdot}00109 \times 12^2)] \times 45000}{9500} = 58{\cdot}521$$

or the area should be $58\frac{1}{2}$ inches.

Having taken the ratio at 12 we should have the thickness in inches equal to the length in feet, or $6{\cdot}73$. Dividing the area $58{\cdot}521$ by this gives a quotient of $8{\cdot}696$ as the breadth. The dimensions of the piece are $6\frac{3}{4} \times 8\frac{3}{4}$. If it be desirable to have the thickness greater than here given, then a second trial may be had with a less ratio.

644.—Top Chord and Diagonals—Dimensions.—By transformation of formula (*301.*) a rule may be arrived at which shall define the breadth of a diagonal or post exactly.

Let $A = hb$, and let h, the thickness, bear a certain relation to b, the breadth; or $nh = b$, n being a constant assumed at will (for example, if $n = 1{\cdot}2$, then $1{\cdot}2h = b$). Then $A = nh^2$. Putting also for r^2 its value $\frac{12l^2}{h^2}$ (l being taken in feet) we have

$$h^2 n = \frac{Wa + \frac{3}{2} Wae \frac{12^2 l^2}{h^2}}{C}$$

$$Ch^2 n = Wa + \frac{3}{2} Wae \frac{12^2 l^2}{h^2}$$

$$Ch^4 n = Wah^2 + (\tfrac{3}{2} \times 12^2 Wael^2)$$

$$Ch^4 n - Wah^2 = \tfrac{3}{2} \times 12^2 Wael^2$$

$$h^4 - \frac{Wa}{Cn} h^2 = \frac{432}{2} W \frac{ael^2}{Cn}$$

Completing the square and reducing gives

$$h = \sqrt{\sqrt{\frac{432}{2} W \frac{ael^2}{Cn} + \left(W \frac{a}{2Cn}\right)^2} + W \frac{a}{2Cn}}$$

Let $W \frac{a}{2Cn}$ be called G; then we have

$$G = W \frac{a}{2Cn} \qquad (302.)$$

and by substitution the above formula becomes

$$h = \sqrt{\sqrt{432\, Gel^2 + G^2} + G} \qquad (303.)$$

which is a rule to ascertain the thickness or smallest diameter of a strut or post, and in which l is in feet and the other dimensions are in inches.

This rule, owing to its complication, will be found to be tedious in practice. For this reason, formula (*301.*) ordinarily, for its greater simplicity, is to be preferred; although, from the necessity of assuming the value of r, a second computation may be required.

645.—Example.—What is the value of h, the thickness of the strut at AQ, *Fig.* 93; the length being 6·73, and the force pressing in the line of its axis being 45,000 pounds.

Putting 10 for a, the factor of safety, putting 1·2 for n, the factor defining the relation of the breadth to the thickness, and taking from Table XX. the values of the constants C and e for Georgia pine, we have $W = 45000$, $a = 10$, $e = 0{\cdot}00109$, $l = 6{\cdot}73$, $C = 9500$ and $n = 1{\cdot}2$.

By formula (*302.*) we have

$$G = W\frac{a}{2Cn} = \frac{45000 \times 10}{2 \times 9500 \times 1 \cdot 2} = 19 \cdot 737$$

Then, by formula (*303.*),

$$h = \sqrt{\sqrt{(432 \times 19 \cdot 737 \times 0 \cdot 00109 \times \overline{6 \cdot 73}^2) + \overline{19 \cdot 737}^2} + 19 \cdot 737}$$
$$= 6 \cdot 943$$

or the thickness of the strut is required to be, say 7 inches. As $nh = b$, therefore

$$b = 1 \cdot 2 \times 6 \cdot 943 = 8 \cdot 332$$

equals the breadth of the strut; and since $hb = A$, therefore

$$A = 6 \cdot 943 \times 8 \cdot 332 = 57 \cdot 849$$

equals the area of the strut; a fraction less than was before found by formula (*301.*). That value would have been the same as this had the value of r been correctly assumed. Its exact value is $11 \cdot 632$ instead of 12, the amount there taken.

646.—Derangement from Shrinkage of Timbers.—Owing to the natural shrinkage of timber in seasoning, the most carefully framed girder will settle or sag more or less, provided adequate measures are not taken to prevent it. The ends of the struts press upon the inside of the chords, while the iron rods have their bearing at the outside. The consequent diminution in height of the girder will be equal to the shrinkage of both the top and bottom chords, and the rods which at first were of the proper length will be found correspondingly long.

By screwing up the nuts upon the rods as the shrinkage progresses, the sagging may be prevented; but this would be inconvenient in most cases. It is better, in constructing the girder, to provide bearings of metal extending through

the depth of each chord, and so shaped that the strut and rod shall each have its bearing upon it. The shrinkage will then have no effect upon the integrity of the frame.

647.—Framed Girder with Unequal Loads, Irregularly Placed.—Let *Fig.* 94 represent such a case, wherein A, B and C are the loads upon the top chord, and D and E the loads on the bottom chord, all located as shown. As

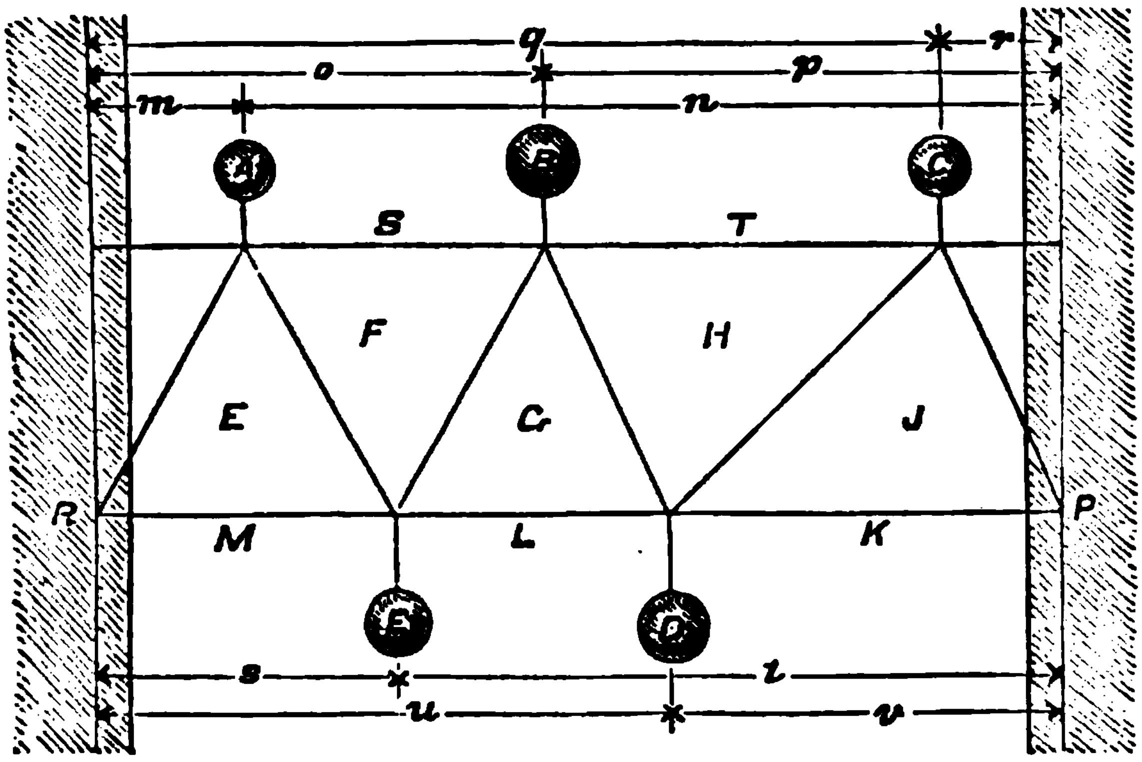

FIG. 94.

in other cases, the first requirement is to know the reactions at the two supports R and P. In a girder symmetrically loaded this involves but little trouble, as the half of the total load equals the reaction at each support. In our present case, we can not thus divide the load, since the reactions are not equal. To obtain the required division of the total load, we must consider each of the several weights separately, dividing it between the two supports according to its distances from them. Thus, putting m and n for the distances of the load A from the two supports, the portion of A bearing upon R is shown by formula (*3.*) (placing A for W),

$$R = A\frac{n}{l}$$

in which A, the weight, is multiplied by n, its distance

from the opposite support, and divided by l, the length or span. In like manner, each of the other weights may be divided, and the portion bearing upon each support found.

Putting the letters o, p, q, r, s, t, u and v to represent the distances shown in the figure, we have, as the total effect upon one of the supports,

$$R = \frac{An}{l} + \frac{Bp}{l} + \frac{Cr}{l} + \frac{Dv}{l} + \frac{Et}{l}$$

$$R = \frac{An + Bp + Cr + Dv + Et}{l} \qquad (304.)$$

and for the total effect upon the other support,

$$P = \frac{Am + Bo + Cq + Du + Es}{l} \qquad (305.)$$

Adding these two formulas, we have as the total effect upon both supports,

$$R + P = \frac{A(m+n) + B(o+p) + C(q+r) + D(u+v) + E(s+t)}{l}$$

Here the sum of the two quantities within each parenthesis is equal to l the length, and consequently

$$R + P = \frac{Al + Bl + Cl + Dl + El}{l}$$

$$R + P = A + B + C + D + E$$

or the sum of the reactions of the two supports is equal to the sum of all the weights. In this we have proof of the accuracy of the two formulas (*304.*) and (*305.*).

648.—Load upon Each Support—Graphical Representation.—The value of R in formula (*304.*) may be readily

found, either arithmetically or graphically. The formula for one weight, $R_{\prime} = A\frac{n}{l}$ (3.), gives $R_{\prime}l = An$, or two equal rectangles. Having three of these quantities, l, A and n, the fourth quantity, $R_{\prime\prime}$ may be graphically found thus:

In *Fig.* 96 let AB, by any convenient scale, equal n. Draw AC at any angle with AB, and equal in length to

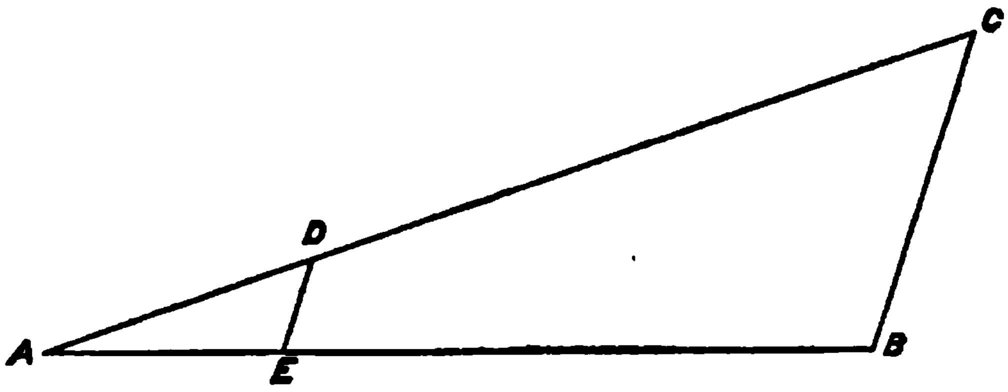

FIG. 96.

l. Lay off AD equal to A. Join B with C, and from D draw DE parallel with CB. AE will equal $R_{\prime}$ the required quantity, for, from similar triangles, we have

$$AC : AB :: AD : AE$$
$$l : n :: A : R_{\prime} = A\frac{n}{l}$$

To obtain the value of R for all of the weights, proceed as in *Fig.* 97, in which the parallel lines FL, GM, HN, JO

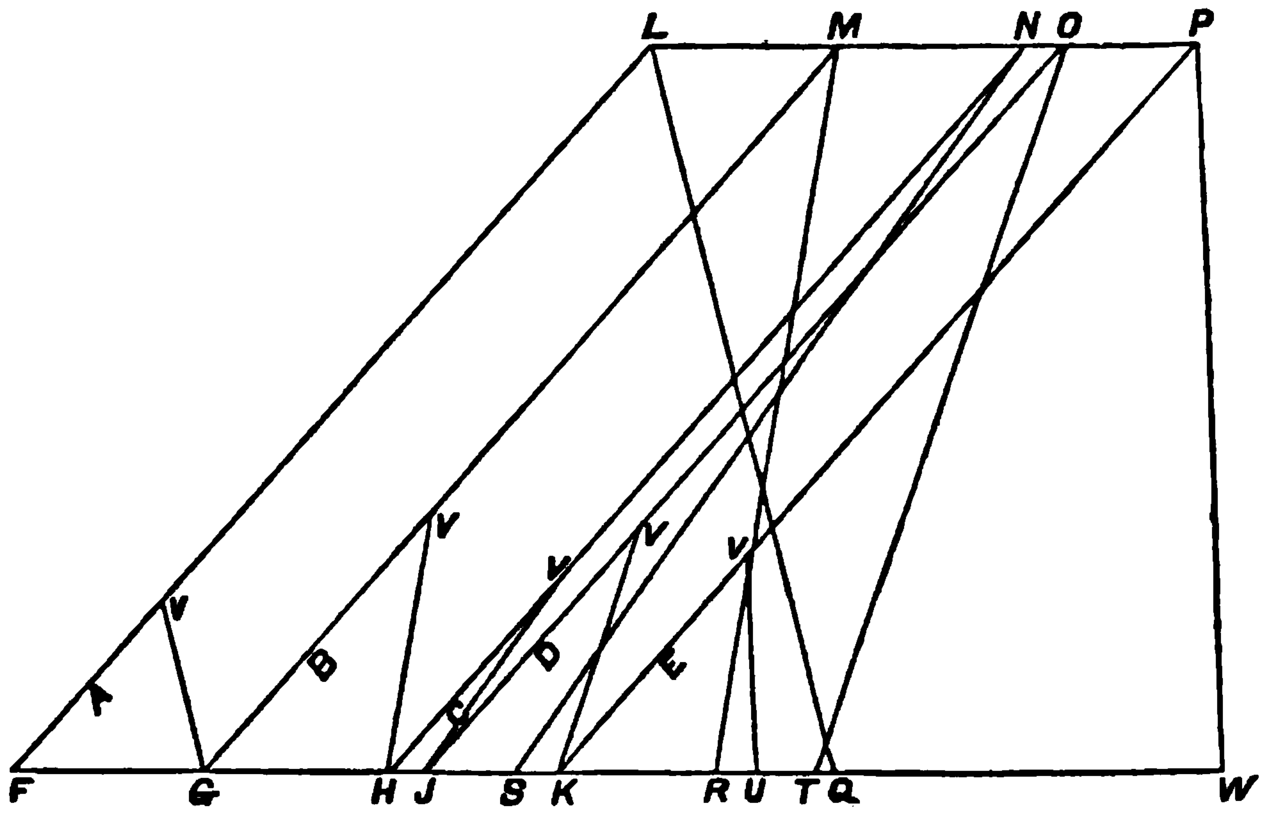

FIG. 97.

and KP are each equal to l, the span RP of *Fig.* 94.

From F lay off upon FL, the first of these lines, the distance FV equal by scale to the weight A of *Fig.* 94, and from F on line FW place FQ equal to n. Connect Q with L. From V draw VG parallel with LQ. FG will represent R_1.

From G draw GM parallel with FL. Make GV equal to the weight B (94), and GR equal to p. Connect R with M, and from V draw VH parallel with MR. GH will represent R_2.

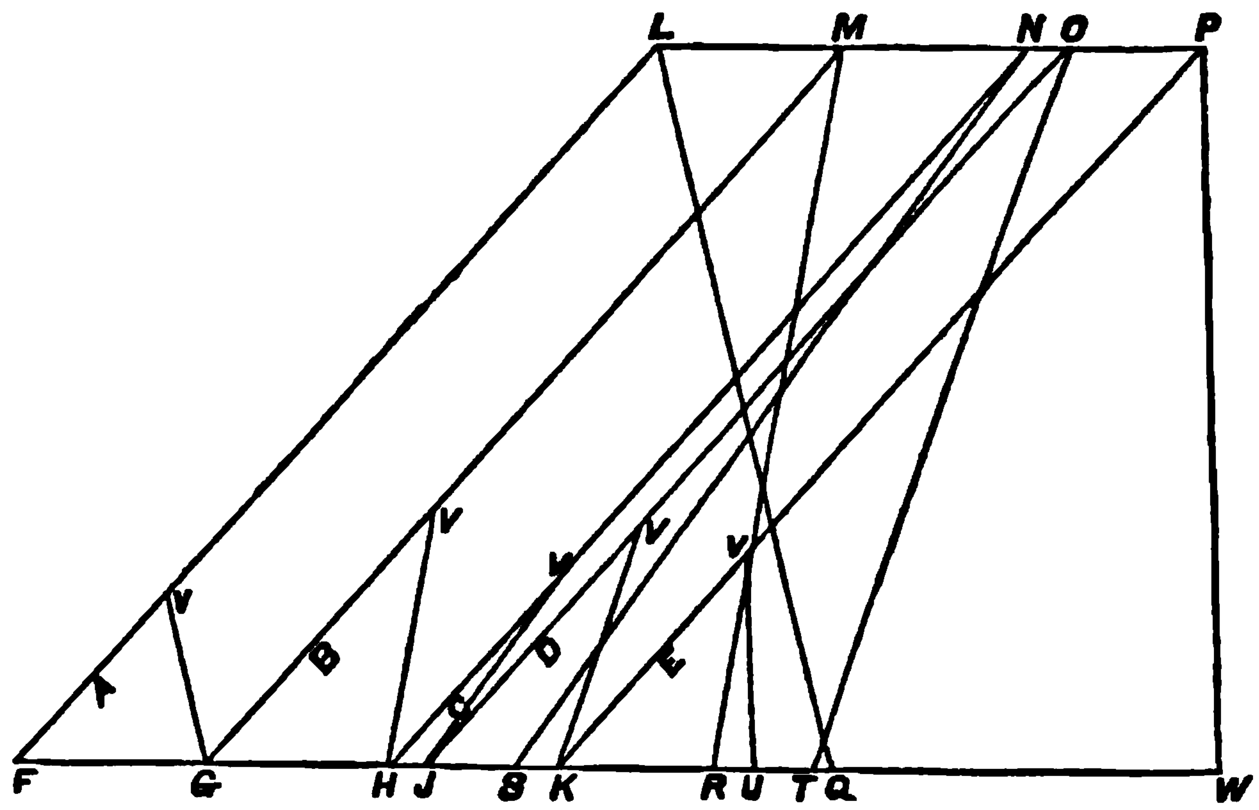

FIG. 97.

From H draw HN parallel with FL. Make HV equal to the weight C (*Fig.* 94), and HS equal to r. Connect S with N, and parallel with NS draw VJ. HJ will represent R_3.

In like manner, with the weight D and distance v of *Fig.* 94, obtain JK equal to R_4; and with the weight E and distance t obtain KU equal to R_5.

We now have the line FU equal to the sum of

$$R_1 + R_2 + R_3 + R_4 + R_5 = R$$

equals that portion of the total load on the girder which presses upon the support R.

Similarly, the amount of pressure upon the support *P* may be obtained. The two, *R* and *P*, should together equal the sum of the weights *A*, *B*, *C*, *D* and *E*.

649.—Girder Irregularly Loaded—Force Diagram.— Having accomplished the division of the total weight, we

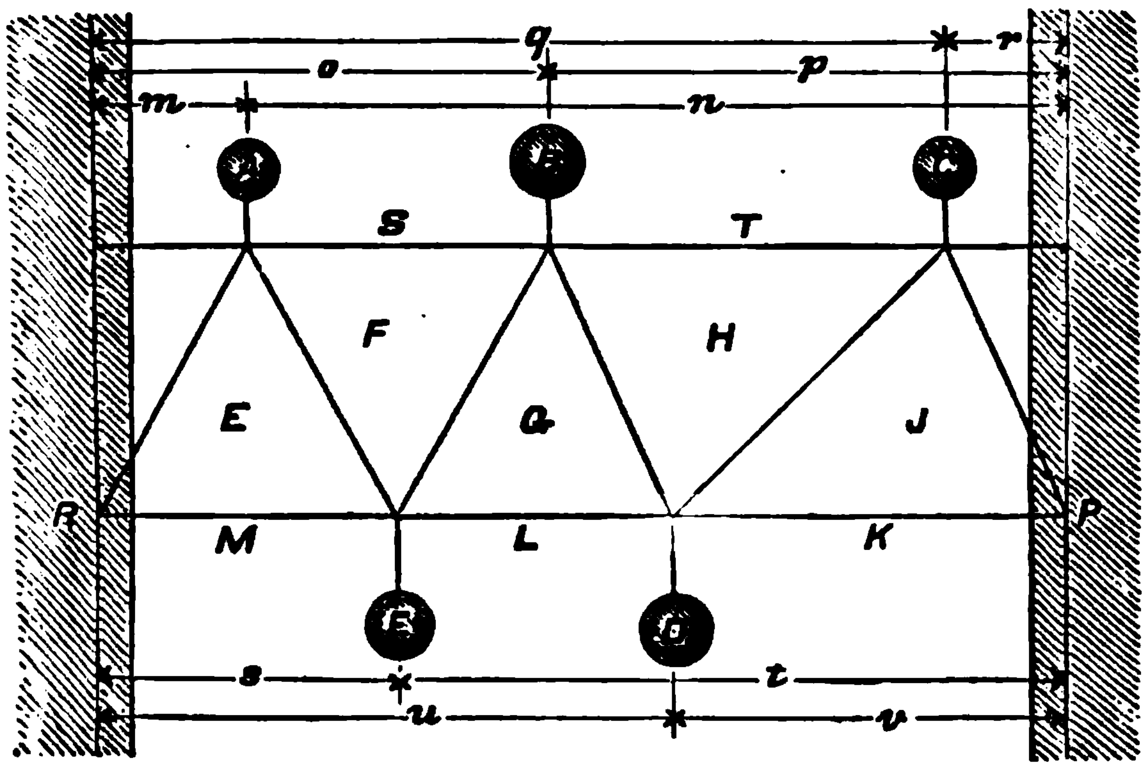

FIG. 94.

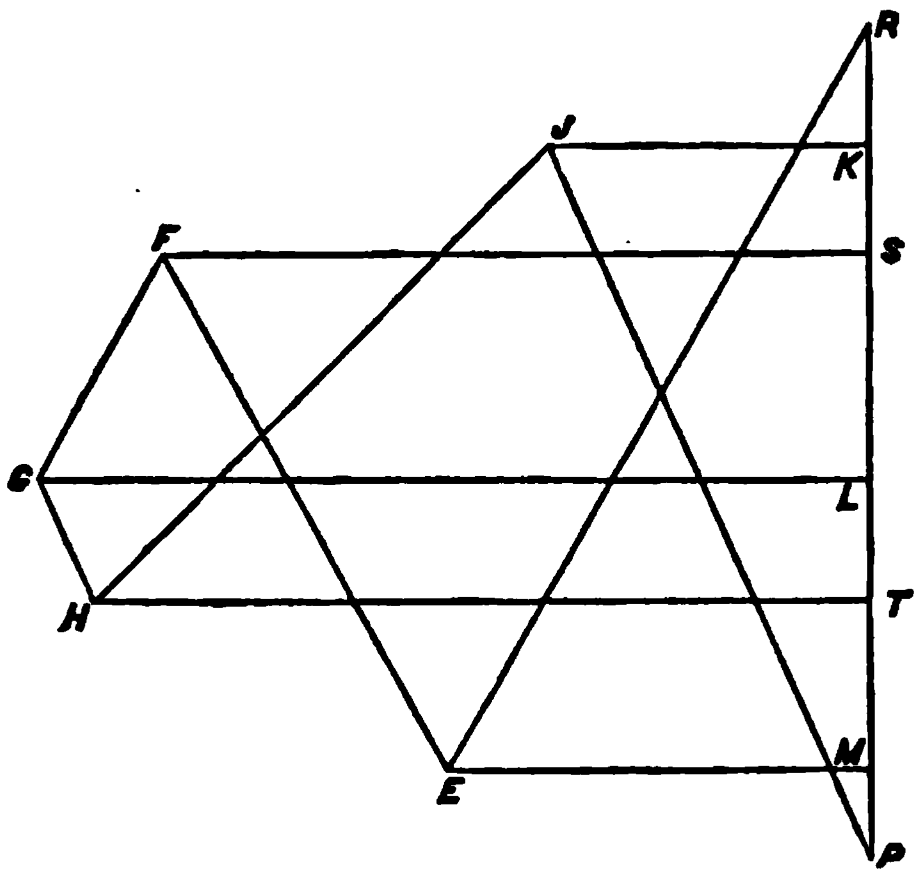

FIG. 95.

may now construct upon the same scale with that of *Fig.* 97, the force diagram, *Fig.* 95, for the girder represented in *Fig.* 94 and described in *Art.* 647. On a vertical, *RP*, make

RM equal to *FU* (97); and *RS* equal to the weight *A*, *ST* equal to the weight *B*, and *TP* equal to the weight *C*, all as in *Fig.* 94. Now, since *RM* equals *FU* (97), equals the reaction of the support *R*, therefore, from *R* draw *RE* parallel with *RE* (94), and from *M* draw *ME* parallel with *ME* (94). From *M* make *ML* equal to the weight *E* (94), and *LK* equal to the weight *D* (94). Draw the other lines all parallel with the corresponding lines of *Fig.* 94, as per *Arts.* **618** and **619**, and the force diagram will be complete.

650.—Load upon Each Support, Arithmetically Obtained. —The reaction of the two supports may be found arithmetically, as before stated, by the use of formulas (*304.*) and (*305.*). Thus, let the several weights *A*, *B*, *C*, *D* and *E* of *Fig.* 94 be rated, by the scale of the diagram, at 15, 23, 17, 22 and 19 parts respectively. These parts may represent hundreds or thousands of pounds, or any other denomination at will. Let l, the span, equal 64, and the several distances n, p, r, v and t measure respectively 54, 34, 8, 26 and 44 by the same scale.

Formula (*304.*) now gives

$$R = \frac{(15 \times 54) + (23 \times 34) + (17 \times 8) + (22 \times 26) + (19 \times 44)}{64} = 49$$

Formula (*305.*) gives

$$P = \frac{(15 \times 10) + (23 \times 30) + (17 \times 56) + (22 \times 38) + (19 \times 20)}{64} = 47$$

$$R + P = 49 + 47 = 96$$

The sum of the weights is

$$W = 15 + 23 + 17 + 22 + 19 = 96$$

the same amount, thus proving the above computation correct.

QUESTIONS FOR PRACTICE.

651.—Given a frame similar to *Fig.* 87, with a span of 40 feet, a height of 23 feet, with the length of the vertical *BC* equal to 15 feet, and with *AF* and *BG* equal. Draw a diagram of forces, and show what the strains are in each line of the frame; the three loads *FG*, *GH* and *HJ* being each equal to 5000 pounds.

652.—According to the rule given in *Art.* **624**, show what should be the height of a framed girder which is 75 feet between bearings.

653.—According to *Art.* **627**, show how many bays the girder of the last article should have.

654.—Show, by the diagram of forces, what are the strains in the several lines of a girder 55 feet long between centres of bearings and 5·27 feet high between axes of chords; the girder to be divided into five equal bays, each being an isosceles triangle as in *Fig.* 93. The load upon the apex of each triangle is 5000 pounds, and that suspended from the lower chord at each point of intersection with the diagonals is 1250 pounds. Letter the girder as in *Fig.* 91.

655.—To test the accuracy of the results obtained in the last article compute the strains arithmetically.

656.—What should be the areas of cross-section of the bottom chord of the girder of *Art.* **654**, at the several bays?

What should be the sizes of the upper chord and of the diagonal struts? The timber is to be of spruce; a, the factor of safety, to be taken at 10, and n at $1 \cdot 2$.

What should be the areas of cross-section of the diagonal rods, taking the safe strength of the metal at 9000 pounds?

In the questions of this *Art.* take the strains given by the diagram of forces.

CHAPTER XXIII.

ROOF TRUSSES.

ART. **657.—Roof Trusses considered as Framed Girders.** —It is proposed, in this chapter, to discuss the subject of roof trusses in so far only as they may be considered to be framed girders, placed in position to carry the roofing material. A full treatise on roofs would include matter extending beyond the limits of a work on the transverse strain. Those desirous of pursuing the subject farther are referred to Tredgold, Bow and others* who have written more fully on roofs.

658.—Comparison of Roof Trusses.—Designs for roof trusses, illustrating various principles of roof construction, are herewith presented.

The designs at *Figs.* 98 to 102 are distinguished from those at *Figs.* 103 to 106, by having a horizontal tie-beam. In the latter group, and in all designs similarly destitute of the horizontal tie at the foot of the rafters, the strains are much greater than in those having the tie, unless the truss be protected by exterior resistance, such as may be afforded by competent buttresses.

To the uninitiated it may appear preferable, in *Fig.* 103, to extend the inclined ties to the rafters, as shown by the dotted lines. But this would not be beneficial: on the con-

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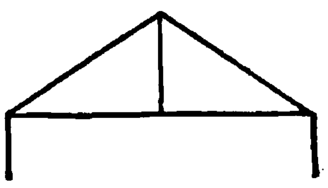
* Tredgold's Carpentry. Bow's Economics of Construction.

trary, it would be injurious. The point of the rafter where the tie would be attached is near the middle of its length, and consequently is a point the least capable of resisting transverse strains. The weight of the roofing itself tends to bend the rafter; and the inclined tie, were it attached to the rafter, would, by its tension, have a tendency to increase this bending. As a necessary consequence, the feet of the rafters would separate, and the ridge descend.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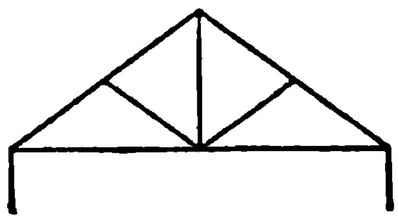
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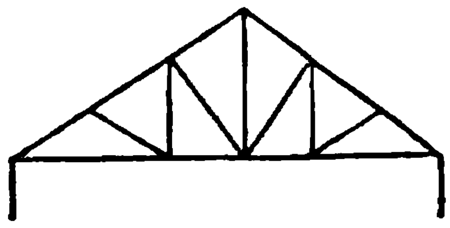
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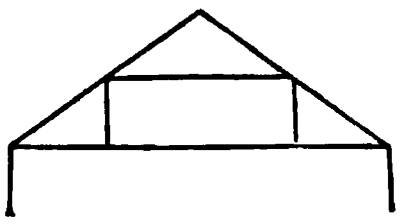
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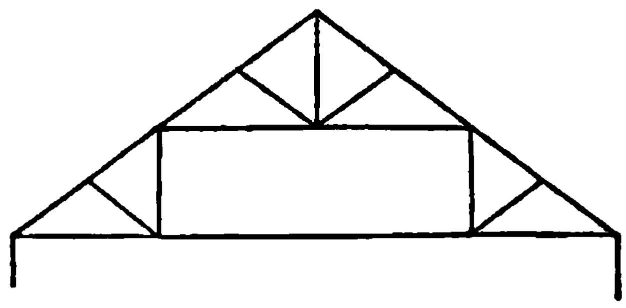
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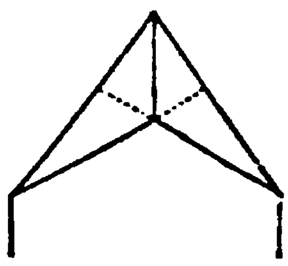
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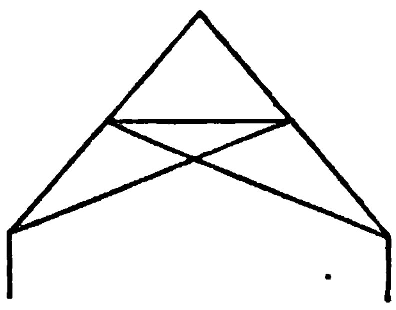
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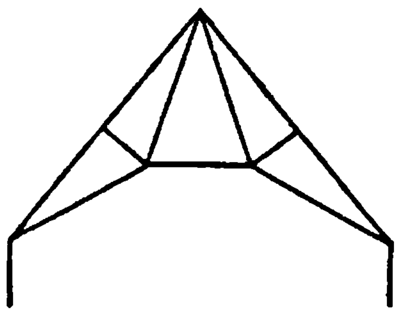
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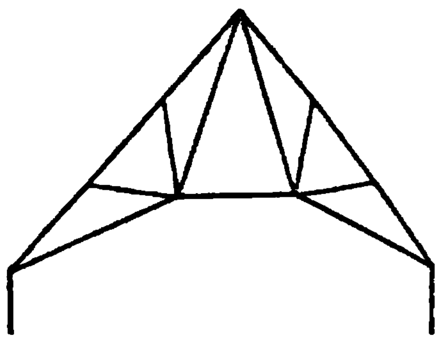
106.

In *Fig.* 104 the inclined ties are extended to the rafters; but here the horizontal strut or straining beam, located at the points of contact between the ties and rafters, counteracts the bending tendency of the rafters and renders these points stable. In this design, therefore, and only in such designs, is it permissible to extend the ties through to the rafters. Even here it is not advisable to do so, because of the increased strain produced. (See *Figs.* 118 and 120.) The design in *Fig.* 103, 105 or 106 is to be preferred to that in *Fig.* 104.

659.—Force Diagram—Load upon Each Support.—By a comparison of the force diagrams hereinafter given, of each of the foregoing designs, we may see that the strains in the trusses without horizontal tie-beams at the feet of the rafters are greatly in excess of those having the tie. In constructing these diagrams, the first step is to ascertain the reaction of, or load carried by, each of the supports at the ends of the truss. In symmetrically loaded trusses, the weight upon each support is always just one half of the whole load.

660.—Force Diagram for Truss in *Fig.* 98.—To obtain the force diagram appropriate to the design in *Fig.* 98, first letter the figure as directed in *Art.* **619**, and as in *Fig.* 107. Then

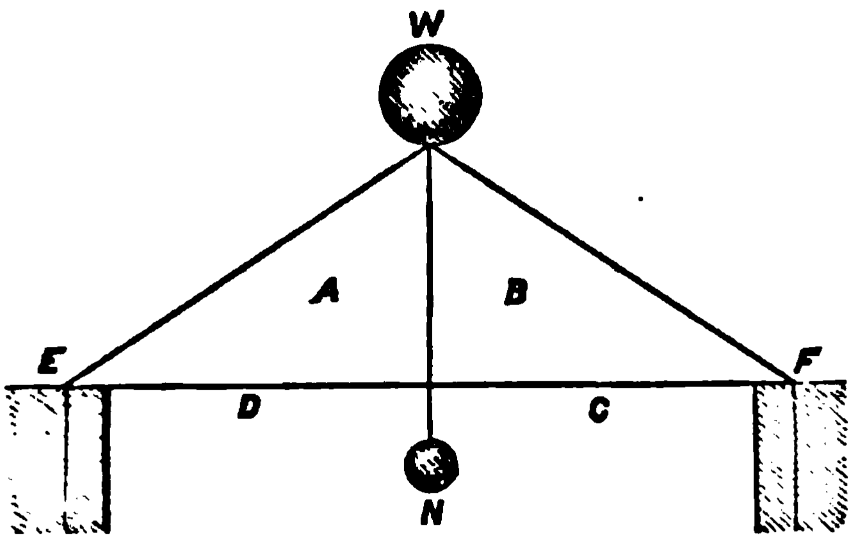

FIG. 107.

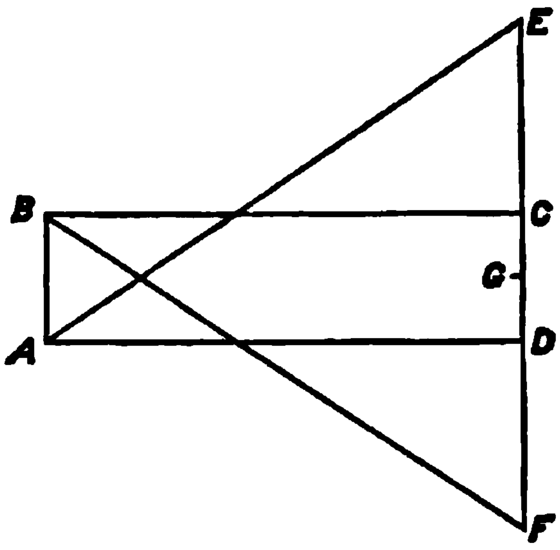

FIG. 108.

draw a vertical line, *EF* (*Fig.* 108), equal to the weight *W* at the apex of the roof; or (which is the same thing in effect) equal to the sum of the two loads of the roof, one extending on each side of *W* half-way to the foot of the rafter. Divide *EF* into two equal parts at *G*. Make *GC* and *GD* each equal to one half of the weight *N*. Now, since *EG* is equal to one half of the upper load, and *GD* to one half of the lower load, therefore their sum, $EG + GD = ED$, is equal to one half of the total load, or to the reaction of each support, *E* or *F*. From *D* draw *DA* parallel with *DA* of *Fig.* 107, and from *E* draw *EA* parallel with *EA* of *Fig.* 107. The three lines of the triangle *AED* rep-

resent the strains, respectively, in the three lines converging at the point *ADE* of *Fig.* 107. Draw the other lines of the diagram parallel with the lines of *Fig.* 107, and as directed in *Arts.* **619** and **620**. The various lines of *Fig.* 108 will represent the forces in the corresponding lines of *Fig.* 107; bearing in mind (*Art.* **619**) that while a line in the force diagram is designated in the usual manner by the letters at the two ends of it, a line of the frame diagram is designated by the two letters between which it passes. Thus, the horizontal lines *AD*, the vertical lines *AB*, and the inclined lines *AE* have these letters at their ends in *Fig.* 108, while they pass between these letters in *Fig.* 107.

661.—Force Diagram for Truss in *Fig.* 99.—For this truss we have, in *Fig.* 109, a like design, repeated and lettered as

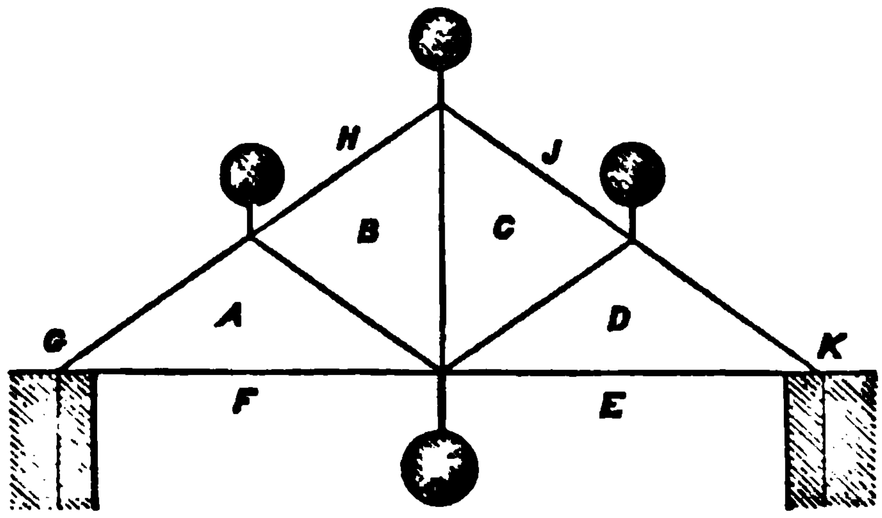

FIG. 109.

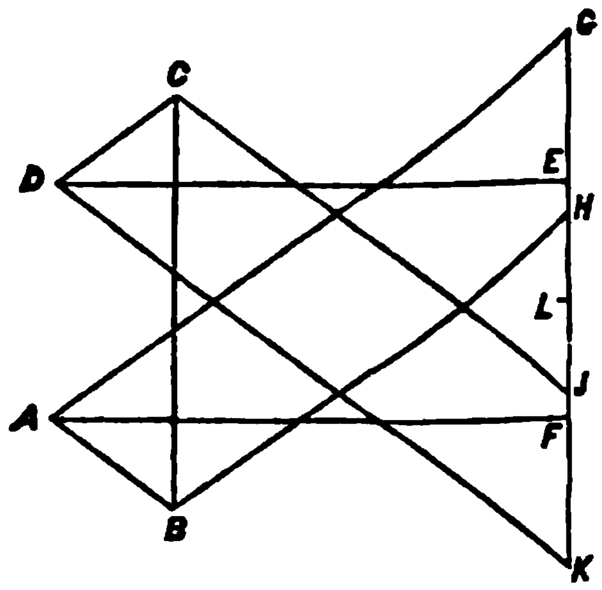

FIG. 110.

required. We here have one load on the tie-beam and three loads above the truss; one on each rafter and one at the ridge. In the force diagram, *Fig.* 110, make *GH*, *HJ* and *JK*, by any convenient scale, equal, respectively, to the weights *GH*, *HJ* and *JK* of *Fig.* 109. Divide *GK* into two equal parts at *L*. Make *LE* and *LF* each equal to one half the weight *EF* (*Fig.* 109). Then *GF* is equal to one half the total load, or to the load upon the support *G* (*Art.* **660**). Complete the diagram by drawing its several lines parallel with the lines of *Fig.* 109, as indicated by the letters (see *Art.* **660**), commencing with *GF*, the load on

the support G (*Fig.* 109). Draw from F and G the two lines FA and GA, parallel with these lines in *Fig.* 109. Their point of intersection defines the point A. From this the several points B, C and D are developed, and the figure completed. Then the lines in *Fig.* 110 will represent the forces in the corresponding lines of *Fig.* 109, as indicated by the lettering. (See *Art.* 619.)

662.—Force Diagram for Truss in *Fig.* 100.—For this truss we have, in *Fig.* 111, a similar design, properly prepared

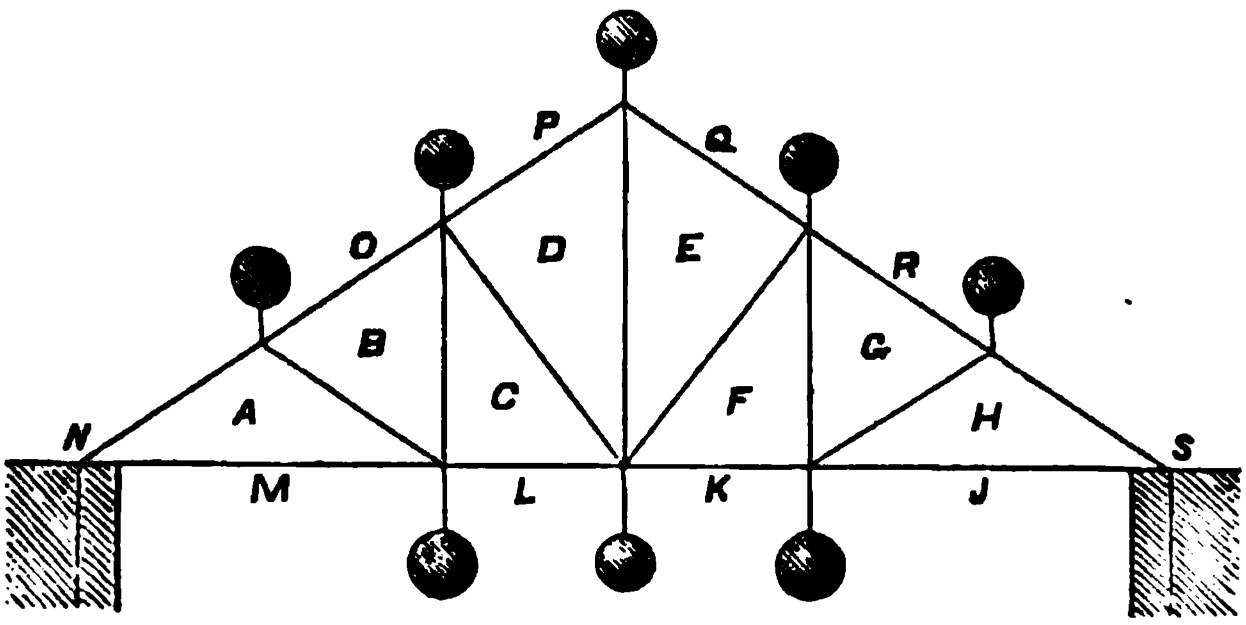

FIG. 111.

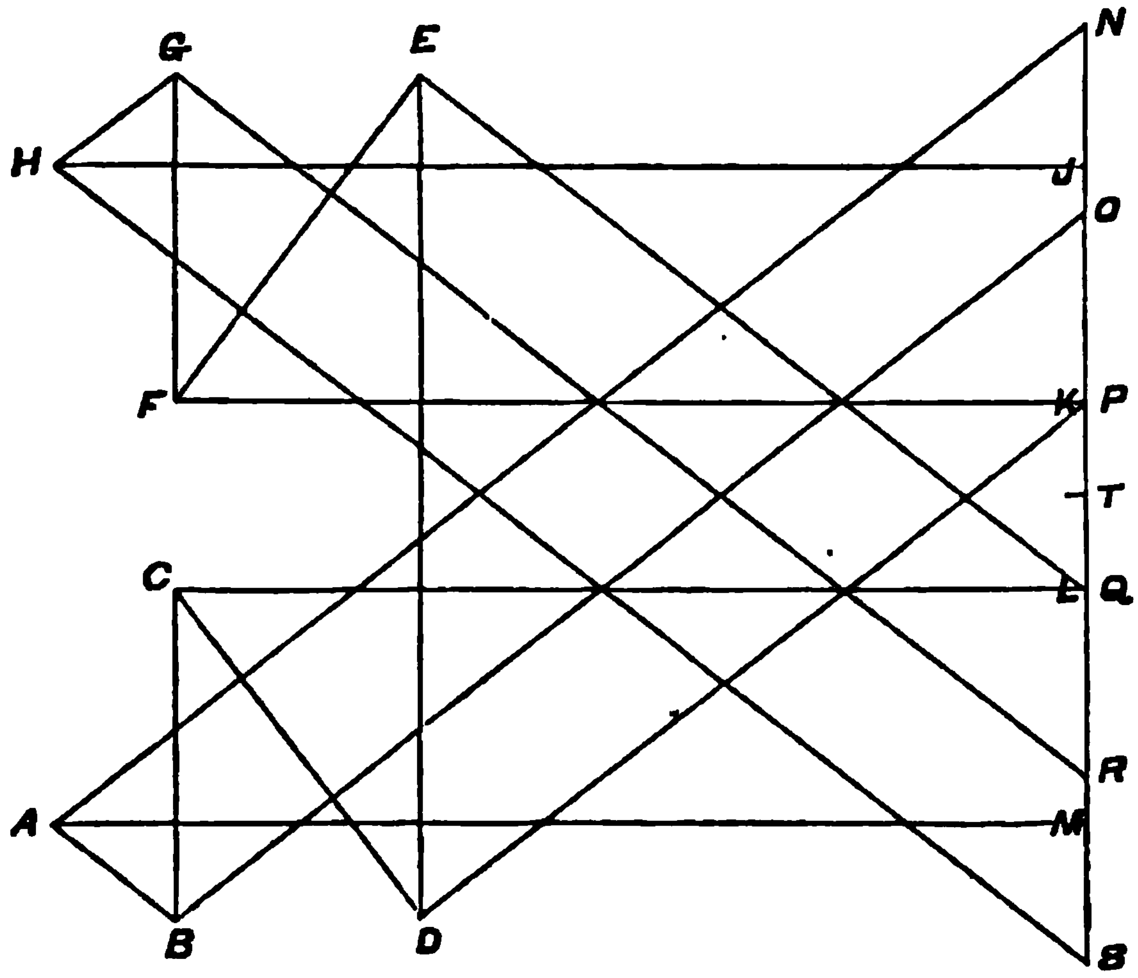

FIG. 112.

by weights and lettering; and in *Fig.* 112 the force diagram appropriate to it.

In the construction of this diagram, proceed as directed in the previous example, by first constructing *NS*, the vertical line of weights; in which line *NO*, *OP*, *PQ*, *QR* and *RS* are made respectively equal to the several weights above the truss in *Fig.* 111. Then divide *NS* into two

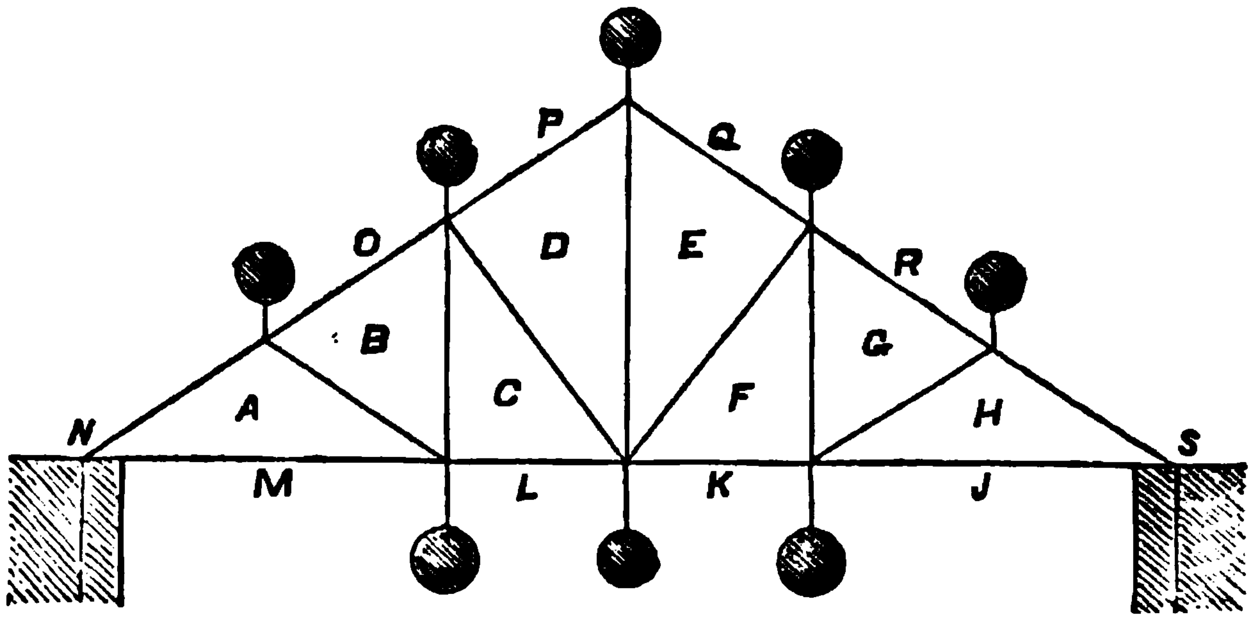

FIG. 111.

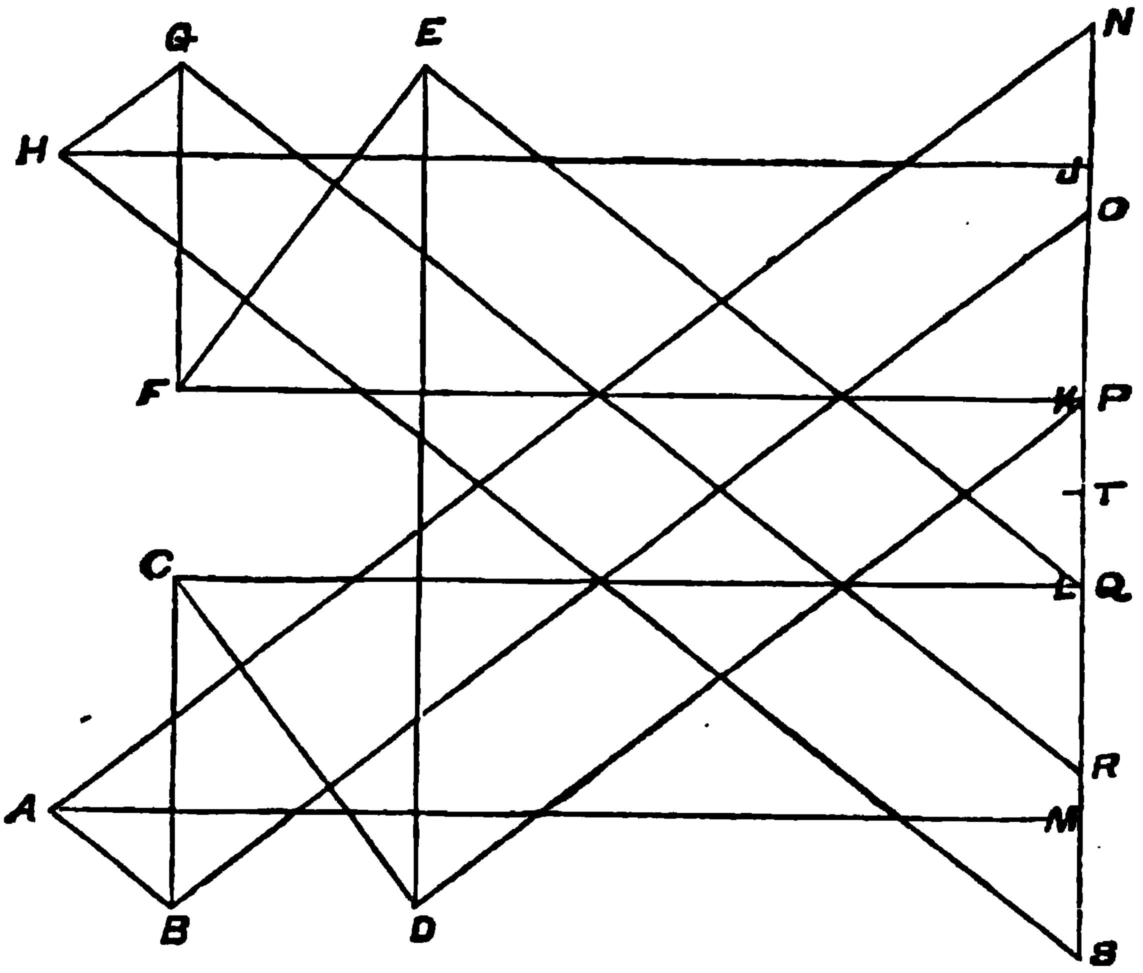

FIG. 112.

equal parts at *T*. Make *TK* and *TL* each equal to the half of the weight *KL*. Make *JK* and *LM* equal to the weights *JK* and *LM* of *Fig.* 111. Now, since *MN* is equal to one half of the weights above the truss, plus one half of the weights below the truss, or half of the whole weight, it is therefore the weight upon the support *N* (*Fig.*

111), and represents the reaction of that support. A horizontal line drawn from *M* will meet the inclined line drawn from *N*, parallel with the rafter *AN* (*Fig.* 111), in the point *A*, and the three sides of the triangle *AMN* (*Fig.* 112) will give the strains in the three corresponding lines meeting at the point *AMN* (*Fig.* 111). The sides of the triangle *HJS* (*Fig.* 112) give likewise the strains in the three corresponding lines meeting at the point *HJS* (*Fig.* 111). Continuing the construction, draw all the other lines of the force diagram parallel with the corresponding lines of *Fig.* 111, and as directed in *Art.* **619.** The completed diagram will measure the strains in all the lines of *Fig.* 111.

663.—Force Diagram for Truss in ***Fig.*** **101.**—For the roof truss at *Fig.* 101 we have, in *Fig.* 113, a repetition of it, and in *Fig.* 114 its force diagram.

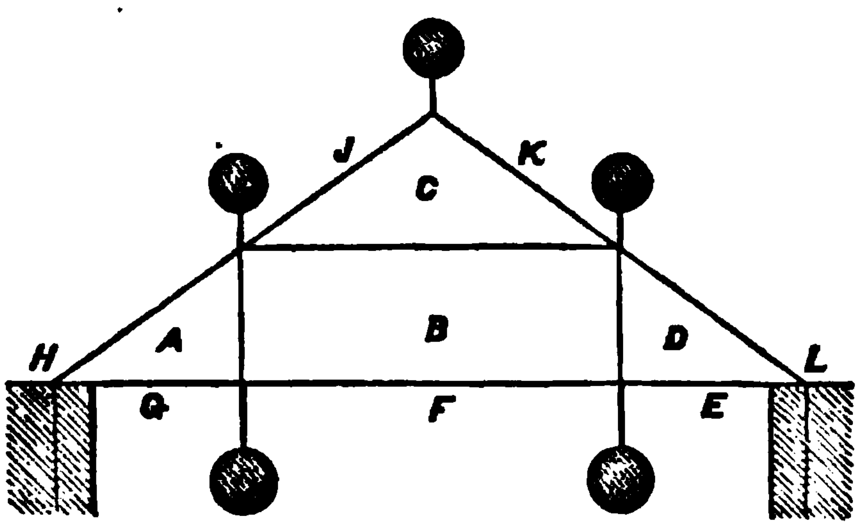

FIG. 113.

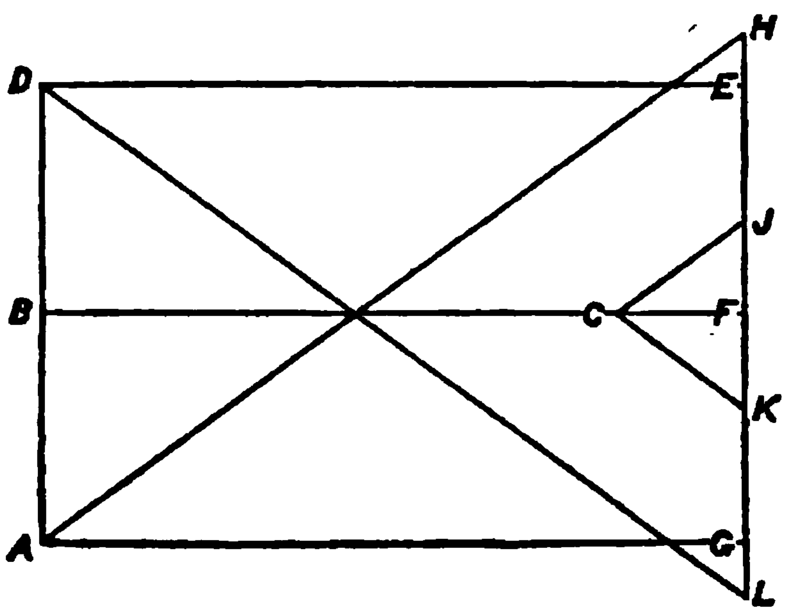

FIG. 114.

The dimensions on the vertical line *HL* (*Fig.* 114) are made respectively equal to the weights in *Fig.* 113, as indicated by the lettering. With *GH* equal to half the whole weight on the truss (*Art.* **660**), the triangle *AGH* is constructed, giving the strains in the three lines concentrating at the point *AGH* (*Fig.* 113). Then, drawing the other lines parallel with the corresponding lines of *Fig.* 113, the completed diagram gives the strains in the several lines of that figure, as indicated by the lettering. (See *Art.* **619**.)

664.—Force Diagram for Truss in *Fig.* **102.**—The roof truss indicated at *Fig.* 102 is repeated in *Fig.* 115, with the addition of the lettering required for the construction of the force diagram, *Fig.* 116.

In this case, there are seven weights, or loads, above the truss, and three below. Divide the vertical line *OV* at *W*, into two equal parts, and place the lower loads in two equal parts on each side of *W*. Owing to the middle one of these loads not being on the tie-beam with the other two, but on the upper tie-beam, the line *GH*, its representative in the force diagram, has to be removed to the vertical *BJ*, and the letter *M* is duplicated. The line *NO* equals half the whole weight of the truss, or $3\frac{1}{2}$ of the upper loads, plus one of the lower loads, plus half of the load at the upper tie-beam. It is therefore the true reaction of the support *NO*, and *AN* is the horizontal strain in the beam there. It will be observed also, that while *HM* and *GM* (*Fig.* 116), which are equal lines, show the strain in the lower tie-beam at the middle of the truss, the lines *CH* and *FG*, also equal but considerably shorter lines, show the strains in the upper tie-beam. Ordinarily in a truss of this design, the strain in the upper beam would be equal to that in the lower one, which becomes true when the rafters and braces above

the upper beam are omitted. In the present case, the thrusts of the upper rafters produce tension in the upper beam

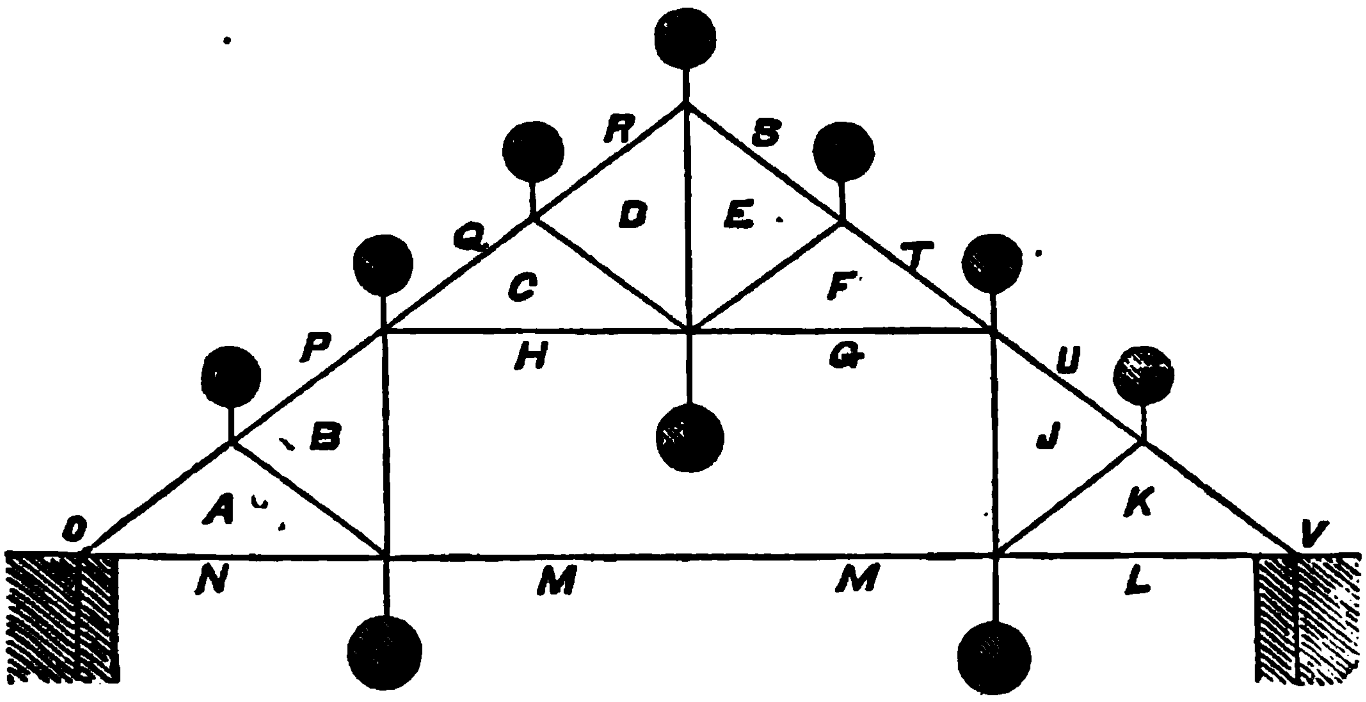

FIG. 115.

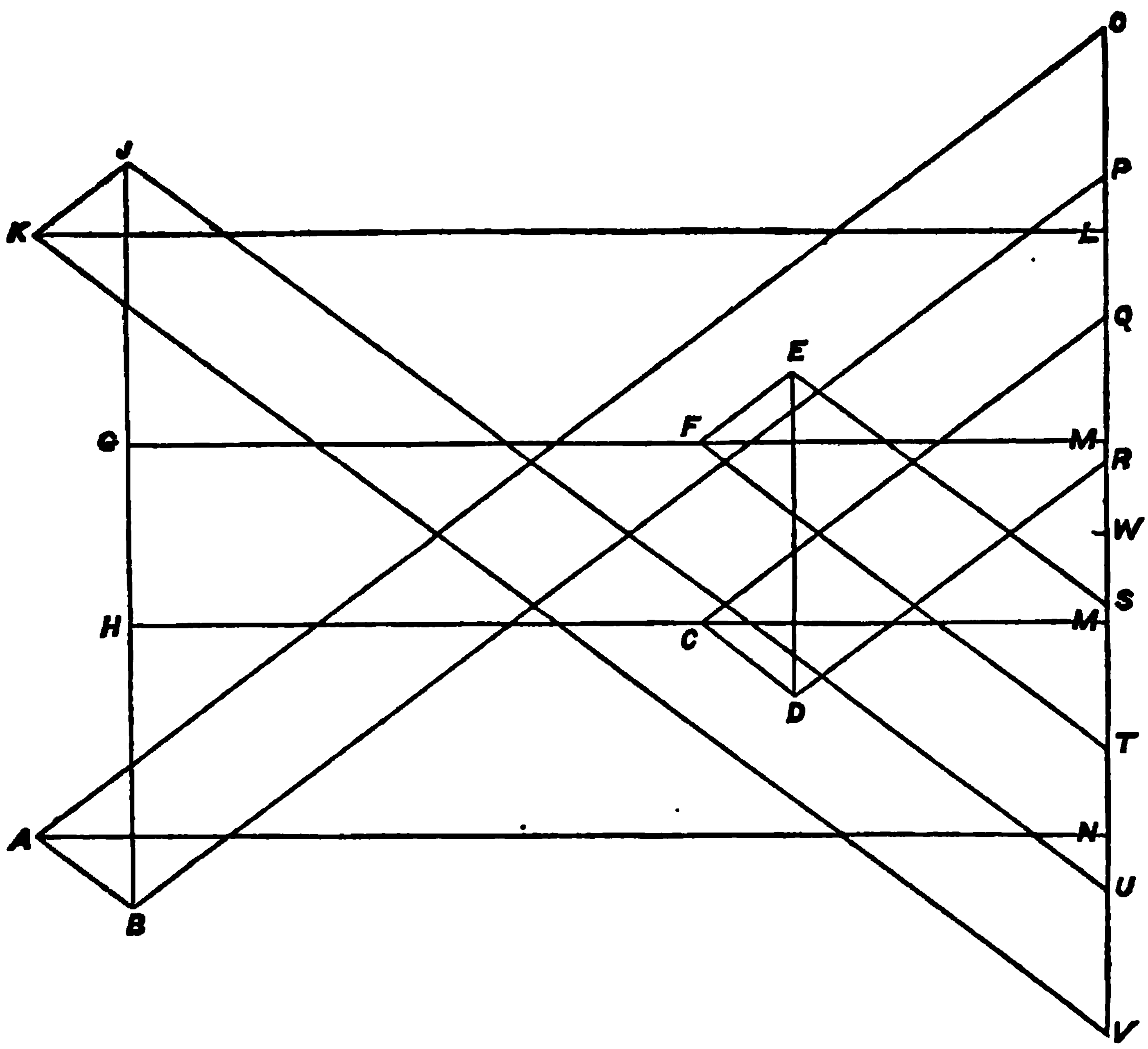

FIG. 116.

equal to *CM* or *FM* of *Fig.* 116, and thus, by counteracting the compression in the beam, reduce it to *CH* or *FG* of the force diagram, as shown.

665.—Force Diagram for Truss in *Fig.* 103.—The force diagram for the roof truss at *Fig.* 103 is given in *Fig.* 118, while *Fig.* 117 is the truss reproduced, with the lettering requisite for the construction of *Fig.* 118.

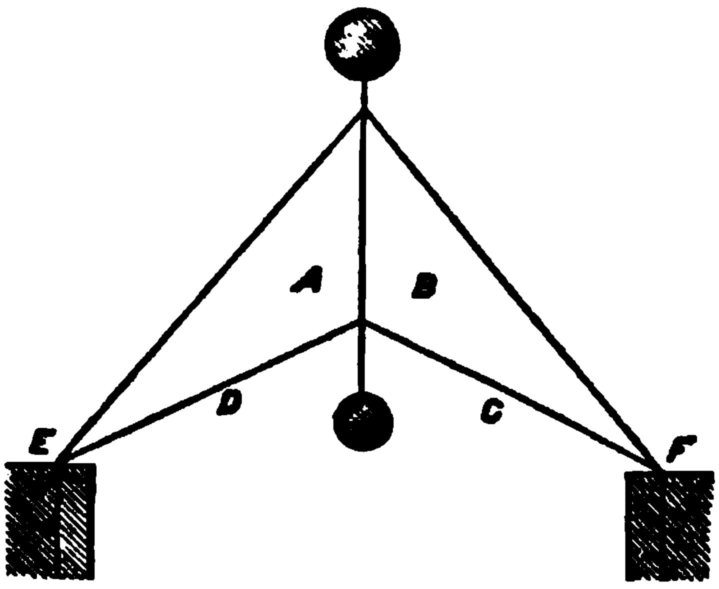

FIG. 117.

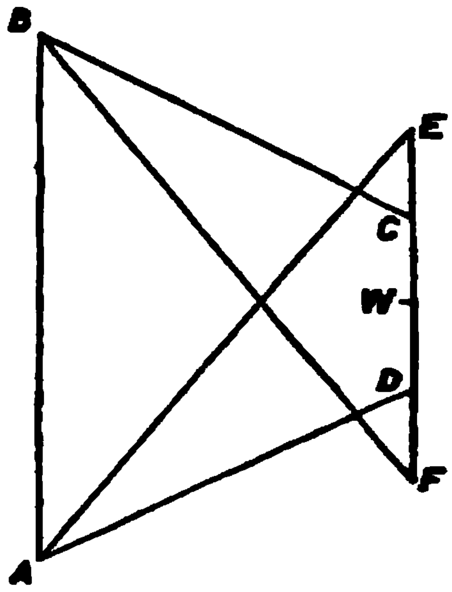

FIG. 118.

The vertical *EF* (*Fig.* 118) represents the load at the ridge. Divide this equally at *W*, and place half the lower weight each side of *W*, so that *CD* equals the lower weight. Then *ED* is equal to half the whole load, and equal to the reaction of the support *E* (*Fig.* 117). The lines in the triangle *ADE* give the strains in the corresponding lines converging at the point *ADE* of *Fig.* 117. The other lines, according to the lettering, give the strains in the corresponding lines of the truss. (See *Art.* 619.)

666.—Force Diagram for Truss in *Fig.* 104.—This truss is reproduced in *Fig.* 119, with the letters proper for use in the force diagram, *Fig.* 120.

Here the vertical *GK*, containing the three upper loads *GH*, *HJ* and *JK*, is divided equally at *W*, and the lower load *EF* is placed half on each side of *W*, and extends from *E* to *F*. Then *FG* represents one half of the whole load of the truss, and therefore the reaction of the support *G* (*Fig.* 119). Drawing the several lines of *Fig.* 120 parallel with the corresponding lines of *Fig.* 119, the force

diagram is complete, and the strains in the several lines of 119 are measured by the corresponding lines of 120. (See *Art.* 619.)

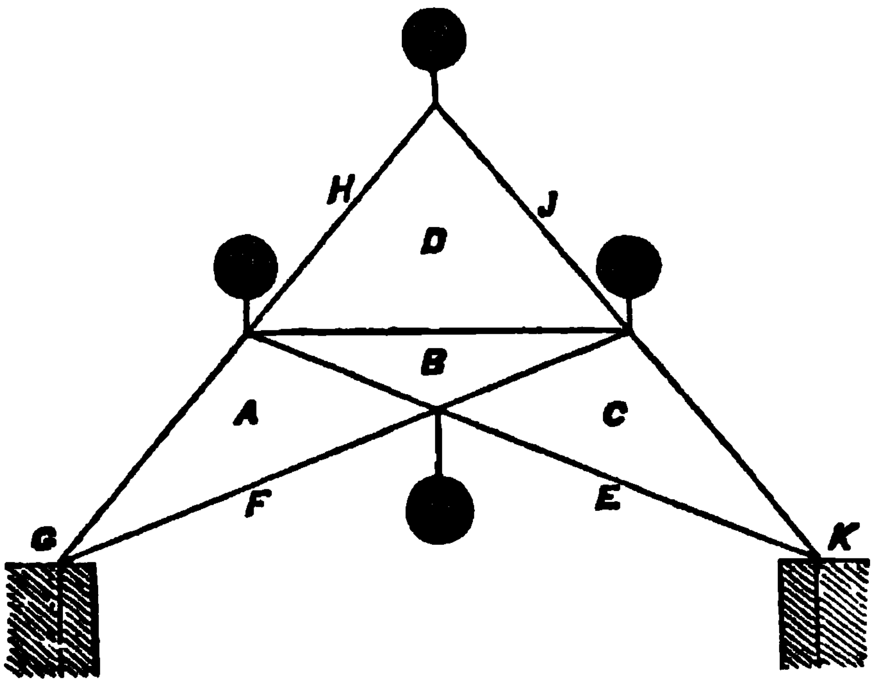

FIG. 119.

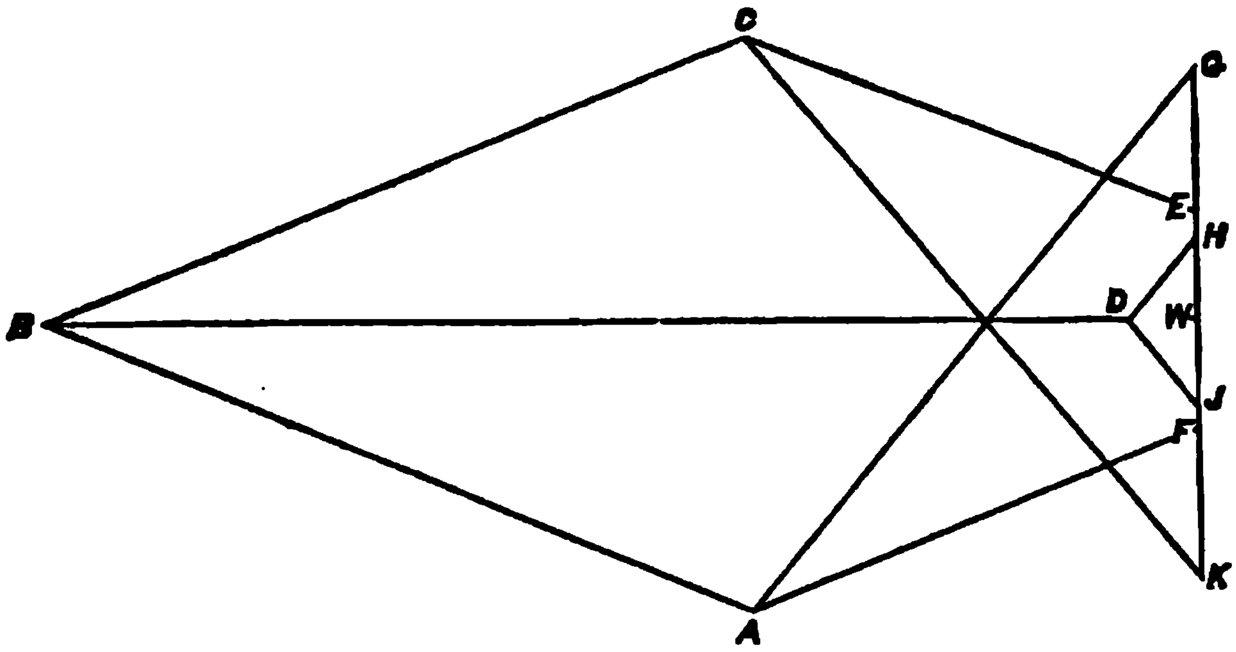

FIG. 120.

A comparison of the force diagram of the truss in *Fig.* 117 with that of the truss in *Fig.* 119 shows much greater strains in the latter, and we thus see that *Fig.* 117, or 103, is the more economical form.

667.—Force Diagram for Truss in *Fig.* 105.—This truss is reproduced and prepared by proper lettering in *Fig.* 121, and its force diagram is given in *Fig.* 122.

Here the vertical *JM* contains the three upper loads *JK*, *KL* and *LM*. Divide *JM* into two equal parts at

G, and make *FG* and *GH* respectively equal to the two loads *FG* and *GH* of *Fig.* 121. Then *HJ* represents one half of the whole weight of the truss, and therefore the reaction of the support *J*. From *H* and *J* draw lines parallel with *AH* and *AJ* of *Fig.* 121, and the sides of the tri-

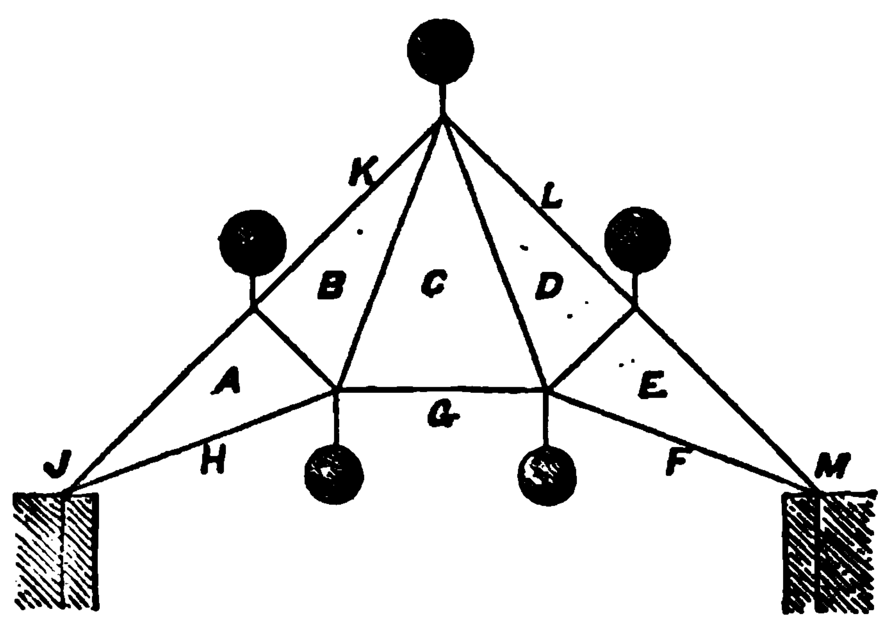

FIG. 121.

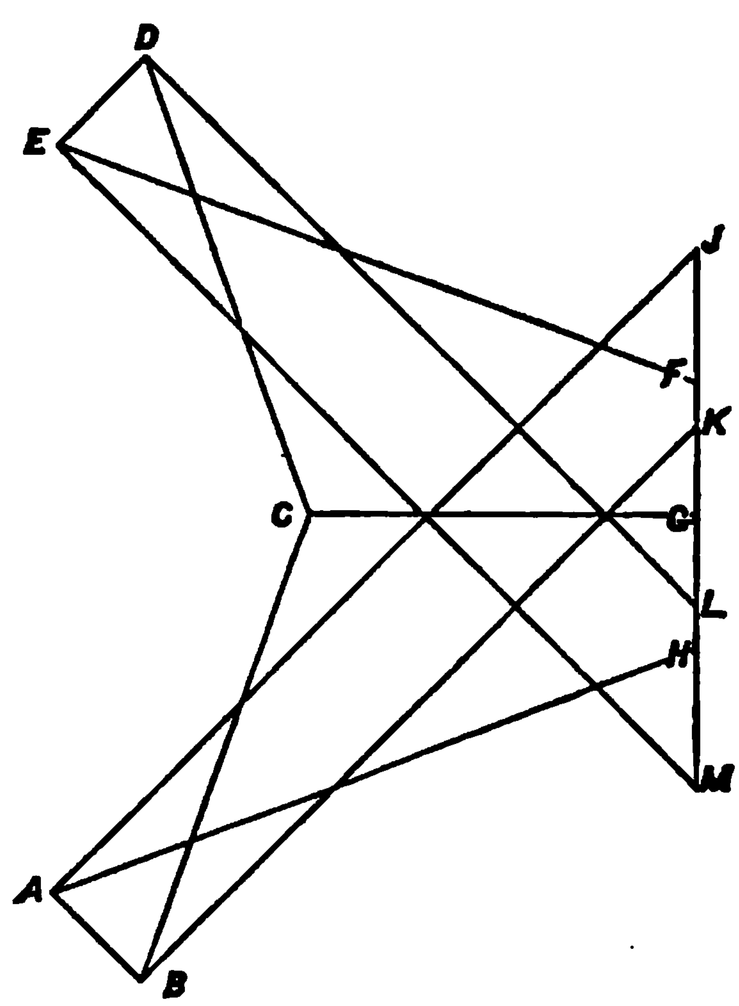

FIG. 122.

angle *AHJ* will give the strains in the three lines concentrating in the point *AHJ* (*Fig.* 121). The other lines of *Fig.* 122 are all drawn parallel with their corresponding lines in *Fig.* 121, as indicated by the lettering. (See *Art.* 619.)

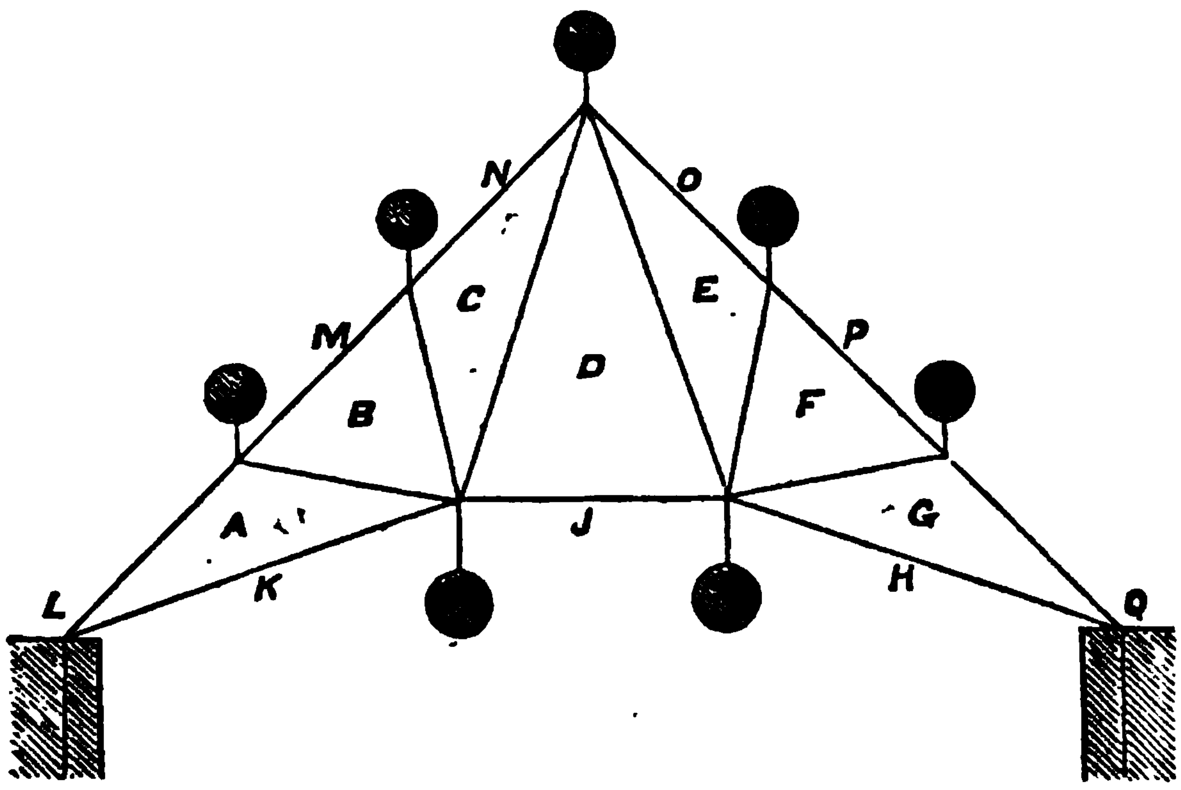

FIG. 123.

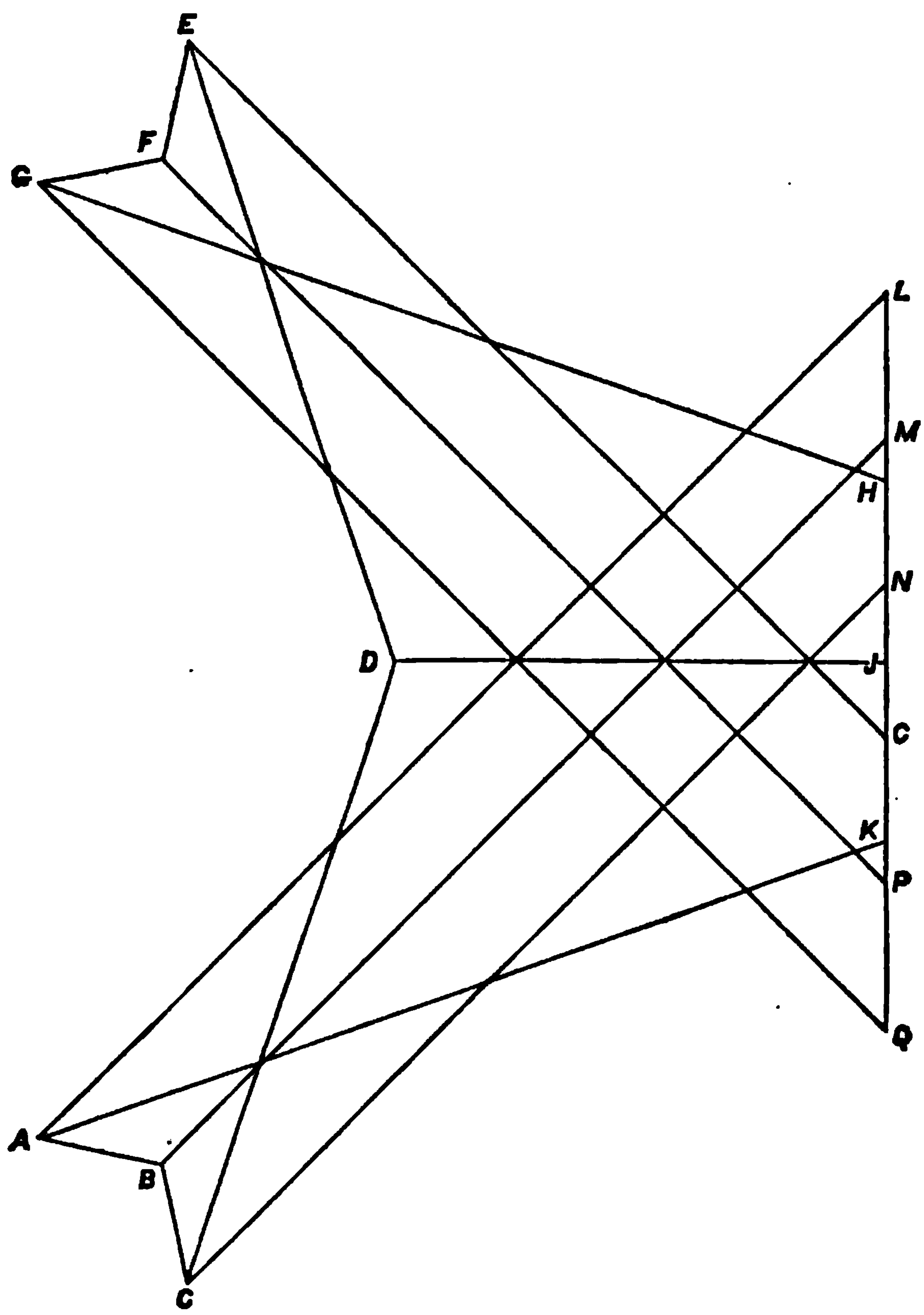

FIG. 124.

668.—Force Diagram for Truss in *Fig.* 106.—This truss is reproduced in *Fig.* 123 with the lettering proper for its force diagram, as given in *Fig.* 124. The five external weights of *Fig.* 123 make up the line *LQ*, and the two internal weights are set, one on each side of *J*, the middle point of *LQ*, extending to *H* and *K*. *KL* equals one half the weight of the whole truss, and equals the reaction of the point of support *L* (*Fig.* 123). The sides of the triangle *AKL*, therefore, give the respective strains in the three lines converging at the point *AKL* of *Fig.* 123. The other lines of *Fig.* 124 are found in the usual manner. (See *Art.* 619.)

669.—Strains in Horizontal and Inclined Ties Compared.—A comparison between a truss with a horizontal tie at the

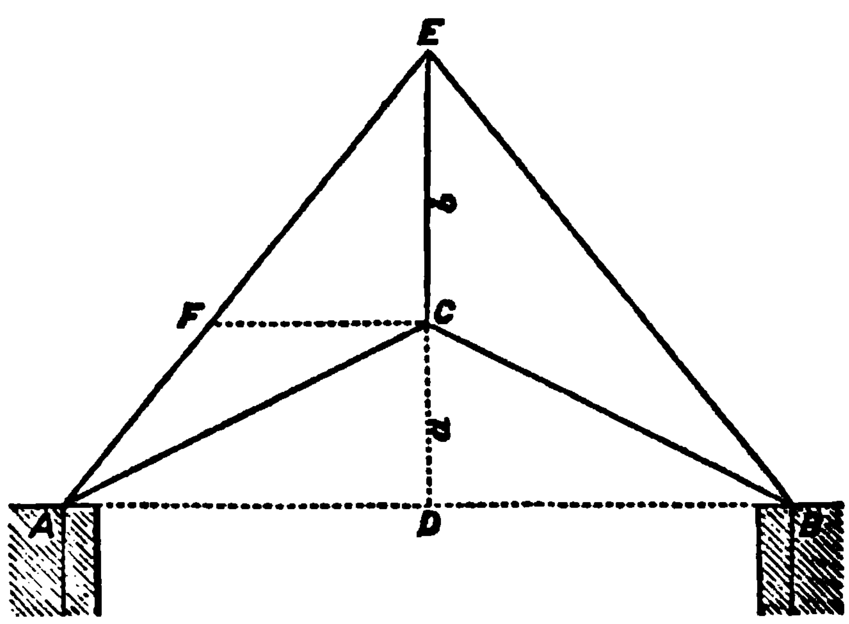

FIG. 125.

feet of the rafters, and one without such tie will now be given. The truss without a horizontal tie shown in *Fig.* 103 is one of the simplest in construction, and is suitable for the comparison. Repeating it in *Fig.* 125, and adding the dotted lines, we have likewise the form of a truss with a horizontal tie. From *Art.* 608 we have, in formula (*293.*), for the horizontal strain,

$$H_{,} = W_{,}\frac{h}{c}$$

in which $W_{,}$ equals the total weight of the truss and its load (*Fig.* 125), *h* equals half the span, equals *AD*, and *c*

equals twice the height, equals $2DE$. By putting $P = \frac{W}{2}$ equals the reaction of one of the supports A or B, and putting d for DE, we have

$$H = 2P\frac{h}{2d} = P\frac{h}{d}$$

or, from *Fig.* 125,

$$DE : AD :: P : H$$

$$d : h :: P : H = P\frac{h}{d}$$

that is to say, when the vertical DE represents half the weight of the truss, then AD may be put to represent the horizontal strain. Draw CF horizontal, and by similar triangles we have

$$DE : AD :: CE : CF \qquad \text{or}$$

$$CE : CF :: P : H = P\frac{CF}{CE}$$

or, with CE put to represent one half the weight of the whole truss, then CF, by the same scale, will measure the horizontal strain.

Under these conditions, CF measures the *horizontal* strain in either truss, whether with or without a tie-beam. If the truss have a horizontal tie AB, then CF measures the tension in this tie. If it be without the tie AB, having instead thereof the raised tie ACB, then still CF measures the *horizontal* strain at A or B, but *not* the strain in the raised tie AC.

The strain in this inclined tie is measured by the line AC, for the three sides of the triangle ACE are in proportion as the strains in these lines respectively (see *Art.* **619**), therefore the strains in the *ties* of the two trusses are comparable by the two lines CF and AC.

The compressive strain in the rafter is also correspondingly increased; for just in proportion as AE exceeds EF, so does the compressive strain in the rafter of a truss with an inclined tie exceed that of one with a horizontal tie.

670.—Vertical Strain in Truss with Inclined Tie.—In *Fig.* 125, if the inclined tie were lowered, so that the point C,

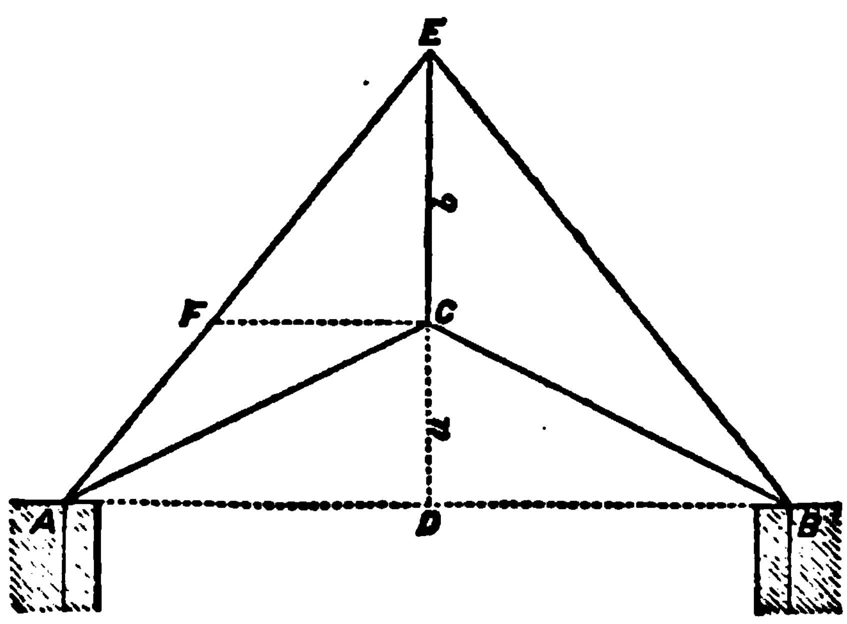

FIG. 125.

descending, should reach the point D; or, if the inclined tie become the horizontal tie AB; then the vertical rod DE would be subject to no strain from the weight of the rafters and the load upon them. In the absence of the horizontal tie, or when the inclined tie is depended upon to resist the spreading of the rafters, the vertical rod CE is strained directly in proportion to CD, the elevation of the tie, and inversely as the height CE. This relation may be shown as follows:

Let P be put for DE (*Fig.* 125) and represent one half the weight of the truss. Then AD will represent the horizontal strain at A; or, representing the span AB by the symbol s, then $\frac{s}{2}$ equals AD equals the horizontal strain. Putting a for CD and d for DE we have the proportion

$$DE : AD :: P : H$$

or $$d : \frac{s}{2} :: P : H = P\frac{s}{2d}$$

and also, $$AD : CD :: H : V$$

$$\frac{s}{2} : a :: H : V = H\frac{2a}{s} = P\frac{a}{d}$$

by substitution, or

$$d : a :: P : V = P\frac{a}{d}$$

This gives the vertical strain in CE, due to the raising of the tie from D to C, but it is not the whole of the strain; it is only so much of the vertical strain as is due to the weight of the roof. The tension thus found in CE is sustained at E by the two rafters, and, passing through them to A and B, creates horizontal and vertical thrusts precisely as did the original weight. The vertical tension thus brought to CE again acts as a weight at E, and, passing down the rafters and through the tie back to C, again adds a load at C. This in turn passes around and returns to C, adding to the load; and so on in an endless round to infinity. But the successive strains thus generated are in a decreasing series, and they may therefore be summed up and defined. Thus, as has just been shown, the vertical effect from the weight of the roof is

$$V = P\frac{a}{d}$$

The vertical effect of this latter is

$$d : a :: V : V' = V\frac{a}{d} = P\left(\frac{a}{d}\right)^2$$

The vertical effect of this is

$$d : a :: V' : V'' = V'\frac{a}{d} = P\left(\frac{a}{d}\right)^2$$

The next term in the series will be

$$V''' = P\left(\frac{a}{d}\right)^3$$

and the sum of all the terms will be

$$V \doteq P\left[\frac{a}{d} + \left(\frac{a}{d}\right)^2 + \left(\frac{a}{d}\right)^3 + \text{etc.}\right]$$

showing that the several values of the fraction by which the weight P is multiplied constitute a geometrical series, with $\frac{a}{d}$ for the first term and $\frac{a}{d}$ for the ratio. Since $\frac{a}{d}$ is less than unity, we have a geometrically decreasing infinite series, the sum of which is equal to the first term divided by one minus the ratio,* or

$$S = \frac{\frac{a}{d}}{1 - \frac{a}{d}} = \frac{a}{d - a}$$

and, since $d - a = b$ of *Fig.* 125,

$$S = \frac{a}{b}$$

We have, therefore, as the total vertical effect due to the elevation of the middle of the tie from D to C,

$$V = P\frac{a}{b}$$

---

* Ray's Algebra, Part Second, Art. 299.

or the vertical effect is directly in proportion to CD, the elevation of the tie, and inversely in proportion to CE, the length of the vertical tie-rod.

671.—Illustrations.—To illustrate the effect of the elevation of the tie-rod, upon the vertical strain in the suspension-rod, let the point C, *Fig.* 125, be elevated $\frac{1}{5}$ of the verti-

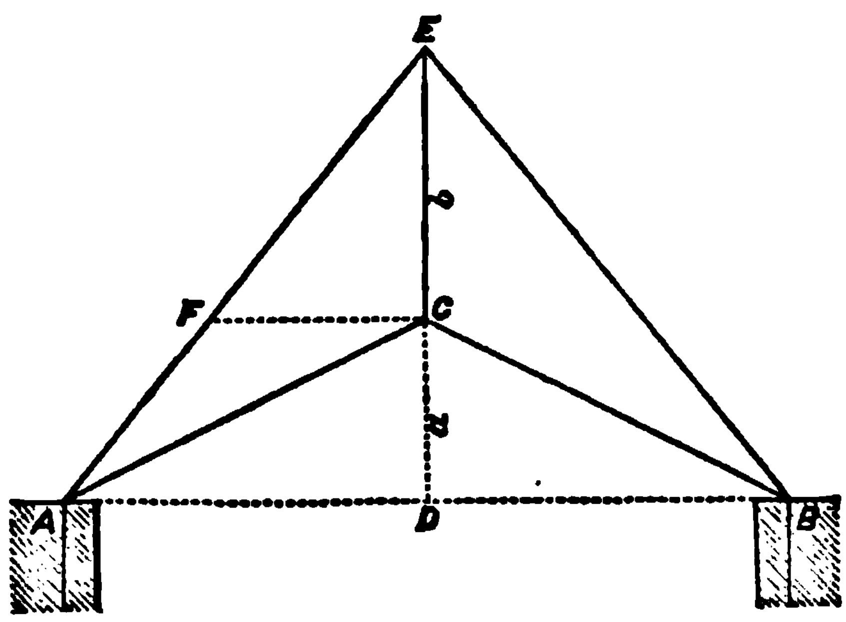

FIG. 125.

cal height of the truss above the horizontal line AB. Here $a = 1$ and $b = 4$, and $\frac{a}{b} = \frac{1}{4}$; or

$$V = P\frac{a}{b} = \frac{1}{4}P$$

When the elevation equals $\frac{1}{4}$ of the entire height, then

$$V = \frac{1}{3}P$$

When the elevation equals $\frac{1}{3}$ of the entire height, then

$$V = \frac{1}{2}P$$

When the elevation equals $\frac{1}{2}$ of the whole height, then

$$V = \frac{1}{1}P = P$$

Thus it is seen, in this last case, that the effect due to the

elevation of the tie-beam is equal to that of doubling the whole weight of the roof, and this increase affects not only the vertical suspension rod at the middle, but also the rafters and inclined ties, as was shown at *Art.* **669**.

When, therefore, in order to gain a small additional height to the interior of a building, it is proposed to raise the middle point of the tie-rod, it would seem advisable to consider whether this small additional height be an adequate compensation for the increased strains thereby induced, and the consequent enhanced cost for material necessary to resist these strains; and also, whether it be not more advisable to raise the walls of the building, rather than the ties of the trusses.

672.—Planning a Roof.—In designing a roof for a building, the first point requiring attention is the location of the trusses. These should be so placed as to secure solid bearings upon the walls; care being taken not to place either of the trusses over an opening, such as those for windows or doors, in the wall below. Ordinarily, trusses are placed so as to be centrally over the piers between the windows; the number of windows consequently ruling in determining the number of trusses and their distances from centres. This distance should be from ten to twenty feet; fifteen feet apart being a suitable medium distance. The farther apart the trusses are placed, the more they will have to carry; not only in having a larger surface to support, but also in that the roof timbers will be heavier; for the size and weight of the roof beams will increase with the span over which they have to reach.

In the roof-covering, itself, the roof-planking may be laid upon jack-rafters, carried by purlins supported by the trusses; or upon roof beams laid directly upon the back of

the principal rafters in the trusses. In either case, proper struts should be provided, and set at proper intervals to resist the bending of the rafter. In case purlins are used, one of these struts should be placed at the location of each purlin.

The number of these points of support rules largely in determining the design for the truss, thus:

For a short span, where the rafter will not require support at an intermediate point, *Fig.* 98 or 103 will be proper.

For a span in which the rafter requires supporting at one intermediate point, take *Fig.* 99, 104 or 105.

For a span with two intermediate points of support for the rafter, take *Fig.* 100 or 106.

For a span with three intermediate points, take *Fig.* 102.

Generally, it is found convenient to locate these points of support at nine to twelve feet apart. They should be sufficiently close to make it certain that the rafter will not be subject to the possibility of bending.

673.—Load upon Roof Truss.—In constructing the force diagram for any truss, it is requisite to determine the points of the truss which are to serve as points of support (see *Figs.* 109, 111, etc.), and to ascertain the amount of strain, or loading, which will occur at every such point.

The points of support along the rafters will be required to sustain the roofing timbers, the planking, the slating, the snow, and the force of the wind. The points along the tie-beam will have to sustain the weight of the ceiling and the flooring of a loft within the roof, if there be one, together with the loading upon this floor. The weight of the truss itself must be added to the weight of roof and ceiling.

674.—Load on Roof per Foot Horizontal.—In any important work, each of the items in *Art.* **673** should be carefully estimated, in making up the load to be carried. For ordinary roofs, the weights may be taken per foot superficial, as follows:

Slate,	about	7·0	pounds.
Roof plank,	"	2·7	"
Roof beams, or jack-rafters,	"	2·3	"
In all,		12	pounds.

This is for the superficial foot of the inclined roof. For the foot horizontal, the augmentation of load due to the angle of the roof will be in proportion to its steepness. In ordinary cases, the twelve pounds of the inclined surface will not be far from fifteen pounds upon the horizontal foot.

For the roof load we may take as follows:

Roofing,	about	15	pounds.
Roof truss,	"	5	"
Snow,	"	20	"
Wind,	"	10	"
Total on roof,		50	pounds

per square foot horizontal.

This estimate is for a roof of moderate inclination, say one in which the height does not exceed ¼ of the span. Upon a steeper roof, the snow would not gather so heavily, but the wind, on the contrary, would exert a greater force. Again, the wind acting on one side of a roof may drift the snow from that side, and perhaps add it to that already lodged upon the opposite side. These two, the wind and the snow, are compensating forces. The action of the snow is vertical: that of the wind is horizontal, or nearly so. The power of the wind in this latitude is not more than thirty

pounds upon a superficial foot of a vertical surface; except, perhaps, on elevated places, as mountain tops for example, where it should be taken as high as fifty pounds per foot of vertical surface.

675.—Load upon Tie-Beam.—The load upon the tie-beam must of course be estimated according to the requirements of each case. If the timber is to be exposed to view, the load to be carried will be that only of the tie-beam and the timber struts resting upon it. If there is to be a ceiling attached to the tie-beam, the weight to be added will be in accordance with the material composing the ceiling. If of wood, it need not weigh more than two or three pounds per foot. If of lath and plaster, it will weigh about nine pounds; and if of iron, from ten to fifteen pounds, according to the thickness of the metal. Again, if there is to be a loft in the roof, the requisite flooring may be taken at five pounds, and the load upon the floor at from twenty-five to seventy pounds, according to the purpose for which it is to be used.

676.—Selection of Design for Roof Truss.—As an example in designing a roof truss: Let it be required to provide trusses for a building measuring 60 x 90 feet to the centre of thickness of the walls, with seven windows upon each side, and with a roof having its height equal to one third of the span. The roofing is to be of plank and slate, the ceiling is to be finished with plastering, and the space within the roof is to be used for the storage of light articles, not to exceed twenty-five pounds to the square foot.

Here, in the first place, we have to determine the number of trusses. As there are seven windows on a side, there should be six trusses, one upon each pier between the windows. The six trusses and the two end walls will afford

eight lines of support for the roofing. There will thus be seven bays of roofing of $\frac{90}{7} = 12\frac{6}{7}$ feet each, and this is the width of roofing to be carried by each truss.

In the next place, the points of support in the truss are to be ascertained. If these are provided at every ten feet horizontally, they will divide the half truss into three spaces, and there will be two intermediate points of support. For this arrangement, such a roof truss as is shown in *Fig.* 100 will be appropriate, but if the space in the roof is required to be quite unobstructed with timber at the middle, then a modification of this design may be used, as in the form shown in *Fig.* 126; each rafter being still divided into three equal parts.

677.—Load on Each Supported Point in Truss.—The horizontal measurement, then, of the roofing to be carried by each supported point in the truss, will be 10 feet along the line of the truss and $12\frac{6}{7}$ feet across the truss (this latter being the width of each bay as above found); or $10 \times 12\frac{6}{7} = 128\frac{4}{7}$ feet. With a weight per foot of 50 pounds, as estimated in *Art.* **674**, we have, for the load upon each supported point of the truss,

$$128\tfrac{4}{7} \times 50 = 6428\tfrac{4}{7}$$

or, say 6500 pounds.

678.—Load on Each Supported Point in Tie-Beam.—The tie-beam having two points of support, we have $\frac{60}{3} = 20$ feet for the length of the surface to be carried. This, multiplied by the width between trusses, gives $20 \times 12\frac{6}{7} = 257\frac{1}{7}$ feet area of surface to be carried by each point of support. We will estimate the weight per foot in this present case as follows:

Load upon the floor,	25	pounds.
Flooring, with timber,	5	"
Plastering,	9	"
Tie-beam, etc.,	1	pound.
Total at tie-beam,	40	pounds.

This gives

$$257\frac{1}{7} \times 40 = 10285\frac{5}{7}$$

or, say 10,300 pounds upon each supported point.

Therefore, the two balls *GH* and *HJ*, suspended from the tie-beam of *Fig.* 126, are to be taken as weighing 10,300 pounds each, while the five balls located above the rafters are to be understood as weighing 6500 pounds each (*Art.* **677**).

679.—Constructing the Force Diagram.—We may now proceed to construct the force diagram, *Fig.* 127, as follows:

Upon the vertical line *KP* lay off in equal parts *KL*, *LM*, *MN*, *NO* and *OP*, according to any convenient scale, each equal to 6500 pounds—the weight of the balls above the rafters (*Art.* **677**). If a scale of 100 parts to the inch be selected for the force diagram, and each part be understood as representing 100 pounds, then $\frac{6500}{100} = 65$, equals the number of parts to assign to each of the distances *KL*, *LM*, etc., and each will be $\frac{65}{100}$ of an inch in length. Dividing *KP* at *H* into two equal parts, lay off on each side of *H* the distances *GH* and *HJ*, each equal, by the scale, to 10,300 pounds. This distance is found by dividing 10,300 by 100; the quotient 103 is the number of parts, and the distances will each be $\frac{103}{100}$, or one inch and $\frac{3}{100}$ of an inch in length.*

* The scale here selected, although sufficient for the purposes of illustration,

HK now represents one half the weight upon the rafters, and *HJ* one half the load upon the tie-beam, and their sum, *JK*, equals one half the total load of the truss, equals the load upon the point of support *K*.

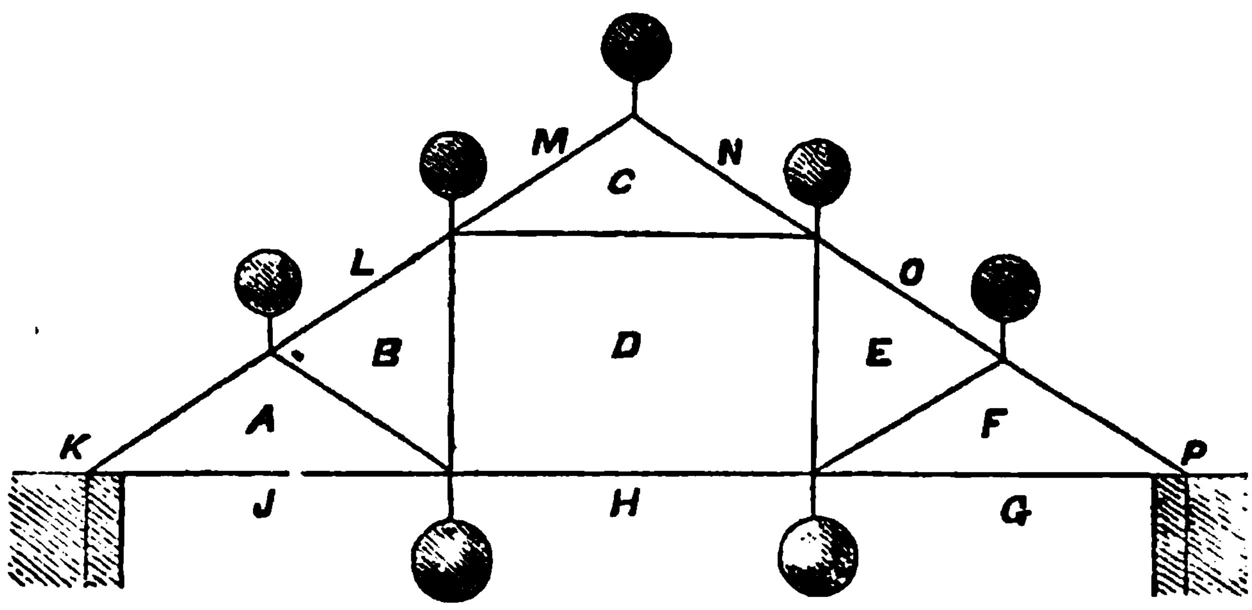

FIG. 126.

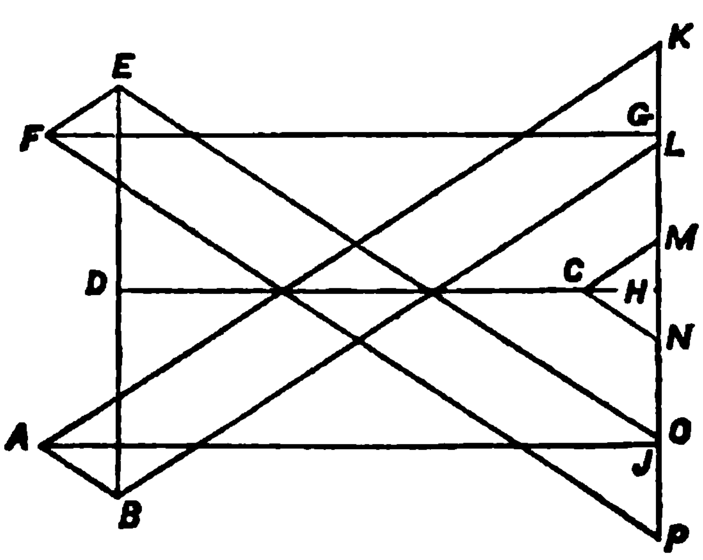

FIG. 127.

From *J*, *H* and *G* draw the horizontal lines *JA*, *HD* and *GF*. From *K*, *L*, *M*, *N*, *O* and *P* draw

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would be too small for a working drawing. For the latter, a scale should be selected as large as can be conveniently used, such as 10 parts to the inch, and 100 pounds to each part. This would give 1000 pounds to the inch, and each of the distances *KL*, *LM*, etc., would measure 6¼ inches.

It must also be remembered that the accuracy of the force diagram depends upon the care with which the distances upon the vertical line are laid off and the lines drawn. The drawing implements should be examined to know that they are true, and each line should be drawn carefully parallel with the corresponding line of the truss. Unless this care is exercised, the results may differ considerably from the truth.

lines, as shown, carefully parallel with the rafters. From *F* and *A* draw the lines *FE* and *AB*, parallel with the two braces. Connect *B* and *E* by the vertical line *BE*, and then the force diagram is complete.

680.—Measuring the Force Diagram.—After drawing the lines of the diagram as above directed, they should all be carefully traced to know that the required conditions are fulfilled, or that each set of lines, drawn parallel, in the diagram of forces, to the lines converging to a point in the truss, forms a closed polygon. (See *Arts.* **618**, **619** and **620**.)

The diagram, by this test, having been found correct, the force in each line of the truss may be measured by applying the scale to the corresponding line of the diagram.

For example, take the strains in one of the rafters. At its lower end, or the part *AK*, its corresponding line *AK* of *Fig.* 127 measures 478 parts, by the same scale with which the weights on the vertical line *KP* were laid off. This, at 100 pounds to the part, gives 47,800 pounds as the strain in the foot of the rafter. The next section of the rafter is designated by the letters *BL*, and the line *BL* (*Fig.* 127) measures 420, and indicates a strain in this part of the rafter of 42,000 pounds. The third or upper portion of the rafter is designated by the letters *CM*, and the corresponding line in *Fig.* 127 measures 58 parts, indicating 5800 pounds as the strain in the upper end of the rafter.

For the brace *AB* we have the line *AB* (*Fig.* 127), measuring 58 parts of the scale, and indicating 5800 pounds as the strain in the brace.

For the vertical *BD* we have the line *BD* (*Fig.* 127) measuring 135 parts of the scale, and indicating 13,500 pounds as the strain in the vertical.

For the horizontal strains, we have for *CD*, the corresponding line in *Fig.* 127, which measures 301 parts, and

gives 30,100 pounds as the strain. For *DH*, the middle portion of the tie-beam, *DH* (*Fig.* 127) measures 350, showing the strain to be 35,000 pounds; and for *AJ*, or one end of the tie-beam, *AJ* (*Fig.* 127) measures 398 parts, and gives 39,800 pounds as the strain.

The strains in the other and corresponding parts of the truss are the same as these, so that we now have all the strains required.

681.—Strains Computed Arithmetically.—Instead of depending solely upon the scale, the lengths of the lines in the force diagram may be computed arithmetically. The sizes measured by the scale, when the diagram is carefully drawn, are sufficiently accurate for all practical purposes; but in some cases, such, for instance, as when the implements for making a correct diagram are not at hand, and in all cases as a check upon the accuracy of the results obtained by the graphic method, to be able to arrive at the correct results arithmetically would be useful. Preparatory to computing the lengths of the lines, it will be observed that the triangle *KAJ*, *Fig.* 127, is precisely proportionate to the triangles formed by the inclination of the rafters of *Fig.* 126 with the vertical and horizontal lines; that all the inclined lines of *Fig.* 127 are drawn at equal angles of elevation; and that the triangles formed by these inclined lines with the vertical and horizontal lines are all homologous.

Since the height of the roof is given at 20 feet, and half the span is 30 feet, therefore the perpendicular and base lines of each triangle are in like proportion—namely, as 20 to 30, or as 1 to $1\frac{1}{2}$.

The perpendicular being the weight in each case, which is known, we may, therefore, by this proportion obtain the base. Having both base and perpendicular, the length of the hypothenuse may be found by Euclid's 47th of 1st book—

the length of the hypothenuse equals the square root of the sum of the squares of the base and perpendicular. If the hypothenuse of one triangle be computed by this method, that of the others (since the triangles are homologous) may be found by the more simple method of proportion.

Taking a triangle having the perpendicular and base equal to 1 and $1\frac{1}{2}$, we find, by the above rule, that its hypothenuse equals $1 \cdot 802776$ nearly. The hypothenuses of the other triangles, therefore, may be found by the proportion:

$$1 : 1 \cdot 802776 :: p : h$$

$$h = 1 \cdot 802776 p$$

and for the base we have

$$1 : 1 \cdot 5 :: p : b$$

$$b = 1 \cdot 5 p$$

With these formulas, the lines in *Fig.* 127 have been computed. The strains in the proposed truss (*Fig.* 126), by both methods, have been found to be as follows:

		BY SCALE.		BY COMPUTATION.	
AK	=	47,800	pounds;	47,864	pounds.
BL	=	42,000	"	42,005	"
CM	=	5,800	"	5,859	"
AB	=	5,800	"	5,859	"
CD	=	30,100	"	30,075	"
DH	=	35,000	"	34,950	"
AJ	=	39,800	"	39,825	"
BD	=	13,500	"	13,550	"

682.—Dimensions of Parts Subject to Tension.—With these forces, and the appropriate rules hereinbefore given, the dimensions of the several parts of the truss may now be determined.

Commencing with the tie-beam, *KP*, it may be observed, preparatory to computing its dimensions, that while this piece, in resisting the thrust of the rafters, is subjected to a tensile strain, it is also subject to a transverse strain from the weight of the ceiling and floor which it has to carry. These two strains, however, are of such a nature that in their effect upon the beam they do not conflict; for the tensile strain from the thrust of the rafters, acting, as it will usually, in the upper half of the beam, serves to counteract the compression produced by the transverse strain in this part of the beam, and the fibres near the middle of the beam, owing to their proximity to the neutral line, being strained very little by the transverse strain, have a large reserve of strength available to assist in resisting the tensile strain. It will be sufficient, therefore, to provide a piece of timber for the tie-beam of sufficient size to resist only one of the two strains; not necessarily that strain, however, which is the greater, but that one which requires the larger piece of timber to resist it.

The computations of dimensions required to resist the two strains will now claim attention.

For the tensile strain we have, by formula (*299.*),

$$\frac{20 \times 39800}{16000} = 49 \cdot 75$$

or say 50 inches area of cross-section, for Georgia pine. For white pine the area should be 65 inches.

The load producing transverse strain is (*Art.* **678**) 10,300 pounds. The rule for determining the proper area of cross-section is to be found in formula (*130.*), which may be modified for this case by substituting rl for δ, the symbol for deflection, and by putting for r the rate 0·04 of an inch. With these substitutions, we have

$$\tfrac{5}{8} Ul^2 = 0 \cdot 04 Fbd^3$$

Fixing upon a proportion for b in terms of d, say, for example, $b = \frac{3}{4}d$, and substituting this value for b, we have

$$\tfrac{5}{8}Ul^2 = 0{\cdot}04 \times \tfrac{3}{4}Fd^4$$

$$\frac{20\frac{5}{6}Ul^2}{F} = d^4$$

If the timber is to be of white pine, then F equals 2900 (Table XX.), and we have

$$d = \sqrt[4]{\frac{20\frac{5}{6} \times 10300 \times 20^2}{2900}} = 13{\cdot}116$$

or the depth will need to be $13\frac{1}{8}$ inches. Three quarters of this, or $9\frac{7}{8}$, will be the breadth. The tie-beam, of white pine, will need to be, therefore, say 10×13 inches. If of Georgia pine, instead of white pine, then 5900, the value of F for Georgia pine, must be substituted for 2900 in the formula, and the results, $8{\cdot}237$ and $10{\cdot}982$, will show, say $8\frac{1}{4} \times 11$ inches as the size of timber required.

The dimensions thus found, to resist the transverse strain, being in excess of those required to resist the tensile strain, are to be adopted as the dimensions of the required tie-beam.

The length of the tie-beam, 60 feet, being greater than can readily be obtained in one piece, it will have to be built up. In doing this, it is necessary that each piece be of the full height of the beam, or that the joints of the make-up be vertical and not horizontal. These vertical laminas should be in pieces of such lengths that no two heading joints occur within five feet of each other, and that these joints shall be as near as practicable to the two vertical suspending rods. The laminas need to be well secured together with proper iron bolts. The feet of the rafters should be provided with iron clamps of sufficient area to resist the horizontal strain there, and should be secured to the tie-beam with bolts of corresponding resistance.

If the iron in the bolts and clamps of the truss be of average good quality, it may be calculated on as resisting effectually 9000 pounds per square inch (see *Art.* **642**). The vertical suspension rods BD and DE, *Fig.* **126**, may also be calculated for a like strain.

683.—Dimensions of Parts Subject to Compression.—The rafters, straining beam and braces are all subject to compression, and their dimensions may now be obtained.

The areas of these pieces may be had by the use of formula (*301.*); or, as this in some cases is objectionable, for the reason that the ratio between the length and thickness has to be assumed in advance, we may find in formula (*303.*) a rule free from this objection, but encumbered with more intricate computations. Formula (*301.*), when used by those having experience in such work, is far preferable, on account of its greater simplicity.

Taking first the rafter, and the portion of it at the foot, where the strain is greatest, 47,800 pounds, we have for its length about 12 feet. If of Georgia pine, its thinnest dimension of cross-section will probably be about 8 inches. Then $r = \frac{12l}{h} = \frac{12 \times 12}{8} = 18$ (see *Art.* **643**). The value of C is 9,500 and the value of e is 0·00109, both by Table XX. Making the symbol for safety, a, equal 10 we have

$$A = \frac{10[1 + (\tfrac{3}{4} \times 0{\cdot}00109 \times 18^2)]\,47800}{9500} = 76{\cdot}97$$

or the area of the rafter should be 77, say $8 \times 9\tfrac{5}{8}$ inches.

If computed by formula (*303.*), putting $n = 1{\cdot}2$, the exact size will be found at $8{\cdot}006 \times 9{\cdot}607 = 76{\cdot}92$ inches area.

684.—Dimensions of Mid-Rafter.—In the rafter at BL the strain is 42,000 pounds. The length and ratio here will be the same as at AK, and the dimensions of AK and BL are therefore in proportion to the weights (*form. 301.*), or

$$47800 : 42000 :: 76 \cdot 92 : A$$

$$A = \frac{42000 \times 76 \cdot 92}{47800} = 67 \cdot 587$$

so that 68 inches of sectional area, or $8 \times 8\frac{1}{2}$ inches, is the size required.

685.—Dimensions of Upper Rafter.—The upper end of the rafter has only the weight at the ridge, 5,800 pounds, to bear. The thickness of the rafter here will probably be but 4 inches. This gives a ratio of $\frac{144}{4} = 36$. With this ratio, with 5,800 for the weight, and with the other quantities as before, a computation by formula (*301.*) will result in showing the required area to be $19 \cdot 04$, or, say 4×5 inches; but, in order to resist effectually the distributed load of the roofing, this part of the rafter should not be less than 4×8 inches.

686.—Dimensions of Brace.—The brace, AB, being of equal length and carrying an equal load with the upper end of the rafter, may be made of the size there found necessary, or, say 4×6 inches.

687.—Dimensions of Straining-Beam.—The straining-beam CD is compressed with a strain of 30,100 pounds,

and its length is 20 feet.

Assuming its thickness to be that of the rafter, we have $r = \frac{12 \times 20}{8} = 30$, and in formula (*301.*)

$$A = \frac{10[1 + (\frac{3}{2} \times 0 \cdot 00109 \times 30^2)]30100}{9500} = 78 \cdot 29$$

or its area should be $78\frac{1}{4}$, or, say $8 \times 10 = 80$ inches.

With this result, the computation of the dimensions of all the pieces of the truss is completed; for the other rafter and brace are in like condition with those computed, and should therefore be of the same dimensions.

QUESTIONS FOR PRACTICE.

688.—In a roof truss similar to that shown in *Fig.* 109, of 42 feet span and 14 feet height, measuring from the axial lines: What will be the strains in the various pieces of the truss, with a load of 5,000 pounds at each of the three points above the rafters, and a load of 10,000 pounds suspended from the centre of the tie-beam?

Draw the appropriate force diagram, and give the strains from measurement.

689.—Draw a force diagram for a roof truss similar to the design in *Fig.* 111, with a span of 54 feet and a height

of 18 feet; the upper weights being taken at 6,000 pounds each, the central weight under the tie-beam at 5,000 pounds, and each of the two other weights at 7,000 pounds.

Show, from the diagram, the strain in each line of the truss.

690.—In a truss similar to that in *Fig.* 121, show, by a force diagram, what would be the strains in each line, when the span is 40 feet and the height 20 feet. The weights *FG* and *GH* are so located as to divide the span into three equal parts, the three loads above the rafters are each 7,000 pounds, and the two loads below each 4,000 pounds. The point *JABK* is to be taken at the middle of the rafter, and the line *AB* is to be drawn at right angles with the rafter.

691.—In a roof with an elevated tie-beam, such as in *Fig.* 125, with a span of 40 feet and height of 20 feet, and with the tie elevated at the middle 8 feet above the level of the feet of the rafters, compute the strain in the suspension-rod at the middle, due to the elevation of the tie; the weight upon one half of the truss being 24,000 pounds.

692.—In a building 119 feet long, and 80 fect wide to the centres of bearings, and having the side walls pierced for seven windows each, state how many roof trusses there should be.

Which of the designs given, having a tie horizontal from the feet of the rafters, would be appropriate for the case?

The roof is to be 25 feet high at middle, and to have the interior space along the middle free from timber. The

load upon the roof is to be taken at 50 pounds per foot horizontal, upon the tie-beam at 40 pounds to the foot, and upon the straining beam at 5 pounds per foot.

Make a force diagram, and from it show the strains in each piece.

Compute the dimensions of the several timbers, which are all to be of Georgia pine; the rafter being 9 inches thick below the straining-beam and 6 inches above, and the iron work being subjected to a tensile strain of 9000 pounds per inch.

CHAPTER XXIV.

TABLES.

ART. **693.—Tables I. to XXI.—Their Utility.**—Rules for determining the required dimensions of the various timbers in floors are included in previous chapters. These rules are carefully reduced to the forms required in practice. In using them, it is only needed to substitute for the various algebraic symbols their proper numerical values, and to perform the arithmetical processes indicated, in order to arrive at the result desired.

To do even this simple work, however, requires care and patience, and these the architect, owing to the multiplicity of detail demanding time and attention in his professional practice, frequently finds it difficult to exercise. To relieve him of this work, the first twenty-one of the following tables have been carefully computed. Tables I. to XXI. afford the data for ascertaining readily the dimensions of the beams and principal timbers required in floors of dwellings and first-class stores. Tables XVII., XVIII. and XIX. refer to beams of rolled-iron; the others to those of wood.

694.—Floor Beams of Wood and Iron (I. to XIX. and XXI.)—In these tables will be found the dimensions of Floor Beams and Headers, of Hemlock, White pine, Spruce and Georgia pine; for Dwellings and for First-class stores.

Tables XVIII. and XIX. exhibit the distances from centres at which Rolled-iron Beams are required to be placed

in Banks, Office Buildings and Assembly-Rooms, and in First-class Stores.

695.—Floor Beams of Wood (I. to VIII.).—In these tables the recorded distance from centres is in inches, and is for a beam one inch thick, or broad. The *required* distance from centres is to be obtained by multiplying the *tabular* distance by the breadth of the given beam.

For example: Let it be required to ascertain the distance from centres at which white pine 3×10 inch beams, 16 feet long in the clear of the bearings, should be placed in a dwelling.

By reference to Table II., "White Pine Floor Beams One Inch Thick, for Dwellings, Office Buildings, and Halls of Assembly," we find, vertically under 10, the depth, and opposite to 16, the length, the dimension $4 \cdot 5$. This is the distance from centres for a beam one inch broad. Then, since the given beam has a breadth of 3 inches,

$$3 \times 4 \cdot 5 = 13 \cdot 5$$

equals the required distance from centres for beams 3 inches broad. Therefore, 3×10 inch white pine beams with 16 feet clear bearing, should, in a dwelling, etc., be placed $13\frac{1}{2}$ inches from centres.

Tables I. to IV. were computed from formula (*143.*),

$$cl^2 = ibd^2$$

which, with $b = 1$, and putting c in inches, becomes

$$c = \frac{12id^2}{l^3} \qquad (306.)$$

Tables V. to VIII. were computed from formula (*149.*),

$$cl^3 = kbd^3$$

which, with $b = 1$, and with c in inches, becomes

$$c = \frac{12kd^3}{l^3} \qquad (307.)$$

696.—Headers of Wood (IX. to XVI.).—(See *Art.* **142.**) The results recorded in these tables show the breadth of headers which carry tail beams one foot long. The tabular breadth, if multiplied by the length in feet of the given tail beam, will give the breadth of the required header.

For example: Let it be required to ascertain the breadth of a Georgia pine header 20 feet long, 15 inches deep, and carrying tail beams 12 feet long, in the floor of a first-class store. By referring to Table XVI., "Georgia Pine Headers for First-class Stores," at the intersection of the vertical column for 15 inches depth and the horizontal line for 20 feet length, we find the dimension 0·85. This is the breadth of the header for each foot in length of the tail beams. As the tail beams in this case are 12 feet long, therefore $12 \times 0{\cdot}85 = 10{\cdot}2$, equals the required breadth of the header in inches.

The first four (IX. to XII.) of these tables were computed from formula (*156.*),

$$b = \frac{fng^3}{4Fr(d-1)^3}$$

which, when reduced (putting $r = 0{\cdot}03$, $f = 90$ and $n = 1$) becomes

$$b = \frac{750g^3}{F(d-1)^3} \qquad (308.)$$

The second four (XIII. to XVI.) of these tables were computed from the same formula, (*156.*), by putting $r = 0{\cdot}04$, $f = 275$ and $n = 1$; which reduction gives

$$b = \frac{1718\tfrac{3}{4} g^2}{F(d-1)^2} \qquad (309.)$$

697.—Elements of Rolled-Iron Beams (XVII.).—Table XVII. contains the dimensions of cross-section and the values of I, the moment of inertia, for 66 of the rolled-iron beams of American manufacture in use. These values are required in using the rules in Chapter XIX., by which the capacities of the beams are ascertained. (See *Arts.* **479** to **482**, **485** to **492**, **501**, **511**, **512**, **514**, **517**, **519**, **521**, **523**, etc.)

The values of I were computed by formula (*213.*)

$$I = \tfrac{1}{12}(bd^3 - b_l d_l^3)$$

698.—Rolled-Iron Beams for Office Buildings, etc. (XVIII.).—Table XVIII. contains the distances from centres, in feet, at which rolled-iron beams should be placed, in the floors of Dwellings, Banks, Office Buildings and Assembly Halls. (See *Arts.* **500** and **501**.)

These distances were computed by formula (*237.*),

$$c = \frac{255{\cdot}0\tfrac{5}{7} I}{l^2} - \frac{y}{420}$$

699.—Rolled-Iron Beams for First-class Stores (XIX.).—Table XIX. contains the distances from centres, in feet, at which rolled-iron beams should be placed, in the floors of First-class Stores. (See *Arts.* **504** and **505**.)

These distances were computed by formula (*239.*),

$$c = \frac{148 \cdot 8 I}{l^2} - \frac{y}{960}$$

700.—Example.—As an example to show the uses of Tables XVIII. and XIX.: Let it be required to know the distances from centres at which 9 inch 84 pound Phœnix rolled-iron beams should be placed, on walls with a span or clear bearing of 18 feet, to form a floor to be used in an Office Building or Assembly Room.

In Table XVIII., the one suitable for this case, at the intersection of the vertical column for 18 feet, with the horizontal line for the given beam named above, we find 4·51, or $4\frac{1}{2}$ feet, the required distance from centres.

For a First-class Store (see Table XIX.), these beams, if of the length stated, should be placed 2·66, or 2 feet and 8 inches from centres.

701.—Constants for Use in the Rules (XX.).—Constants for use in the rules in previous chapters are to be found in Table XX.

These constants, for the 13 American woods named and for mahogany, have been computed from experiments made by the author in 1874 and 1876 expressly for this work (*Arts.* **704** to **707**). For the values of *B* and *F*, the lowest and highest of the two series of experiments are taken, and the average given for use in the rules.

The constants for the other woods named in the table have been computed for this work from experiments made by Barlow, and recorded in his work on the Strength of Materials.

The constant *F*, for American wrought-iron, was com-

puted by the author from six tests made by Major Anderson on rolled-iron beams at the Trenton Iron Works, and from two tests made at the works of the Phœnix Iron Co. of Philadelphia. The beams upon which these tests were made were from 6 to 15 inches deep and from 12 to 27 feet long.

The values of F for the other metals, and of B for all the metals, have been computed from tests made by trustworthy experimenters, such as Hodgkinson, Fairbairn, Kirkaldy, Major Wade and others. The average of these values may be used in the rules, for good ordinary metal. For any important work, however, constants should be derived from tests expressly made for the work, upon fair specimens of the particular kind of metal proposed to be used.

702.—Solid Timber Floors (XXI.).—The depths required for beams when placed close to each other, side by side, without spaces between them, may be found in Table XXI.

This is not an economical method of construction. More timber is required than in the ordinary plan of narrow, deep beams, set apart. But a solid floor has the important characteristic of resisting the action of fire nearly as long, if not quite, as a floor made with rolled-iron beams and brick arches.

A floor of timber as usually made, with spaces between the beams, resists a conflagration but a very short time. The beams laid up like kindling-wood, with spaces between, afford little resistance to the flames; but, when laid close, they, by the solidity obtained, prevent the passage of the air. The fire, thus retarded and confined to the room in which it originated, may be there extinguished before doing

serious damage. Floors built solid should be plastered upon the underside. The plastering lath should be nailed to narrow furring strips, half an inch thick, and the plastering pressed between the lath so as to fill the half inch space with mortar. The mortar used should contain a large portion of plaster of Paris, and be finished smooth with it. Owing to the fire-proof quality of this material, it will protect the lath a long time. Thus constructed, a solid floor will possess great endurance in resisting a conflagration.

The timbers should be attached to each other by dowels. These will serve, like cross-bridging, to distribute the pressure from a concentrated weight to the contiguous beams.

The depths given in Table XXI. were computed by formulas (*311.*) and (*312.*). These were reduced from formula (*130.*), which is

$$\tfrac{5}{8}Ul^3 = Fbd^3\delta$$

In this formula $U = cfl$, c and l being taken in feet. If c be taken in inches, then for c we have $\frac{c}{12}$, and $U = \frac{c}{12}fl$. Putting rl for δ (*Art.* **313**) we have

$$\frac{5cfl^3}{8 \times 12} = Fbd^3r$$

In a solid floor the breadth of the beams will equal the distances from centres, or $b = c$ (c now being in inches). In the formula these cancel each other; or

$$\frac{5fl^3}{8 \times 12} = Fd^3r \qquad \text{and}$$

$$d^3 = \frac{fl^3}{19 \cdot 2Fr} \qquad (310.)$$

For dwellings and halls of assembly, we have taken (*Art.* **115**) f at 90, or 70 for the superincumbent load and 20 for the materials of construction. In a solid floor, however, the weight of the timbers differs too much to permit an average of it to be used as a constant in the formula. The weight of the plastering, furring and floor-plank is constant, and may be taken at 12 pounds. To this add 70 for the superincumbent load, and the sum, 82, plus the weight of the beam, will equal f, the total load.

The weight of the beam will equal the weight of a foot superficial, inch thick, of the timber, multiplied by the depth of the beam; or, putting y equal to the weight of one foot, inch thick, of the timber, we have its total weight equal to yd; or, $f = 82 + yd$. Substituting this value for f in formula (*310.*), and putting $r = 0{\cdot}03$, then we have

$$d^3 = \frac{(82 + yd)l^3}{19{\cdot}2 \times 0{\cdot}03F} \qquad \text{or}$$

$$d^3 = \frac{(82 + yd)l^3}{{\cdot}576F} \qquad (311.)$$

This formula is general for floors of dwellings, office buildings, and halls of assembly. As the symbol for the depth is found on both sides of this equation, the depth for any given length can not be directly obtained by it; a modification is needed to make the formula practicable.

An inspection of the formula shows that the depth will be very nearly in direct proportion to the length. By a simple transformation of the symbols, a formula is obtained which will give the length for any given depth. By an application of this formula to the two extremes of depth and length for each kind of material, the relative values of d and l may be found. The results for the two extremes in each case will differ but little. An average may be used as a constant for all practical lengths, without appreciable

error. The values of d have been computed for the four woods named below, and the average value found to be for

Georgia pine,	$d = 0 \cdot 314l$
Spruce,	$d = 0 \cdot 365l$
White pine,	$d = 0 \cdot 389l$
Hemlock,	$d = 0 \cdot 39l$

An average value of y, the weight per foot superficial, inch thick, may be taken as follows: for

Georgia pine,	$y = 4$
Spruce,	$y = 2\frac{1}{2}$
White pine,	$y = 2\frac{1}{8}$
Hemlock,	$y = 2$

With these values of y and d, formula (*311.*) becomes practicable, and will give the required depth for any given length of floor beams, of the four woods named, for the solid floors of dwellings, office buildings, and halls of assembly.

For the floors of first-class stores, taking 250 pounds as the superincumbent load and 13 pounds as the weight of the plastering, flooring, etc., and putting $r = 0 \cdot 04$ we have, in formula (*310.*),

$$d^2 = \frac{(263 + yd)l^2}{\cdot 768F} \qquad (312.)$$

This formula is general for floors of first-class stores. The values of d have been computed for the extremes of lengths, and an average found to be as follows: for

Georgia pine,	$d = \cdot 4l$
Spruce,	$d = \cdot 472l$
White pine,	$d = \cdot 502l$
Hemlock,	$d = \cdot 506l$

With these values of *d*, and the above values of *y*, formula (*312.*) will give the depths of solid floors for first-class stores.

The depths of solid floors in Table XXI., for dwellings, office-buildings and halls of assembly, were computed by formula (*311.*), and those for first-class stores by formula (*312.*)

703.—Weights of Building Materials (XXII.).—Table XXII. contains the weight per cubic foot of various building materials.

704.—Experiments on American Woods (XXIII. to XLVI.).—Tables XXIII. to XLVI., inclusive, contain the results of experiments upon six of our American woods such as are more commonly used as building material.

These experiments, as well as those of 1874 (*Art.* **701**), were made upon a testing machine constructed for the author, and after his plan, by the Fairbanks Scale Co. It is a modification of the Fairbanks scale, a system of levers working on knife edges, and arranged with gearing and frame by which a very gradual pressure is brought to bear upon the piece tested, which pressure is sustained by the platform of the scale and thus measured.

By an application of clock-work, devised by Mr. R. F. Hatfield, son of the author, the poise upon the scale beam is kept in motion by the pressure upon the platform, and is arrested at the instant of rupture of the piece tested. For the moderate pressures (under 2000 pounds) required, this machine is found to work satisfactorily.

705.—Experiments by Transverse Strain (XXIII. to XXXV., XLII. and XLIII.).—Tables XXIII. to XXXV.

contain tests by Transverse Strain, upon six of the thirteen woods tested by the author for this work.

At intervals, as shown, the pressure was removed and the set, if any, measured. It was found that in many instances a decided set had occurred before the increments of deflection had ceased being equal for equal additions of weight. It was thus made plain that some modification of this rule for determining the limit of elasticity must be made. To fix this limit clearly inside of any doubtful line, 25 per cent of the deflection obtained, while the increments of deflection remained equal for equal additions of weight, was deducted, and the remainder taken as the deflection at the limit of elasticity.

With this deflection, the values of the constants e and a in Table XX. were computed (*Art.* **701**).

The load upon a beam, determined by the rules with the constants restricted within this limit, will not, it is confidently believed, be subject to set; or if, as is claimed by Professor Hodgkinson, any deflection, however small, will produce a set, that this set will be so slight and of such a nature as not to be injurious, or worthy of consideration.

A *résumé* of the results of Tables XXIII. to XXXV. is given in Tables XLII. and XLIII.

The values of F and B, given in Table XX., were derived, not alone from the results given in these tables, but also from results of the other experiments made in 1874. (*Art.* **701**.)

706. — Experiments by Tensile and Sliding Strains (XXXVI. to XXXIX., XLIV. and XLV.).—Tables XXXVI. and XXXVII. contain tests of the resistance to tensile strain of six of the more common American woods.

A *résumé* of the results is given in Table XLIV.

Tables XXXVIII. and XXXIX. give tests made to show the resistance to sliding of the fibres in six of the more common American woods. These experiments were made to ascertain the power of the several woods to resist a force tending to separate the fibres by sliding, in the longitudinal direction of the fibres. The rafter of a roof, when stepped into an indent in the tie-beam, exerts a thrust tending to split off the upper part of the end of the tie-beam. A pin through a tenon, when subjected to strain, tends to split out the part of the tenon in front of it. These are instances in which rupture may occur by the sliding of the fibres longitudinally, and a knowledge of the power of the various woods to resist it, as shown in these tables, and as condensed in Table XLV., will be useful in apportioning parts subject to this strain. The symbol *G*, in Table XX., represents in pounds the sliding resistance to rupture per square inch superficial, and is equal to the average of the results of the experiments in Table XLV. A discussion to show the application of these results is omitted as being uncalled for in a work on the Transverse Strain. For its treatment, see "American House Carpenter," Arts. 301 to 303, where *H*, the value of each wood, is taken at $\frac{1}{8}$ of the resistance to rupture.

707.—Experiments by Crushing Strain (XL., XLI. and XLVI.).—Tables XL. and XLI. contain tests of resistance to crushing, in the direction of the fibres, of six of the more common of our American woods. The pieces submitted to this test were from one to two diameters high.

A *résumé* of the results is given in Table XLVI.

TABLES.

TABLE I.

HEMLOCK FLOOR BEAMS ONE INCH THICK, FOR DWELLINGS, OFFICE BUILDINGS, AND HALLS OF ASSEMBLY.

DISTANCE FROM CENTRES (*in inches*).

For Beams Thicker than One Inch, see Arts. **693** *and* **695**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF BEAM (*in inches*).								
	6	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
7	11·3								
8	7·6	12·0							
9	5·3	8·4	12·6						
10	3·9	6·1	9·2	13·1					
11	2·9	4·6	6·9	9·8					
12	..	3·6	5·3	7·6	10·4				
13	..	2·8	4·2	5·9	8·2	10·9			
14	..	..	3·3	4·8	6·5	8·7	11·3		
15	..	..	2·7	3·9	5·3	7·1	9·2	11·7	
16	..	..	..	3·2	4·4	5·8	7·6	9·6	
17	..	..	..	2·7	3·6	4·9	6·3	8·0	10·0
18	..	..	..	..	3·1	4·1	5·3	6·7	8·4
19	..	..	..	..	2·6	3·5	4·5	5·7	7·2
20	..	..	..	..	..	3·0	3·9	4·9	6·1
21	..	..	..	..	..	..	3·3	4·2	5·3
22	..	..	..	..	..	..	2·9	3·7	4·6
23	..	..	..	..	..	..	..	3·2	4·0
24	..	..	..	..	..	..	..	2·8	3·6

TABLE II.

WHITE PINE FLOOR BEAMS ONE INCH THICK, FOR DWELLINGS, OFFICE BUILDINGS, AND HALLS OF ASSEMBLY.

DISTANCE FROM CENTRES (*in inches*).

For Beams Thicker than One Inch, see Arts. **693** *and* **695**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF BEAM (*in inches*).								
	6	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
7	11·7								
8	7·8	12·4							
9	5·5	8·7	13·0						
10	4·0	6·4	9·5						
11	3·0	4·8	7·1	10·2					
12	..	3·7	5·5	7·8	10·7				
13	..	2·9	4·3	6·2	8·4	11·2			
14	..	..	3·5	4·9	6·8	9·0	11·7		
15	..	..	2·8	4·0	5·5	7·3	9·5		
16	..	..	..	3·3	4·5	6·0	7·8	10·0	
17	..	..	..	2·8	3·8	5·0	6·5	8·3	10·4
18	..	..	..	..	3·2	4·2	5·5	7·0	8·7
19	..	..	..	..	2·7	3·6	4·7	5·9	7·4
20	..	..	..	..	..	3·1	4·0	5·1	6·4
21	..	..	..	..	..	2·7	3·5	4·4	5·5
22	..	..	..	..	..	..	3·0	3·8	4·8
23	..	..	..	..	..	..	..	3·4	4·2
24	..	..	..	..	..	..	..	2·9	3·7

TABLE III.

SPRUCE FLOOR BEAMS ONE INCH THICK, FOR DWELLINGS, OFFICE BUILDINGS, AND HALLS OF ASSEMBLY.

DISTANCE FROM CENTRES (*in inches*).

For Beams Thicker than One Inch, see Arts. **693** *and* **695**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF BEAM (*in inches*).								
	6	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
7	14·1								
8	9·4								
9	6·6	10·5							
10	4·8	7·7	11·5						
11	3·6	5·8	8·6	12·3					
12	2·8	4·4	6·6	9·4					
13	..	3·5	5·2	7·4	10·2				
14	..	2·8	4·2	6·0	8·2	10·9			
15	..	..	3·4	4·8	6·6	8·8	11·5		
16	..	..	2·8	4·0	5·5	7·3	9·4		
17	..	..	..	3·3	4·6	6·1	7·9	10·0	
18	..	..	..	2·8	3·8	5·1	6·6	8·4	10·5
19	..	..	..	..	3·3	4·3	5·6	7·2	9·0
20	..	..	..	..	2·8	3·7	4·8	6·2	7·7
21	..	..	..	..	..	3·2	4·2	5·3	6·6
22	..	..	..	..	..	2·8	3·6	4·6	5·8
23	..	..	..	..	..	..	3·2	4·0	5·1
24	..	..	..	..	..	..	2·8	3·6	4·4

TABLE IV.

GEORGIA PINE FLOOR BEAMS ONE INCH THICK, FOR DWELLINGS, OFFICE BUILDINGS, AND HALLS OF ASSEMBLY.

DISTANCE FROM CENTRES (*in inches*).

For Beams Thicker than One Inch, see Arts. **693** *and* **695**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF BEAM (*in inches*).								
	6	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
9	11·2								
10	8·2	13·0							
11	6·1	9·7							
12	4·7	7·5	11·2						
13	3·7	5·9	8·8						
14	3·0	4·7	7·0	10·0					
15	..	3·8	5·7	8·2	11·2				
16	..	3·2	4·7	6·7	9·2				
17	..	2·6	3·9	5·6	7·7	10·2			
18	..	..	3·3	4·7	6·5	8·6	11·2		
19	..	..	2·8	4·0	5·5	7·3	9·5		
20	..	..	..	3·4	4·7	6·3	8·2	10·4	
21	..	..	..	3·0	4·1	5·4	7·0	9·0	11·2
22	..	..	..	..	3·5	4·7	6·1	7·8	9·7
23	..	..	..	..	3·1	4·1	5·4	6·8	8·5
24	..	..	..	..	2·7	3·6	4·7	6·0	7·5

TABLE V.

HEMLOCK FLOOR BEAMS ONE INCH THICK, FOR FIRST-CLASS STORES.

DISTANCE FROM CENTRES (*in inches*).

For Beams Thicker than One Inch, see Arts. **693** *and* **695.**

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF BEAM (*in inches*).										
	8	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
8	7·8										
9	5·5	7·8									
10	4·0	5·7	7·8								
11	3·0	4·3	5·9								
12	2·3	3·3	4·5	6·0							
13	1·8	2·6	3·6	4·7	6·2						
14	..	2·1	2·8	3·8	4·9	6·3					
15	..	1·7	2·3	3·1	4·0	5·1	6·4				
16	..	..	1·9	2·5	3·3	4·2	5·2	6·4			
17	..	..	..	2·1	2·8	3·5	4·4	5·4	6·5		
18	..	..	..	1·8	2·3	2·9	3·7	4·5	5·5	6·6	
19	..	..	..	..	2·0	2·5	3·1	3·8	4·7	5·6	6·6
20	..	..	..	..	..	2·1	2·7	3·3	4·0	4·8	5·7
21	..	..	..	..	..	1·9	2·3	2·8	3·5	4·1	4·9
22	..	..	..	..	..	..	2·0	2·5	3·0	3·6	4·3
23	..	..	..	..	..	..	..	2·2	2·6	3·2	3·7
24	..	..	..	..	..	..	..	1·9	2·3	2·8	3·3
25	..	..	..	..	..	..	..	..	2·0	2·5	2·9
26	..	..	..	..	..	..	..	..	..	2·2	2·6

TABLE VI.

WHITE PINE FLOOR BEAMS ONE INCH THICK, FOR FIRST-CLASS STORES.

DISTANCE FROM CENTRES (*in inches*).

For Beams Thicker than One Inch, see Arts. **693** *and* **695**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF BEAM (*in inches*).										
	8	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
8	8·1										
9	5·7	8·1									
10	4·1	5·9									
11	3·1	4·4	6·1								
12	2·4	3·4	4·7	6·2							
13	1·9	2·7	3·7	4·9	6·4						
14	..	2·2	3·0	3·9	5·1	6·5					
15	..	1·7	2·4	3·2	4·1	5·3	6·6				
16	..	..	2·0	2·6	3·4	4·3	5·4	6·7			
17	..	..	..	2·2	2·8	3·6	4·5	5·6	6·8		
18	..	..	..	1·8	2·4	3·1	3·8	4·7	5·7	6·8	
19	..	..	..	..	2·0	2·6	3·2	4·0	4·8	5·8	6·9
20	..	..	..	..	..	2·2	2·8	3·4	4·1	5·0	5·9
21	..	..	..	..	..	1·9	2·4	3·0	3·6	4·3	5·1
22	..	..	..	..	..	..	2·1	2·6	3·1	3·7	4·4
23	..	..	..	..	..	..	1·8	2·2	2·7	3·3	3·9
24	..	..	..	..	..	..	..	2·0	2·4	2·9	3·4
25	..	..	..	..	..	..	..	..	2·1	2·5	3·0
26	..	..	..	..	..	..	..	..	1·9	2·3	2·7

TABLE VII.

SPRUCE FLOOR BEAMS ONE INCH THICK, FOR FIRST-CLASS STORES.

DISTANCE FROM CENTRES (*in inches*).

For Beams Thicker than One Inch, see Arts. **693** *and* **695**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF BEAM (*in inches*).										
	8	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
9	6·9										
10	5·0	7·1									
11	3·8	5·4	7·3								
12	2·9	4·1	5·7	7·5							
13	2·3	3·2	4·4	5·9							
14	1·8	2·6	3·6	4·7	6·2						
15	..	2·1	2·9	3·9	5·0	6·4					
16	..	1·7	2·4	3·2	4·1	5·2	6·5				
17	..	..	2·0	2·6	3·4	4·4	5·5	6·7			
18	..	..	..	2·2	2·9	3·7	4·6	5·7	6·9		
19	..	..	..	1·9	2·5	3·1	3·9	4·8	5·8		
20	..	..	..	..	2·1	2·7	3·4	4·1	5·0	6·0	
21	..	..	..	..	1·8	2·3	2·9	3·6	4·3	5·2	6·2
22	..	..	..	..	..	2·0	2·5	3·1	3·8	4·5	5·4
23	..	..	..	..	..	..	2·2	2·7	3·3	3·9	4·7
24	..	..	..	..	..	..	1·9	2·4	2·9	3·5	4·1
25	..	..	..	..	..	..	..	2·1	2·6	3·1	3·6
26	..	..	..	..	..	..	..	1·9	2·3	2·7	3·2

TABLE VIII.

GEORGIA PINE FLOOR BEAMS ONE INCH THICK, FOR FIRST-CLASS STORES.

DISTANCE FROM CENTRES (*in inches*).

For Beams Thicker than One Inch, see Arts. **693** *and* **695**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF BEAM (*in inches*).										
	8	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
11	6·3										
12	4·9	7·0									
13	3·8	5·5									
14	3·1	4·4	6·0								
15	2·5	3·6	4·9	6·5							
16	2·1	2·9	4·0	5·4	7·0						
17	1·7	2·4	3·4	4·5	5·8						
18	..	2·1	2·8	3·8	4·9	6·2					
19	..	1·8	2·4	3·2	4·2	5·3	6·6				
20	..	..	2·1	2·7	3·6	4·5	5·7				
21	..	..	1·8	2·4	3·1	3·9	4·9	6·0			
22	..	..	..	2·1	2·7	3·4	4·2	5·2	6·3		
23	..	..	..	1·8	2·3	3·0	3·7	4·6	5·5	6·7	
24	..	..	..	..	2·1	2·6	3·3	4·0	4·9	5·9	
25	..	..	..	..	1·8	2·3	2·9	3·6	4·3	5·2	6·2
26	..	..	..	..	..	2·1	2·6	3·2	3·8	4·6	5·5

TABLE IX.

HEMLOCK HEADERS FOR DWELLINGS, OFFICE BUILDINGS, AND HALLS OF ASSEMBLY.

THICKNESS OF HEADER (*in inches*) FOR TAIL BEAMS ONE FOOT LONG.

For Tail Beams Longer than One Foot, see Arts. **693** *and* **696**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF HEADER (*in inches*).								
	6	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
5	·27	·16	·10	·07	·05				
6	·46	·27	·17	·11	·08	·06			
7	·73	·43	·27	·18	·13	·09	·07	·05	
8	1·10	·63	·40	·27	·19	·14	·10	·08	·06
9	1·56	·90	·57	·38	·27	·20	·15	·11	·09
10	2·14	1·24	·78	·52	·37	·27	·20	·16	·12
11	..	1·65	1·04	·70	·49	·36	·27	·21	·16
12	..	2·14	1·35	·90	·63	·46	·35	·27	·21
13	..	..	1·72	1·15	·81	·59	·44	·34	·27
14	..	..	2·14	1·44	1·01	·73	·55	·43	·33
15	..	..	2·63	1·77	1·24	·90	·68	·52	·41
16	..	..	3·20	2·14	1·50	1·10	·82	·63	·50
17	..	..	..	2·57	1·81	1·32	·99	·76	·60
18	..	..	..	3·05	2·14	1·56	1·17	·90	·71
19	..	..	..	..	2·52	1·84	1·38	1·06	·84
20	..	..	..	..	2·94	2·14	1·61	1·24	·98

TABLE X.

WHITE PINE HEADERS FOR DWELLINGS, OFFICE BUILDINGS, AND HALLS OF ASSEMBLY.

THICKNESS OF HEADER (*in inches*) FOR TAIL BEAMS ONE FOOT LONG.

For Tail Beams Longer than One Foot, see Arts. **693** *and* **696.**

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF HEADER (*in inches*).								
	6	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
5	·26	·15	·09	·06					
6	·45	·26	·16	·11	·08	·06			
7	·71	·41	·26	·17	·12	·09	·07	·05	
8	1·06	·61	·39	·26	·18	·13	·10	·08	·06
9	1·51	·87	·55	·37	·26	·19	·14	·11	·09
10	2·07	1·20	·75	·51	·35	·26	·19	·15	·12
11	..	1·59	1·00	·67	·47	·34	·26	·20	·16
12	..	2·07	1·30	·87	·61	·45	·34	·26	·20
13	..	2·63	1·66	1·11	·78	·57	·43	·33	·26
14	..	..	2·07	1·39	·97	·71	·53	·41	·32
15	..	..	2·54	1·70	1·20	·87	·66	·51	·40
16	..	..	3·09	2·07	1·45	1·06	·80	·61	·48
17	..	..	..	2·48	1·74	1·27	·95	·74	·58
18	..	..	..	2·95	2·07	1·51	1·13	·87	·69
19	..	..	..	3·46	2·43	1·77	1·33	1·03	·81
20	..	..	..	..	2·84	2·07	1·55	1·20	·94

TABLE XI.

SPRUCE HEADERS FOR DWELLINGS, OFFICE BUILDINGS, AND HALLS OF ASSEMBLY.

THICKNESS OF HEADER (*in inches*) FOR TAIL BEAMS ONE FOOT LONG.

For Tail Beams Longer than One Foot, see Arts. **693** *and* **696**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF HEADER (*in inches*).								
	6	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
5	·21	·12	·08	·05					
6	·37	·21	·13	·09	·06	·05			
7	·59	·34	·21	·14	·10	·07	·06		
8	·88	·51	·32	·21	·15	·11	·08	·06	·05
9	1·25	·72	·46	·31	·21	·16	·12	·09	·07
10	1·71	·99	·62	·42	·29	·21	·16	·12	·10
11	2·28	1·32	·83	·56	·39	·29	·21	·17	·13
12	..	1·71	1·08	·72	·51	·37	·28	·21	·17
13	..	2·18	1·37	·92	·65	·47	·35	·27	·21
14	..	2·72	1·71	1·15	·81	·59	·44	·34	·27
15	..	..	2·11	1·41	·99	·72	·54	·42	·33
16	..	..	2·56	1·71	1·20	·88	·66	·51	·40
17	..	..	3·07	2·06	1·44	1·05	·79	·61	·48
18	..	..	..	2·44	1·71	1·25	·94	·72	·57
19	..	..	..	2·87	2·02	1·47	1·10	·85	·67
20	..	..	..	3·35	2·35	1·71	1·29	·99	·78

TABLE XII.

GEORGIA PINE HEADERS FOR DWELLINGS, OFFICE BUILDINGS, AND HALLS OF ASSEMBLY.

THICKNESS OF HEADER (*in inches*) FOR TAIL BEAMS ONE FOOT LONG.

For Tail Beams Longer than One Foot, see Arts. **693** *and* **696**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF HEADER (*in inches*).								
	6	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
5	·13	·07	·05						
6	·22	·13	·08	·05					
7	·35	·20	·13	·09	·06				
8	·52	·30	·19	·13	·09	·07	·05		
9	·74	·43	·27	·18	·13	·09	·07	·05	
10	1·02	·59	·37	·25	·17	·13	·10	·07	·06
11	1·35	·78	·49	·33	·23	·17	·13	·10	·08
12	1·76	1·02	·64	·43	·30	·22	·17	·13	·10
13	2·23	1·29	·81	·55	·38	·28	·21	·16	·13
14	..	1·61	1·02	·68	·48	·35	·26	·20	·16
15	..	1·99	1·25	·84	·59	·43	·32	·25	·20
16	..	2·41	1·52	1·02	·71	·52	·39	·30	·24
17	..	..	1·82	1·22	·86	·62	·47	·36	·28
18	..	..	2·16	1·45	1·02	·74	·56	·43	·34
19	..	..	2·54	1·70	1·20	·87	·66	·50	·40
20	..	..	2·96	1·99	1·39	1·02	·76	·59	·46

TABLE XIII.

HEMLOCK HEADERS FOR FIRST-CLASS STORES.

THICKNESS OF HEADER (*in inches*) FOR TAIL BEAMS ONE FOOT LONG.

For Tail Beams Longer than One Foot, see Arts. **693** *and* **696.**

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF HEADER (*in inches*).										
	8	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
5	·22	·15	·11	·08	·06						
6	·39	·26	·18	·13	·10	·08	·06	·05			
7	·61	·41	·29	·21	·16	·12	·10	·08	·06	·05	
8	·92	·61	·43	·31	·24	·18	·14	·11	·09	·08	·06
9	1·30	·87	·61	·45	·34	·26	·20	·16	·13	·11	·09
10	1·79	1·20	·84	·61	·46	·36	·28	·22	·18	·15	·12
11	2·38	1·60	1·12	·82	·61	·47	·37	·30	·24	·20	·17
12	3·09	2·07	1·46	1·06	·80	·61	·48	·39	·31	·26	·22
13	..	2·63	1·85	1·35	1·01	·78	·61	·49	·40	·33	·27
14	..	3·29	2·31	1·68	1·27	·97	·77	·61	·50	·41	·34
15	..	..	2·84	2·07	1·56	1·20	·94	·75	·61	·51	·42
16	..	..	3·45	2·51	1·89	1·46	1·14	·92	·74	·61	·51
17	..	..	..	3·02	2·27	1·75	1·37	1·10	·89	·74	·61
18	..	..	..	3·58	2·69	2·07	1·63	1·30	1·06	·87	·73
19	..	..	..	4·21	3·16	2·44	1·92	1·53	1·25	1·03	·86
20	..	..	..	..	3·69	2·84	2·24	1·79	1·46	1·20	1·00

TABLE XIV.

WHITE PINE HEADERS FOR FIRST-CLASS STORES.

THICKNESS OF HEADER (*in inches*) FOR TAIL BEAMS ONE FOOT LONG.

For Tail Beams Longer than One Foot, see Arts. **693** *and* **696**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF HEADER (*in inches*).										
	8	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
5	·22	·14	·10	·07	·06						
6	·37	·25	·18	·13	·10	·07	·06	·05			
7	·59	·40	·28	·20	·15	·12	·09	·07	·06	·05	
8	·88	·59	·42	·30	·23	·18	·14	·11	·09	·07	·06
9	1·26	·84	·59	·43	·32	·25	·20	·16	·13	·11	·09
10	1·73	1·16	·81	·59	·45	·34	·27	·22	·18	·14	·12
11	2·30	1·54	1·08	·79	·59	·46	·36	·29	·23	·19	·16
12	2·99	2·00	1·40	1·02	·77	·59	·47	·37	·30	·25	·21
13	..	2·54	1·79	1·30	·98	·75	·59	·47	·39	·32	·27
14	..	3·18	2·23	1·63	1·22	·94	·74	·59	·48	·40	·33
15	..	..	2·74	2·00	1·50	1·16	·91	·73	·59	·49	·41
16	..	..	3·33	2·43	1·82	1·40	1·10	·88	·72	·59	·49
17	..	..	..	2·91	2·19	1·69	1·33	1·06	·86	·71	·59
18	..	..	..	3·46	2·60	2·00	1·57	1·26	1·02	·84	·70
19	..	..	..	4·07	3·05	2·35	1·85	1·48	1·20	·99	·83
20	..	..	..	..	3·56	2·74	2·16	1·73	1·40	1·16	·97

TABLE XV.

SPRUCE HEADERS FOR FIRST-CLASS STORES.

THICKNESS OF HEADER (*in inches*) FOR TAIL BEAMS ONE FOOT LONG.

For Tail Beams Longer than One Foot, see Arts. **693** *and* **696**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF HEADER (*in inches*).										
	8	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
5	·18	·12	·08	·06	·05						
6	·31	·21	·15	·11	·08	·06	·05				
7	·49	·33	·23	·17	·13	·10	·08	·06	·05		
8	·73	·49	·34	·25	·19	·15	·11	·09	·07	·06	·05
9	1·04	·70	·49	·36	·27	·21	·16	·13	·11	·09	·07
10	1·43	·96	·67	·49	·37	·28	·22	·18	·15	·12	·10
11	1·91	1·28	·90	·65	·49	·38	·30	·24	·19	·16	·13
12	2·47	1·66	1·16	·85	·64	·49	·39	·31	·25	·21	·17
13	3·15	2·11	1·48	1·08	·81	·62	·49	·39	·32	·26	·22
14	..	2·63	1·85	1·35	1·01	·78	·61	·49	·40	·33	·27
15	..	3·24	2·27	1·66	1·25	·96	·75	·60	·49	·40	·34
16	..	..	2·76	2·01	1·51	1·16	·92	·73	·60	·49	·41
17	..	..	3·31	2·41	1·81	1·40	1·10	·88	·71	·59	·49
18	..	..	..	2·86	2·15	1·66	1·30	1·04	·85	·70	·58
19	..	..	..	3·37	2·53	1·95	1·53	1·23	1·00	·82	·69
20	..	..	..	3·93	2·95	2·27	1·79	1·43	1·16	·96	·80

TABLE XVI.

GEORGIA PINE HEADERS FOR FIRST-CLASS STORES.

THICKNESS OF HEADER (*in inches*) FOR TAIL BEAMS ONE FOOT LONG.

For Tail Beams Longer than One Foot, see Arts. **693** *and* **696**.

LENGTH BETWEEN BEARINGS (*in feet*).	DEPTH OF HEADER (*in inches*).										
	8	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
5	·11	·07	·05								
6	·18	·12	·09	·06	·05						
7	·29	·20	·14	·10	·08	·06	·05				
8	·43	·29	·20	·15	·11	·09	·07	·05			
9	·62	·41	·29	·21	·16	·12	·10	·08	·06	·05	
10	·85	·57	·40	·29	·22	·17	·13	·11	·09	·07	·06
11	1·13	·76	·53	·39	·29	·22	·18	·14	·11	·09	·08
12	1·47	·98	·69	·50	·38	·29	·23	·18	·15	·12	·10
13	1·87	1·25	·88	·64	·48	·37	·29	·23	·19	·16	·13
14	2·33	1·56	1·10	·80	·60	·46	·36	·29	·24	·20	·16
15	2·87	1·92	1·35	·98	·74	·57	·45	·36	·29	·24	·20
16	..	2·33	1·64	1·19	·90	·69	·54	·43	·35	·29	·24
17	..	2·80	1·96	1·43	1·08	·83	·65	·52	·42	·35	·29
18	..	3·32	2·33	1·70	1·28	·98	·77	·62	·50	·41	·35
19	..	..	2·74	2·00	1·50	1·16	·91	·73	·59	·49	·41
20	..	..	3·20	2·33	1·75	1·35	1·06	·85	·69	·57	·47

TABLE XVII.

ELEMENTS OF ROLLED-IRON BEAMS.

See Art. **697**.

Name.	d = Depth.	Weight per Yard.	b = Breadth.	Average Thickness of Flange.	Thickness of Web.	b_1	d_1	$I = \frac{1}{12}(bd^3 - b_1d_1^3)$
Phœnix....	4	18	2·	·268	·21	1·790	3·464	4·467
Paterson...	4	18	2·25	·281	·156	2·094	3·438	4·909
Phœnix....	4	30	2·75	·400	·25	2·500	3·200	7·840
Trenton...	4	30	2·75	·400	·25	2·500	3·200	7·840
Buffalo....	4	30	2·75	·400	·25	2·500	3·200	7·840
Paterson...	4	30	2·75	·400	·25	2·500	3·200	7·840
Trenton...	4	37	3·	·456	·312	2·688	3·088	9·404
Paterson...	4	37	3·	·456	·312	2·688	3·088	9·404
Phœnix....	5	30	2·75	·350	·25	2·500	4·300	12·082
Trenton...	5	30	2·75	·350	·25	2·500	4·300	12·082
Buffalo....	5	30	2·75	·350	·25	2·500	4·300	12·082
Paterson...	5	30	2·75	·350	·25	2·500	4·300	12·082
Phœnix....	5	36	3·	·389	·3	2·700	4·222	14·317
Trenton...	5	40	3·	·454	·312	2·688	4·092	15·902
Paterson...	5	40	3·	·454	·312	2·688	4·092	15·902
Phœnix....	6	40	2·75	·500	·25	2·500	5·000	23·458
Trenton...	6	40	3·	·454	·25	2·750	5·091	23·761
Buffalo....	6	40	3·	·454	·25	2·750	5·091	23·761
Paterson...	6	40	3·	·454	·25	2·750	5·091	23·761
Buffalo....	6	50	3·25	·532	·312	2·938	4·935	29·074
Trenton...	6	50	3·5	·500	·3	3·200	5·000	29·667
Paterson...	6	50	3·5	·500	·3	3·200	5·000	29·667
Phœnix....	7	55	3·5	·484	·35	3·150	6·032	42·430
Trenton...	7	60	3·5	·540	·375	3·125	5·920	46·012
Buffalo....	7	60	3·5	·540	·375	3·125	5·920	46·012
Paterson...	7	60	3·5	·540	·375	3·125	5·920	46·012
Buffalo....	8	65	3·5	·560	·375	3·125	6·880	64·526
Phœnix....	8	65	4·	·507	·35	3·650	6·986	66·963
Trenton...	8	65	4·	·554	·3	3·700	6·892	69·729
Paterson...	8	65	4·	·554	·3	3·700	6·892	69·729
Buffalo....	9	70	3·5	·500	·437	3·063	8·000	81·937
Trenton...	8	80	4·5	·606	·375	4·125	6·788	84·485
Paterson...	8	80	4·5	·610	·37	4·130	6·780	84·735

TABLE XVII.—(*Continued.*)

ELEMENTS OF ROLLED-IRON BEAMS.

See Art. **697**.

NAME.	d = DEPTH.	WEIGHT PER YARD.	b = BREADTH.	AVERAGE THICKNESS OF FLANGE.	THICKNESS OF WEB.	b_1	d_1	$I = \frac{1}{12}(bd^3 - b_1d_1^3)$
Phœnix....	9	70	3·5	·660	·31	3·190	7·680	92·207
Trenton...	9	70	3·5	·672	·3	3·200	7·656	92·958
Paterson...	9	70	3·5	·672	·3	3·200	7·656	92·958
Phœnix....	9	84	4·	·667	·4	3·600	7·667	107·793
Buffalo....	9	90	4·	·643	·5	3·500	7·714	109·117
Paterson...	9	85	4·	·697	·384	3·616	7·605	110·461
Trenton...	9	85	4·	·701	·38	3·620	7·597	110·732
Buffalo	10½	90	4·437	·551	·437	4·000	9·397	151·436
Paterson...	9	125	4·5	·928	·58	3·920	7·143	154·320
Trenton...	9	125	4·5	·937	·57	3·930	7·125	154·917
Buffalo....	10½	105	4·5	·656	·5	4·000	9·187	175·645
Phœnix....	10½	105	4·5	·724	·44	4·060	9·052	183·164
Phœnix....	9	150	5·375	1·005	·6	4·775	6·990	190·630
Trenton...	10½	105	4·5	·795	·375	4·125	8·909	191·040
Paterson...	10½	105	4·5	·795	·375	4·125	8·909	191·040
Trenton...	10½	135	5·	·945	·47	4·530	8·609	241·478
Paterson...	10½	135	5·	·945	·47	4·530	8·609	241·478
Phœnix....	12	125	4·75	·777	·49	4·260	10·446	279·351
Buffalo ...	12¼	125	4·5	·797	·5	4·000	10·656	286·019
Paterson...	12¼	125	4·79	·768	·48	4·310	10·714	292·050
Trenton...	12¼	125	4·8	·778	·47	4·330	10·693	294·136
Phœnix....	12	170	5·5	1·010	·59	4·910	9·980	385·284
Paterson...	12¼	170	5·5	·980	·6	4·900	10·280	398·936
Trenton...	12 5/16	170	5·5	·981	·6	4·900	10·351	402·538
Buffalo....	12¼	180	5·375	1·089	·625	4·750	10·072	418·945
Buffalo....	15	150	4·875	·761	·562	4·313	13·477	491·307
Paterson...	15 3/16	150	5·	·731	·56	4·440	13·725	502·883
Phœnix....	15	150	4·75	·882	·5	4·250	13·235	514·870
Trenton...	15 3/16	150	5·	·822	·5	4·500	13·542	528·223
Phœnix....	15	200	5·312	1·098	·65	4·662	12·803	678·684
Buffalo....	15	200	5·375	1·118	·625	4·750	12·763	688·775
Paterson...	15⅛	200	5·5	1·048	·65	4·850	13·028	692·166
Trenton...	15⅛	200	5·75	1·060	·6	5·150	13·004	714·205

TABLE XVIII.

ROLLED-IRON BEAMS IN DWELLINGS, OFFICE BUILDINGS, AND HALLS OF ASSEMBLY.

DISTANCES FROM CENTRES (*in feet*).

See Arts. **694, 698** *and* **700.**

Name.	Depth.	Weight per Yard.	Length (*in feet*) between Bearings.																
			6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
Phœnix....	4	18	5·23	3·28	2·18	1·52													
Paterson...	4	18	5·75	3·61	2·40	1·67	1·21												
Phœnix....	4	30	..	5·76	3·83	2·67	1·93	1·43											
Trenton ...	4	30	..	5·76	3·83	2·67	1·93	1·43											
Buffalo	4	30	..	5·76	3·83	2·67	1·93	1·43											
Paterson...	4	30	..	5·76	3·83	2·67	1·93	1·43											
Trenton....	4	37	..	6·91	4·60	3·20	2·31	1·71	1·30										
Paterson...	4	37	..	6·91	4·60	3·20	2·31	1·71	1·30										
Phœnix....	5	30	..	..	5·95	4·16	3·01	2·24	1·71	1·33									
Trenton....	5	30	..	..	5·95	4·16	3·01	2·24	1·71	1·33									
Buffalo.....	5	30	..	..	5·95	4·16	3·01	2·24	1·71	1·33									
Paterson...	5	30	..	..	5·95	4·16	3·01	2·24	1·71	1·33									
Phœnix....	5	36	..	..	7·05	4·92	3·57	2·66	2·03	1·58	1·24								
Trenton....	5	40	..	..	7·83	5·47	3·96	2·95	2·25	1·75	1·38								
Paterson...	5	40	..	..	7·83	5·47	3·96	2·95	2·25	1·75	1·38								
Phœnix....	6	40	..	..	..	8·11	5·89	4·40	3·37	2·63	2·09	1·68	1·37						
Trenton....	6	40	..	..	..	8·22	5·97	4·46	3·41	2·66	2·11	1·70	1·38						
Buffalo.....	6	40	..	..	..	8·22	5·97	4·46	3·41	2·66	2·11	1·70	1·38						
Paterson...	6	40	..	..	..	8·22	5·97	4·46	3·41	2·66	2·11	1·70	1·38						
Buffalo.....	6	50	..	..	..	..	7·30	5·45	4·17	3·26	2·58	2·08	1·69	1·39					
Trenton....	6	50	..	..	..	..	7·45	5·57	4·26	3·33	2·64	2·12	1·73	1·42					
Paterson...	6	50	..	..	..	..	7·45	5·57	4·26	3·33	2·64	2·12	1·73	1·42					
Phœnix....	7	55	..	..	..	..	..	8·00	6·13	4·79	3·81	3·08	2·51	2·07	1·72	1·45			
Trenton....	7	60	..	..	..	..	..	8·67	6·65	5·20	4·13	3·33	2·72	2·25	1·87	1·57			
Buffalo	7	60	..	..	..	..	..	8·67	6·65	5·20	4·13	3·33	2·72	2·25	1·87	1·57			
Paterson...	7	60	..	..	..	..	..	8·67	6·65	5·20	4·13	3·33	2·72	2·25	1·87	1·57			
Buffalo....	8	65	..	..	..	..	..	..	9·37	7·34	5·84	4·72	3·86	3·19	2·67	2·24	1·90	1·62	
Phœnix....	8	65	..	..	..	..	..	..	..	7·62	6·07	4·91	4·01	3·32	2·77	2·33	1·98	1·69	
Trenton....	8	65	..	..	..	..	..	..	..	7·94	6·33	5·11	4·19	3·46	2·89	2·44	2·07	1·77	
Paterson...	8	65	..	..	..	..	..	..	..	7·94	6·33	5·11	4·19	3·46	2·89	2·44	2·07	1·77	
Buffalo....	9	70	..	..	..	..	..	..	..	9·35	7·45	6·03	4·94	4·09	3·42	2·88	2·45	2·09	1·80
Trenton....	8	80	..	..	..	..	..	..	..	9·62	7·66	6·19	5·07	4·20	3·50	2·95	2·50	2·14	1·83
Paterson...	8	80	..	..	..	..	..	..	..	9·65	7·69	6·21	5·09	4·21	3·52	2·96	2·51	2·14	1·84

TABLE XVIII.—(*Continued.*)

ROLLED-IRON BEAMS IN DWELLINGS, OFFICE BUILDINGS, AND HALLS OF ASSEMBLY.

Distances from Centres (*in feet*).

See Arts. **[illegible]4, [illegible]** *and* **700.**

TABLE XIX.

ROLLED-IRON BEAMS IN FIRST-CLASS STORES.

DISTANCES FROM CENTRES (*in feet*).

See Arts. **694, 697, 699** *and* **700.**

NAME.	DEPTH.	WEIGHT PER YARD.	LENGTH (*in feet*) BETWEEN BEARINGS. 6	7	8	9	10	11	12	13	14	15	16	17	18	19
Phœnix	4	18	3·06	1·92	1·28											
Paterson	4	18	3·36	2·11	1·41											
Phœnix	4	30	5·37	3·37	2·25	1·57	1·14									
Trenton	4	30	5·37	3·37	2·25	1·57	1·14									
Buffalo	4	30	5·37	3·37	2·25	1·57	1·14									
Paterson	4	30	5·37	3·37	2·25	1·57	1·14									
Trenton	4	37	6·44	4·04	2·69	1·88	1·36									
Paterson	4	37	6·44	4·04	2·69	1·88	1·36									
Phœnix	5	30	8·29	5·21	3·48	2·43	1·77	1·32								
Trenton	5	30	8·29	5·21	3·48	2·43	1·77	1·32								
Buffalo	5	30	8·29	5·21	3·48	2·43	1·77	1·32								
Paterson	5	30	8·29	5·21	3·48	2·43	1·77	1·32								
Phœnix	5	36	..	6·17	4·12	2·88	2·09	1·56								
Trenton	5	40	..	6·86	4·58	3·20	2·32	1·74	1·33							
Paterson	5	40	..	6·86	4·58	3·20	2·32	1·74	1·33							
Phœnix	6	40	..	..	6·77	4·75	3·45	2·58	1·98	1·55						
Trenton	6	40	..	..	6·86	4·81	3·49	2·61	2·00	1·57						
Buffalo	6	40	..	..	6·86	4·81	3·49	2·61	2·00	1·57						
Paterson	6	40	..	..	6·86	4·81	3·49	2·61	2·00	1·57						
Buffalo	6	50	..	..	8·40	5·88	4·27	3·20	2·45	1·92	1·52					
Trenton	6	50	..	..	8·57	6·00	4·36	3·26	2·50	1·96	1·56					
Paterson	6	50	..	..	8·57	6·00	4·36	3·26	2·50	1·96	1·56					
Phœnix	7	55	..	..	..	8·60	6·26	4·69	3·60	2·82	2·24	1·81	1·48			
Trenton	7	60	..	..	..	9·33	6·78	5·08	3·90	3·05	2·43	1·97	1·61			
Buffalo	7	60	..	..	..	9·33	6·78	5·08	3·90	3·05	2·43	1·97	1·61			
Paterson	7	60	..	..	..	9·33	6·78	5·08	3·90	3·05	2·43	1·97	1·61			
Buffalo	8	65	..	..	..	..	9·53	7·15	5·49	4·30	3·43	2·78	2·28	1·89	1·58	
Phœnix	8	65	..	..	..	..	9·90	7·42	5·70	4·47	3·56	2·88	2·36	1·96	1·64	
Trenton	8	65	..	..	..	..	..	7·73	5·94	4·65	3·71	3·01	2·46	2·04	1·71	
Paterson	8	65	..	..	..	..	..	7·73	5·94	4·65	3·71	3·01	2·46	2·04	1·71	
Buffalo	9	70	..	..	..	..	..	9·09	6·98	5·48	4·37	3·54	2·90	2·41	2·08	1·70
Trenton	8	80	..	..	..	..	..	9·36	7·19	5·64	4·50	3·64	2·99	2·48	2·07	1·75
Paterson	8	80	..	..	..	..	..	9·39	7·21	5·66	4·51	3·65	2·99	2·48	2·08	1·75

TABLE XIX.—(*Continued.*)

ROLLED-IRON BEAMS IN FIRST-CLASS STORES.

DISTANCES FROM CENTRES (*in feet*).

See Arts. **694, 697, 699** *and* **700.**

Name.	Depth.	Weight per Yard.	Length (*in feet*) between Bearings.																			
			11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Phœnix..	9	70	10·23	7·87	6·17	4·93	3·99	3·28	2·72	2·28	1·93	1·64										
Trenton .	9	70	10·32	7·93	6·22	4·97	4·02	3·30	2·74	2·30	1·94	1·66										
Paterson.	9	70	10·32	7·93	6·22	4·97	4·02	3·30	2·74	2·30	1·94	1·66										
Phœnix..	9	84	..	9·19	7·21	5·76	4·66	3·83	3·18	2·66	2·25	1·92	1·64									
Buffalo ..	9	90	..	9·30	7·30	5·82	4·72	3·87	3·21	2·69	2·27	1·94	1·66									
Paterson.	9	85	..	9·42	7·39	5·90	4·78	3·92	3·26	2·73	2·31	1·97	1·69									
Trenton..	9	85	..	9·45	7·41	5·92	4·79	3·93	3·26	2·74	2·31	1·97	1·69									
Buffalo...	10½	90	..	..	10·16	8·12	6·58	5·41	4·49	3·77	3·19	2·72	2·34	2·02	1·76							
Paterson.	9	125	..	..	10·32	8·24	6·67	5·48	4·54	3·81	3·22	2·74	2·35	2·03	1·76							
Trenton..	9	125	..	..	10·36	8·27	6·70	5·50	4·56	3·82	3·23	2·75	2·36	2·03	1·76							
Buffalo...	10½	105	..	..	..	9·42	7·63	6·27	5·21	4·37	3·70	3·16	2·71	2·35	2·04	1·78						
Phœnix..	10½	105	..	..	..	9·82	7·97	6·54	5·44	4·56	3·86	3·30	2·83	2·45	2·13	1·86						
Phœnix..	9	150	..	..	..	10·18	8·25	6·77	5·62	4·71	3·98	3·39	2·91	2·51	2·17	1·90						
Trenton..	10½	105	..	..	..	10·25	8·31	6·83	5·68	4·76	4·03	3·44	2·96	2·56	2·23	1·95	1·71					
Paterson.	10½	105	..	..	..	10·25	8·31	6·83	5·68	4·76	4·03	3·44	2·96	2·56	2·23	1·95	1·71					
Trenton..	10½	135	..	..	..	..	10·50	8·63	7·17	6·02	5·10	4·35	3·74	3·23	2·81	2·46	2·16	1·90				
Paterson.	10½	135	..	..	..	..	10·50	8·63	7·17	6·02	5·10	4·35	3·74	3·23	2·81	2·46	2·16	1·90				
Phœnix..	12	125	..	..	..	..	..	10·02	8·33	7·00	5·93	5·07	4·36	3·77	3·29	2·88	2·53	2·23	1·98			
Buffalo...	12¼	125	..	..	..	..	..	10·26	8·53	7·17	6·07	5·19	4·47	3·87	3·37	2·95	2·59	2·29	2·03	1·81		
Paterson.	12¼	125	..	..	..	..	..	10·48	8·71	7·32	6·21	5·30	4·56	3·95	3·44	3·01	2·65	2·34	2·08	1·85		
Trenton..	12¼	125	..	..	..	..	..	10·55	8·78	7·37	6·25	5·34	4·60	3·98	3·47	3·04	2·67	2·36	2·09	1·86		
Phœnix..	12	170	..	..	..	..	..	..	..	9·65	8·18	6·99	6·01	5·21	4·53	3·97	3·49	3·08	2·74	2·43	2·17	1·95
Paterson.	12¼	170	..	..	..	..	..	..	..	10·00	8·48	7·24	6·23	5·40	4·70	4·12	3·62	3·20	2·84	2·53	2·26	2·02
Trenton..	12 3/16	170	..	..	..	..	..	..	..	10·09	8·56	7·31	6·29	5·45	4·75	4·16	3·66	3·23	2·87	2·55	2·28	2·04
Buffalo ..	12¼	180	..	..	..	..	..	..	..	10·50	8·90	7·60	6·54	5·67	4·94	4·32	3·80	3·36	2·98	2·65	2·37	2·12
Buffalo..	15	150	..	..	..	..	..	..	..	..	10·50	8·98	7·74	6·71	5·85	5·13	4·52	4·00	3·56	3·17	2·84	2·55
Paterson.	15 3/16	150	..	..	..	..	..	..	..	..	10·75	9·20	7·92	6·87	5·99	5·26	4·63	4·10	3·65	3·25	2·91	2·61
Phœnix..	15	150	..	..	..	..	..	..	..	..	11·01	9·42	8·12	7·04	6·14	5·39	4·75	4·20	3·74	3·33	2·98	2·68
Trenton..	15 3/16	150	..	..	..	..	..	..	..	..	11·30	9·67	8·33	7·23	6·30	5·53	4·87	4·32	3·84	3·42	3·07	2·75
Phœnix..	15	200	..	..	..	..	..	..	..	..	..	..	10·70	9·28	8·09	7·10	6·25	5·54	4·92	4·39	3·93	3·53
Buffalo..	15	200	..	..	..	..	..	..	..	..	..	..	10·86	9·42	8·22	7·21	6·35	5·62	5·00	4·46	3·99	3·59
Paterson.	15⅛	200	..	..	..	..	..	..	..	..	..	..	10·91	9·46	8·26	7·24	6·38	5·65	5·02	4·48	4·01	3·61
Trenton..	15⅛	200	..	..	..	..	..	..	..	..	..	..	11·27	9·77	8·53	7·48	6·59	5·84	5·19	4·63	4·15	3·73

TABLE XX.

See Arts. **701, 705** *and* **706.**

The larger figures give the average, for use in the rules.	*See Table XLII.* *Formula (10.)* $B = \frac{Wl}{bd^2}$	*See Table XLIII.* *Formula (113.)* $F = \frac{Wl^3}{bd^3\delta}$	*Formula (117.)* $e = \frac{d\delta}{72l^2}$	*Formula (118.)* $a = \frac{B}{72Fe}$	*See Table XLIV.* Resistance to Rupture by Tension, per sq. inch sectional area, $= T$.	*See Table XLV.* Resistance to Rupture by Sliding, per sq. in. superficial, $= G$.	*See Table XLVI.* Resistance to Crushing, per sq. in. sec. area, short blocks, $= C$.
	500	4807	·001069	1·3514	11671	5383	8170
Georgia Pine.........	850	5900	·00109	1·8357	16000	6400	9500
	1176	6990	·001112	2·1013	21742	7050	11503
	952	4470	·001239	2·3874	11487	7391	11009
Locust..................	1200	5050	·0015	2·2002	24800	8800	11700
	1406	5650	·001764	1·9593	33882	10485	12582
	460	1704	·000791	4·7400	12453	8105	6531
White Oak	650	3100	·00086	3·3863	19500	9400	8000
	875	4444	·00093	2·9405	31194	11025	9775
	417	2209	·0008646	3·0324	15719	3460	7166
Spruce................	550	3500	·00098	2·2271	19500	3950	7850
	722	4819	·0010987	1·8940	22069	4709	8408
	420	2026	·0010156	2·8350	9786	3204	5879
White Pine...........	500	2900	·0014	1·7105	12000	3500	6650
	643	3766	·001791	1·3240	13754	3870	7502
	280	1660	·000937	2·5002	6164	2427	5213
Hemlock...............	450	2800	·00095	2·3496	8700	2750	5700
	707	4000	·000971	2·5282	9871	3095	6281
	580	3368	·0009375	2·5512			
Whitewood............	600	3450	·00096	2·5161			
	700	3556	·0009896	2·7628			
	442	2300	·0008854	3·0146			
Chestnut..............	480	2550	·00103	2·5382			
	520	2824	·001177	2·1728			
	860	3800	·001042	3·0166			
Ash.....................	900	4000	·00111	2·8153			
	960	4248	·001177	2·6667			
	1067	4962	·0013854	2·1558			
Maple..................	1100	5150	·0014	2·1190			
	1167	5333	·0014063	2·1612			
	1040	3704	·001198	3·2552			
Hickory...............	1050	3850	·0013	2·9138			
	1100	4000	·001406	2·7165			
	616	2800	·001563	1·9549			
Cherry..................	650	2850	·001563	2·0266			
	746	2933	·001563	2·2601			
	725	3619	·00099	2·8105			
Black-Walnut.........	750	3900	·00104	2·5682			
	824	4211	·001094	2·4842			
	507	3273	·001146	1·8773			
Mahogany, St. Dom...	650	3600	·00116	2·1618			
	790	3894	·001177	2·3940			
	813	4545	·001042	2·3843			
" Bay Wood.	850	4750	·00109	2·2802			
	920	5000	·001146	2·2300			
Oak, English..........	394	2022	·0009014	3·0024			
" "	557	3350	·0007256	3·1826			
" Dantzic..........	490	2697	·0009014	2·7994			
" Adriatic..........	460	2249	·0008104	3·5054			
Oaks, Average of......	475	2580	·000834	3·0660			
Oak, Canadian	589	4466	·000612	2·9930			

TABLE XX.—(*Continued.*)

See Arts. **701, 705** *and* **706.**

The larger figures give the average, for use in the rules.	*See Table XLII.* *Formula* (10.) $B = \frac{Wl}{bd^2}$	*See Table XLIII.* *Formula* (113.) $F = \frac{Wl^3}{bd^3\delta}$	*Formula* (117.) $e = \frac{d\delta}{72l^2}$	*Formula* (118.) $a = \frac{B}{72Fe}$	*See Table XLIV.* Resistance to Rupture by Tension, per sq. inch Sectional Area, $= T$.	*See Table XLV.* Resistance to Rupture by Sliding, per sq. in. Superficial, $= G$.	*See Table XLVI.* Resistance to Crushing, per sq. in. Sec. Area, Short Blocks, $= C$.
Ash	675	3810	·0007177	3·4285			
Beach	519	3134	·000582	3·9520			
Elm	338	1590	·0009552	3·0910			
Pitch Pine	544	2836	·0006428	4·1446			
Red Pine	447	4259	·0004279	3·4066			
Fir, New England	367	3454	·0005277	2·7966			
" Riga	369	3080	·0004932	3·3738			
" "	350	2293	·0006813	3·1117			
" Average of Riga	360	2686	·0005868	3·1723			
" Mar Forest	381	1858	·0008174	3·4843			
" " "	421	2013	·0007762	3·7423			
" Av'ge of Mar Forest	408	1961	·0007903	3·6564			
Larch	284	1422	·0010686	2·5958			
"	277	2078	·0006265	2·9552			
"	376	2437	·0006412	3·3420			
" Average of	330	2093	·000744	2·9433			
Norway Spar	491	3375	·0006173	3·2732			
	2000	41500			20000		80000
Cast-Iron, American	2500	50000	......	......	27000	......	120000
	3000	58500			45000		170000
	1600	27700			13000		80000
" English	2100	40000	......	......	17000	......	100000
	2600	53200			26000		140000
	2400	55500			40000		40000
Wrought-Iron, Amer.	2600	62000	......	......	60000	......	70000
	2800	69000			80000		100000
	1600	53000			30000		40000
" English	1900	60000	......	......	50000	......	50000
	2200	67000			65000		65000
		60000					
" Swedish	......	65000					
		71500					
	3200	67000					
Steel Bars	6000	70000					
	7200	74000					
					115780		
" Chrome	......	......	......	......	155500		
					190262		
	122						
Blue Stone Flagging	200						
	251						
	33						1000
Sandstone	59	......	......	......	......	......	3000
	94						6000
	20						1000
Brick, Common	33	......	......	......	......	......	2000
	43						3000
" Pressed	37	......	......	......	......	......	4000
Marble, Eastchester	147	......	......	......	......	......	20000

TABLE XXI.

SOLID TIMBER FLOORS.

DEPTH OF BEAM (*in inches*).

See Art. **702**.

28	9·47	10·95	11·65	11·65	11·85	13·92	14·82	14·91
30	9·83	11·36	12·09	12·08	12·28	14·42	15·35	15·44

TABLE XXII.

MATERIALS USED IN THE CONSTRUCTION OR LOADING OF BUILDINGS.

WEIGHTS PER CUBIC FOOT.

As per Barlow, Gallier, Haswell, Hurst, Rankine, Tredgold, Wood and the Author.

MATERIAL.	FROM	TO	AVERAGE.
WOODS.			
Acacia	41	51	46
Alder	35	51	38
Apple-tree	49	51	50
Ash	41	57	49
Beech	39	53	46
Birch	35	49	42
Box	59	65	62
" French	..	..	88
Brazil-wood	..	..	64
Cedar	27	35	31
" Canadian	47	57	52
" Palestine	30	38	34
" Virginia Red	..	..	40
Cherry	32	46	39
Chestnut, Horse	29	41	35
" Sweet	27	55	41
Cork	..	..	15
Cypress	27	41	34
" Spanish	..	..	40
Deal, Christiania	..	..	44
" English	..	..	29
" (Norway Spruce)	21	33	27
Dogwood	..	..	47
Ebony	69	83	76
Elder	..	..	43
Elm	33	59	46
Fir (Norway Spruce)	21	33	27
" (Red Pine)	30	44	37
" Riga	..	..	47
Gum, Blue	..	..	53
" Water	..	..	62
Hackmatack	..	..	37
Hemlock	21	31	26
Hickory	40	58	49
Lance-wood	41	63	52
Larch	31	35	33
" Red	31	54	43
" White	..	..	23
Lignum-vitæ	41	83	62
Locust	41	51	46
Logwood	..	..	57
Mahogany, Honduras	35	40	38
Mahogany, St. Domingo	45	65	55
Maple	33	49	41
Mulberry	35	55	45
Oak, Adriatic	..	..	62
" Black Bog	60	66	63
" Canadian	..	..	54
" Dantzic	..	..	47
" English	38	70	54
" Live	57	79	68
" Red	47	54	51
" White	43	57	50
Olive	..	..	58
Orange	..	..	44
Pear-tree	40	44	42
Pine, Georgia (pitch)	38	58	48
" Mar Forest	..	..	43
" Memel and Riga	29	35	32
" Red	..	..	37
" Scotch	27	51	39
" White	21	35	28
" Yellow	27	39	33
Plum	41	49	45
Poplar	23	37	30
Quince	..	..	44
Redwood	..	..	28
Rosewood	..	..	45
Sassafras	..	..	30
Satinwood	55	59	57
Spruce	24	36	30
Sycamore	36	40	38
Teak	41	61	51
Tulip-tree	..	..	30
Vine	77	83	80
Walnut, Black	26	40	33
" White	40	58	49
Whitewood	25	29	27
Yew	..	..	50
METALS.			
Bismuth, Cast	..	..	614
Brass, Cast	487	525	506
" (Gun-metal)	..	..	544
" Plate	528	534	531
Bronze	508	524	516

TABLE XXII.—(*Continued.*)

MATERIALS USED IN THE CONSTRUCTION OR LOADING OF BUILDINGS.

Weights per Cubic Foot.

As per Barlow, Gallier, Haswell, Hurst, Rankine, Tredgold, Wood and the Author.

Material.	From	To	Average.
Copper, Cast	537	549	543
" Hammered	...	...	556
" Plate	...	...	544
Gold	...	...	1206
" Standard	...	...	1108
Gun-metal	...	...	509
Iron, Bar	475	487	481
" Cast	434	474	454
" Malleable	...	...	475
" Wrought	474	486	480
Lead, Cast	...	...	709
" English Cast	...	...	717
" Milled	...	...	713
Mercury at 32°	...	...	851
" " 60°	...	...	849
" " 212°	...	...	887
Nickel, Cast	...	...	488
Pewter	...	...	453
Platina, Crude	...	...	975
" Pure	...	...	1345
" " Rolled	...	...	1379
Plumbago	...	...	142
Silver, Parisian Standard	...	...	636
" Pure Cast	...	...	655
" " " Hammered	...	...	658
" Standard	...	...	644
Steel	486	492	489
Tin, Cast	456	468	462
Zinc, Cast	429	449	439
STONES, EARTHS, Etc.			
Alabaster	165	180	173
Asphalt, Gritted	...	...	156
Asphaltum	57	103	80
Barytes, Sulphate of	250	304	277
Basalt	155	187	171
Bath Stone	122	156	139
Béton Coignet	124	134	129
Blue Stone, Common	...	...	160
Brick	85	119	102
" Fire-	...	...	138
" N. R. common hard	...	...	107
" " Salmon	...	...	100
" Philadelphia Front	...	...	105

Material.	From	To	Average.
Brick-work	96	112	104
" dry	...	...	100
" in Cement	...	...	112
" in Mortar	100	120	110
Caen Stone	...	...	130
Cement, Portland	...	...	81
" Roman, Cast	...	...	100
" " and Sand, equal parts	...	...	113
Chalk	116	174	145
Clay	119	125	122
" with Gravel	...	...	160
Coal, Anthracite	90	102	96
" Bituminous	76	90	83
" Cannel	77	81	79
" Cumberland	...	...	85
Coke	46	62	54
Concrete, Cement	125	135	130
Coquina	...	...	106
Earth, Common	95	125	110
" Loamy	...	...	126
" with Gravel	...	...	126
Emery	...	...	250
Feldspar	...	...	160
Flagging, Silver Gray	...	...	185
Flint	...	...	163
Glass, Crown	155	165	160
" Flint	171	195	183
" Green	...	...	165
" Plate	153	173	163
" White	167	181	174
Granite	158	172	165
" Aberdeen	...	...	164
" Egyptian Red	...	...	166
" Guernsey	...	...	185
" Quincy	...	...	166
Gravel	90	120	105
Grindstone	...	...	134
Gypsum	135	145	140
Lime, Unslaked	...	...	52
Limestone	139	199	169
" Aubigné	...	...	146
" Limerick	...	...	162
Marble	161	178	170
" Brocatel	...	...	166
" Carrara	...	...	170

TABLE XXII.—(*Continued.*)

MATERIALS USED IN THE CONSTRUCTION OR LOADING OF BUILDINGS.

WEIGHTS PER CUBIC FOOT.

As per Barlow, Gallier, Haswell, Hurst, Rankine, Tredgold, Wood and the Author.

"	Marietta, O....	...	...	162	" Sea..................	...	...	64
"	Middletown, Ct..	...	...	150	Whalebone..................	...	...	61

TABLE XXIII.

TRANSVERSE STRAINS IN GEORGIA PINE.

Length 1·6 Feet between Bearings.

See Arts. **704** *and* **705.**

Number of Experiment.	1	2	3	4	5	6	7	8	9
Depth (*in inches*).	1·04	1·04	1·03	1·04	1·03	1·03	1·03	1·03	1·02
Breadth (*in inches*).	1·05	1·04	1·03	1·03	1·04	1·04	1·04	1·03	1·04
Pressure (*in pounds*).	Deflection (*in inches*).								
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
25	·020	·015	·020	·015	·015	·020	·010	·020	·015
50	·040	·030	·040	·030	·025	·035	·025	·040	·030
75	·055	·040	·060	·045	·045	·050	·035	·055	·045
100	·070	·050	·080	·060	·065	·065	·050	·070	·060
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
100	·070	·050	·080	·060	·065	·065	·050	·070	·060
125	·085	·065	·095	·075	·080	·080	·070	·090	·075
150	·100	·080	·110	·090	·100	·090	·085	·110	·085
175	·115	·095	·125	·105	·115	·105	·100	·130	·100
200	·130	·110	·140	·120	·130	·120	·115	·150	·115
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
200	·130	·110	·140	·120	·130	·120	·115	·150	·115
225	·145	·120	·160	·135	·145	·135	·125	·170	·130
250	·160	·135	·175	·150	·160	·150	·140	·190	·145
275	·175	·150	·190	·165	·180	·160	·160	·210	·160
300	·190	·160	·210	·180	·195	·175	·175	·225	·175
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
300	·190	·160	·210	·180	·195	·175	·175	·225	·175
325	·210	·175	·230	·200	·215	·190	·190	·245	·190
350	·225	·190	·250	·215	·230	·210	·210	·265	·205
375	·240	·205	·265	·230	·245	·225	·225	·285	·220
400	·255	·220	·280	·245	·260	·240	·245	·310	·235
0	·000	·000	·005	·000	·000	·005	·005	·005	·005
400	·255	·220	·280	·245	·260	·240	·240	·310	·240
425	·275	·235	·300	·265	·280	·255	·255	·330	·255
450	·290	·250	·320	·280	·295	·270	·270	·355	·275
475	·310	·265	·340	·295	·315	·285	·285	·385	·290
500	·330	·280	·380	·315	·335	·300	·300	·415	·305
0	·000	·010	·030	·000	·005	·005	·005	·030	·005
500	·330	·280	·380	·315	·340	·300	·300	·430	·305
525	·350	·295	....	·330	·360	·315	·320	·470	·325
550	·370	·310	....	·350	·380	·330	·335	....	·345
575	·390	·330	....	·375	·405	·350	·355	....	·370
600	·415	·350	....	·395	·435	·375	·375	....	·400
0	·020	·020	....	·015	·025	·020	·025	....	·025
600	·420	·350	....	·405	·440	·380	·380	....	·410
625	·460	·370	....	·435	·475	·405	·405	....	·455
650	·560	·400	....	·470	·535	·430	·430		
675	....	·440	....	....	....	·460	·475		
700	....	·480							
Breaking Weight (*in pounds*).	674·	719·	518·	662·	652·	699·	685·	536·	642·

TABLE XXIV.

TRANSVERSE STRAINS IN LOCUST.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705**.

NUMBER OF EXPERIMENT.	10	11	12	13	14	15	16	17	18	19	20
DEPTH (*in inches*).	1·03	1·08	1·05	1·08	1·07	1·05	1·07	1·08	1·07	1·09	1·08
BREADTH (*in inches*).	1·02	1·05	1·08	1·02	1·09	1·08	1·08	1·04	1·08	1·08	1·03
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).										
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
25	·015	·015	·015	·015	·010	·015	·015	·015	·015	·015	·015
50	·035	·030	·030	·030	·025	·030	·030	·035	·030	·030	·030
75	·055	·050	·045	·045	·035	·050	·045	·050	·045	·045	·045
100	·075	·065	·060	·060	·050	·065	·060	·070	·060	·060	·060
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
100	·075	·065	·060	·060	·050	·065	·060	·070	·060	·060	·060
125	·095	·080	·075	·075	·065	·080	·075	·090	·075	·075	·080
150	·110	·100	·090	·090	·080	·095	·090	·110	·090	·090	·095
175	·125	·115	·105	·110	·090	·115	·105	·125	·105	·100	·110
200	·145	·130	·115	·125	·105	·130	·120	·145	·120	·115	·125
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
200	·145	·130	·115	·125	·105	·130	·120	·145	·120	·115	·125
225	·165	·150	·130	·140	·120	·145	·135	·165	·130	·130	·140
250	·180	·170	·145	·155	·135	·165	·150	·190	·145	·145	·155
275	·200	·185	·160	·175	·145	·180	·165	·210	·160	·160	·170
300	·220	·200	·175	·190	·160	·195	·180	·230	·175	·175	·190
0	·000	·005	·000	·000	·000	·000	·000	·010	·000	·000	·005
300	·220	·205	·175	·190	·160	·195	·180	·230	·175	·175	·190
325	·240	·220	·190	·210	·175	·210	·195	·250	·190	·185	·205
350	·260	·240	·205	·225	·190	·230	·210	·270	·205	·200	·220
375	·275	·265	·220	·240	·205	·245	·220	·290	·220	·215	·240
400	·295	·290	·235	·255	·220	·265	·235	·315	·235	·230	·255
0	·005	·020	·005	·005	·000	·005	·005	....	·000	·005	·010
400	·295	·295	·235	·255	·220	·265	·235	....	·235	·230	·255
425	·335	....	·250	·275	·235	·280	·250	....	·250	·245	·275
450	....	....	·265	·295	·250	·295	·265	....	·265	·260	·290
475	....	....	·280	·310	·265	·315	·280	....	·280	·275	·310
500	....	....	·290	·325	·280	·335	·295	....	·295	·290	·330

TABLE XXIV.—(*Continued.*)

TRANSVERSE STRAINS IN LOCUST.

Length 1·6 Feet between Bearings.

See Arts. **704** *and* **705**.

Number of Experiment.	10	11	12	13	14	15	16	17	18	19	20
Depth (*in inches*).	1·03	1·08	1·05	1·08	1·07	1·05	1·07	1·08	1·07	1·09	1·08
Breadth (*in inches*).	1·02	1·05	1·08	1·02	1·09	1·08	1·08	1·04	1·08	1·08	1·03
Pressure (*in pounds*).	Deflection (*in inches*).										
0	....	....	·005	·005	·005	·015	·005	....	·005	·005	·015
500	....	....	·290	·325	·280	·335	·295	....	·295	·290	·330
525	....	....	·305	·345	·295	·355	·310	....	·310	·305	·350
550	....	....	·320	·365	·310	·375	·325	....	·325	·320	·370
575	....	....	·335	·385	·325	·395	·340	....	·340	·335	·390
600	....	....	·350	·405	·345	·415	·360	....	·355	·350	·415
0	....	....	·010	·020	·010	·020	·015	....	·010	·015	·025
600	....	....	·350	·405	·345	·420	·360	....	·355	·355	·415
625	....	....	·365	·430	·365	·455	·375	....	·375	·370	·440
650	....	....	·385	·455	·385	·490	·390	....	·390	·390	·460
675	....	....	·400	·485	·405	·525	·405	....	·405	·410	·465
700	....	....	·415	·510	·425	·560	·420	....	·425	·430	·510
0	....	....	·015	·035	·030	·040	·020	....	·025	·025	·050
700	....	....	·415	·515	·430	·565	·420	....	·425	·435	·520
725	....	....	·435	·540	·455	·595	·440	....	·445	·455	
750	....	....	·455	·575	·475	·635	·460	....	·465	·480	
775	....	....	·475	·610	·505	....	·480	....	·490	·500	
800	....	....	·495	·640	·535	....	·500	....	·515	·520	
0	....	....	·035	·075	·060	....	·035	....	·045	·045	
800	....	....	·505	·650	·545	....	·505	....	·520	·530	
825	....	....	·530	·690	·580	....	·530	....	·545	·560	
850	....	....	·555	·725	·615	....	·550	....	·570	·585	
875	....	....	·585	....	·645	....	·575	....	·600	·615	
900	....	....	·615	....	·675	....	·605	....	·630	·650	
0	....	....	·065	....	....	....	·050	....	·075		
900	....	....	·635	....	....	....	·610	....	·640		
925	....	....	·665	....	....	....	·640	....	·675		
950	....	....	·695	....	....	....	·670				
975	....	....	·735	....	....	....	·700				
1000	....	....	....	....	....	....	·735				
Breaking Weight (*in pounds*).	449·	425·	1046·	956·	1001·	860·	1037·	402·	937·	1027·	715·

TABLE XXV.

TRANSVERSE STRAINS IN WHITE OAK.

Length 1·6 Feet between Bearings.

See Arts. **704** *and* **705**.

Number of Experiment.	21	22	23	24	25	26	27	28	29	30
Depth (*in inches*).	1·06	1·06	1·08	1·07	1·08	1·06	1·09	1·08	1·07	1·06
Breadth (*in inches*).	1·08	1·08	1·06	1·06	1·06	1·08	1·07	1·07	1·06	1·08
Pressure (*in pounds*).	Deflection (*in inches*).									
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
25	·030	·040	·035	·040	·045	·045	·035	·040	·035	·045
50	·060	·075	·065	·080	·090	·085	·075	·080	·070	·085
75	·090	·115	·095	·115	·135	·125	·115	·125	·105	·125
100	·120	·155	·130	·150	·180	·170	·155	·165	·140	·165
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
100	·120	·155	·130	·150	·185	·170	·155	·165	·140	·170
125	·150	·195	·165	·190	·235	·215	·200	·215	·175	·215
150	·180	·235	·195	·230	·295	·260	·245	·260	·210	·260
175	·215	·280	·225	·275	·355	·310	·285	·310	·240	·310
200	·245	·325	·265	·315	·410	·355	·330	·360	·275	·365
0	·000	·020	·010	·020	·025	·020	·015	·020	·010	·025
200	·250	·330	·265	·325	·410	·365	·345	·360	·280	·365
225	·285	·380	·310	·380	·480	·420	·395	·420	·320	·420
250	·320	·440	·350	·430	·555	·480	·450	·485	·360	·480
275	·360	·500	·390	·480	·635	·545	·515	·560	·405	·545
300	·400	·560	·440	·540	·715	·615	·580	·640	·450	·615
0	·030	·060	·040	·065	·100	·075	·065	·080	·045	·080
300	·415	·580	·440	·560	·735	·635	·595	·660	·455	·640
325	·465	·650	.490	·620	....	·705	·665	....	·510	·725
350	·515	·715	·545	·680	....	....	....	....	·565	
375	·570	....	·605	·760	....	....	....	....	·625	
400	·630	....	·670	....	....	....	....	....	·690	
0	....	....	·105	....	....	....	....	....	·105	
400	....	....	·690	....	....	....	....	....	·710	
425	....	....	·760							
Breaking Weight (*in pounds*).	520·	404·	510·	475·	368·	430·	426·	391·	504·	401·

TABLE XXVI.

TRANSVERSE STRAINS IN SPRUCE.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705**.

NUMBER OF EXPERIMENT.	31	32	33	34	35	36	37	38	39	40
DEPTH (*in inches*).	1·09	1·05	1·08	1·04	1·07	1·04	1·03	1·07	1·08	1·04
BREADTH (*in inches*).	1·04	1·08	1·03	1·07	1·04	1·08	1·07	1·04	1·04	1·08
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).									
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
25	·020	·025	·040	·025	·025	·030	·025	·025	·025	·020
50	·040	·045	·075	·050	·050	·060	·045	·045	·045	·040
75	·060	·070	·110	·070	·075	·090	·065	·070	·065	·060
100	·080	·090	·145	·095	·100	·120	·085	·090	·085	·080
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
100	·080	·090	·145	·095	·100	·120	·085	·090	·085	·080
125	·100	·115	·180	·115	·125	·145	·110	·115	·105	·100
150	·125	·135	·215	·135	·150	·175	·135	·140	·125	·125
175	·145	·155	·250	·155	·170	·200	·155	·160	·145	·145
200	·165	·175	·285	·175	·190	·230	·175	·180	·165	·165
0	·000	·000	·010	·000	·010	·005	·005	·000	·000	·005
200	·165	·175	·285	·180	·190	·230	·175	·180	·170	·165
225	·185	·200	·325	·200	·215	·265	·195	·200	·190	·190
250	·210	·225	·370	·220	·240	·295	·220	·225	·210	·210
275	·230	·245	·415	·245	·260	·330	·240	·250	·235	·235
300	·250	·265	·465	·270	·285	·370	·260	·270	·255	·255
0	·005	·005	·045	·005	·010	·025	·005	·005	·005	·005
300	·250	·270	·475	·275	·285	·375	·260	·275	·255	·255
325	·275	·295	·530	·300	·310	·410	·285	·300	·275	·280
350	·300	·320	·600	·330	·335	·450	·310	·325	·300	·305
375	·330	·350	·680	·355	·360	·495	·335	·355	·325	·330
400	·380	·390	·760	·385	·390	·540	·365	·395	·350	·360
0	·035	·030	....	·020	·020	·070	·020	·025	·010	·020
400	·390	·400	....	·395	·395	·570	·370	·410	·355	·370
425	·460	·440	....	·425	·430	·620	·400	·455	·385	·400
450	....	·495	....	·455	·460	·680	·440	....	·445	·440
475	....	....	....	·505	....	....	·485	....	....	·500
500	....	....	....	·565	....	....	....	....	....	·570
0	....	....	....	....	....	....	....	....	....	·070
500	....	....	....	....	....	....	....	....	....	·590
BREAKING WEIGHT (*in pounds*).	445·	487·	400·	502·	470·	465·	498·	441·	475·	527·

TABLE XXVII.

TRANSVERSE STRAINS IN SPRUCE.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705**.

NUMBER OF EXPERIMENT.	41	42	43	44	45	46	47	48	49	50
DEPTH (*in inches*).	1·52	1·55	1·56	1·56	1·56	1·55	1·55	1·56	1·55	1·57
BREADTH (*in inches*).	1·09	1·10	1·06	1·10	1·10	1·09	1·08	1·10	1·10	1·09
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).									
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
25	·010	·010	·010	·010	·010	·010	·010	·010	·005	·005
50	·020	·020	·020	·015	·020	·020	·020	·015	·010	·010
75	·025	·030	·025	·025	·030	·025	·030	·025	·020	·020
100	·035	·040	·035	·035	·035	·035	·035	·030	·025	·025
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
100	·035	·040	·035	·035	·035	·035	·035	·030	·025	·025
125	·045	·050	·045	·040	·045	·045	·040	·035	·030	.035
150	·050	·060	·055	·050	·055	·050	·050	·040	·040	·040
175	·060	·070	·065	·060	·065	·060	·055	·050	·045	·045
200	·065	·075	·070	·065	·070	·065	·065	·055	·055	·055
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
200	·065	·075	·070	·065	·070	·065	·065	·055	·055	·055
225	·070	·085	·080	·075	·080	·070	·070	·065	·060	·060
250	·080	·095	·090	·085	·090	·080	·080	·070	·065	·065
275	·090	·100	·100	·090	·095	·085	·090	·075	·075	·075
300	·100	·110	·110	·095	·105	·090	·095	·085	·080	·080
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
300	·100	·110	·110	·095	·105	·090	·095	·085	·080	·080
325	·105	·115	·120	·105	·115	·100	·100	·090	·085	·085
350	·115	·120	·130	·115	·125	·105	·110	·100	·090	·095
375	·120	·130	·140	·125	·130	·110	·120	·105	·100	·100
400	·125	·140	·150	·135	·140	·120	·125	·110	·105	·110
0	·000	·000	·005	·000	·000	·000	·000	·000	·000	·000
400	·125	·140	·150	·135	·140	·120	·125	·110	·105	·110
425	·135	·150	·160	·140	·145	·125	·130	·120	·110	·120
450	·145	·160	·170	·145	·155	·135	·140	·125	·120	·125
475	·150	·165	·180	·155	·165	·140	·150	·130	·125	·130
500	·160	·175	·190	·165	·175	·150	·155	·140	·130	·135
0	·005	·000	·010	·000	·000	·000	·000	·000	·000	·000
500	·160	·175	·190	·165	·175	·150	·155	·140	·130	·135
525	·170	·180	·205	·170	·185	·155	·160	·145	·140	·145
550	·175	·190	·215	·180	·195	·160	·170	·150	·145	·155
575	·185	·200	·225	·185	·205	·170	·180	·160	·150	·160
600	·190	·210	·240	·195	·215	·180	·185	·170	·160	·170

TABLE XXVII.—(*Continued.*)

TRANSVERSE STRAINS IN SPRUCE.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705.**

NUMBER OF EXPERIMENT.	41	42	43	44	45	46	47	48	49	50
DEPTH (*in inches*).	1·52	1·55	1·56	1·56	1·56	1·55	1·55	1·56	1·55	1·57
BREADTH (*in inches*).	1·09	1·10	1·06	1·10	1·10	1·09	1·08	1·10	1·10	1·09
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).									
0	·005	·005	·015	·005	·005	·005	·005	·005	·000	·000
600	·190	·210	·240	·195	·215	·180	·185	·170	·160	·165
625	·195	·215	·250	·205	·225	·185	·195	·175	·170	·175
650	·205	·225	·265	·215	·235	·195	·200	·185	·175	·180
675	·215	·230	·275	·220	·245	·200	·210	·190	·180	·190
700	·225	·240	·290	·230	·255	·210	·220	·200	·190	·195
0	·005	·005	·025	·010	·010	·005	·005	·005	·005	·000
700	·225	·240	·290	·230	·255	·210	·220	·200	·190	·195
725	·235	·250	·305	·240	·265	·220	·230	·210	·200	·205
750	·245	·260	·320	·245	·275	·230	·240	·220	·210	·215
775	·255	·265	·335	·255	·290	·240	·250	·230	·220	·225
800	·265	·275	·350	·265	·305	·250	·255	·240	·230	·235
0	·010	·010	·040	·010	·025	·010	·010	·015	·010	·005
800	·265	·275	·350	·275	·310	·250	·255	·240	·230	·235
825	·275	·285	·370	·285	·330	·260	·265	·250	·240	·245
850	·285	·295	·385	·295	·345	·270	·275	·260	·255	·255
875	·300	·305	·405	·310	·365	·280	·285	·275	·270	·265
900	·310	·315	·420	·320	·405	·290	·295	·290	·290	·275
0	·020	·020	·060	·025	....	·025	·020	·030	·025	·015
900	·315	·320	·430	·325	....	·295	·295	·295	·295	·275
925	·365	·330	·460	·340	....	·310	·310	·310	·315	·290
950	....	·345	....	·355	....	·325	·320	·325	·340	·305
975	....	·360	....	·370	....	·340	·335	·345	·470	·320
1000	....	·370	....	·385	....	·365	·350	·365	....	·335
0	....	·030	....	·045	....	·050	·030	·060	....	·035
1000	....	·380	....	·395	....	·375	·355	·400	....	·345
1025	....	·390	....	....	....	·400	·375	·450	....	·365
1050	....	·405	....	....	....	·430	·400	....	....	·385
1075	....	....	....	....	....	....	....	....	....	·415
BREAKING WEIGHT (*in pounds*).	950·	1074·	926·	1001·	900·	1067·	1071·	1028·	977·	1078·

TABLE XXVIII.

TRANSVERSE STRAINS IN SPRUCE.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705**.

NUMBER OF EXPERIMENT.	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
DEPTH (*in inches*).	2·01	2·00	1·99	1·99	2·02	1·99	1·98	2·01	2·01	2·02
BREADTH (*in inches*).	1·08	1·08	1·08	1·08	1·08	1·06	1·08	1·09	1·07	1·09
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).									
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
50	·015	·015	·010	·010	·010	·010	·010	·010	·015	·005
100	·025	·025	·020	·015	·020	·020	·020	·020	·025	·015
150	·035	·035	·030	·025	·025	·025	·025	·025	·035	·020
200	·040	·045	·035	·030	·035	·030	·035	·035	·045	·030
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
200	·040	·045	·035	·030	·035	·030	·035	·035	·045	·030
250	·050	·055	·045	·040	·045	·035	·040	·040	·055	·040
300	·060	·065	·050	·050	·055	·045	·050	·045	·070	·050
350	·070	·075	·060	·055	·065	·050	·055	·055	·080	·055
400	·075	·085	·065	·065	·070	·060	·065	·060	·090	·065
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
400	·075	·085	·065	·065	·070	·060	·065	·060	·090	·065
450	·080	·095	·070	·070	·080	·065	·070	·070	·100	·070
500	·090	·105	·080	·080	·090	·070	·080	·075	·110	·080
550	·100	·115	·090	·085	·100	·080	·085	·080	·120	·090
600	·110	·125	·095	·095	·110	·085	·095	·090	·130	·100
0	·000	·005	·000	·000	·000	·000	·000	·000	·005	·000
600	·110	·125	·095	·095	·110	·085	·095	·090	·135	·100
650	·120	·135	·105	·100	·115	·090	·100	·095	·140	·110
700	·130	·145	·110	·110	·125	·100	·105	·105	·150	·115
750	·135	·155	·120	·115	·135	·105	·115	·110	·160	·125
800	·145	·165	·125	·125	·145	·115	·125	·120	·175	·135
0	·005	·010	·005	·000	·005	·000	·005	·005	·010	·005
800	·145	·165	·125	·125	·145	·115	·125	·120	·175	·135
850	·155	·175	·135	·130	·155	·120	·130	·125	·185	·140
900	·165	·185	·140	·140	·165	·130	·140	·130	·195	·150
950	·175	·195	·150	·145	·175	·135	·145	·140	·210	·160
1000	·185	·210	·160	·155	·185	·145	·155	·145	·225	·170

TABLE XXVIII.—(*Continued.*)

TRANSVERSE STRAINS IN SPRUCE.

Length 1·6 Feet between Bearings.

See Arts. **704** *and* **705**.

Number of Experiment.	51	52	53	54	55	56	57	58	59	60
Depth (*in inches*).	2·01	2·00	1·99	1·99	2·02	1·99	1·98	2·01	2·01	2·02
Breadth (*in inches*).	1·08	1·08	1·08	1·08	1·08	1·06	1·08	1·09	1·07	1·09
Pressure (*in pounds*).	Deflection (*in inches*).									
0	·010	·015	·005	·005	·010	·000	·010	·010	·015	·010
1000	·190	·210	·160	·155	·190	·145	·155	·145	·225	·170
1050	·200	·225	·165	·160	·200	·150	·160	·150	·235	·180
1100	·210	·240	·175	·170	·210	·160	·170	·160	·250	·195
1150	·230	·255	·185	·180	·220	·165	·175	·170	·270	·205
1200	·245	·270	·195	·190	·235	·175	·185	·175	·285	·215
0	·030	·025	·010	·010	·025	·010	·010	·010	·030	·020
1200	·250	·275	·195	·190	·240	·180	·185	·175	·285	·220
1250	·270	·290	·205	·200	·255	·190	·195	·185	·300	·230
1300	·295	·305	·215	·210	·270	·200	·205	·195	·320	·245
1350	·325	·325	·230	·220	·285	·210	·215	·205	·345	·260
1400	·360	·355	·240	·235	·305	·220	·225	·215	·365	·275
0	·070	·060	·020	·020	·040	·020	·020	·020	·055	·030
1400	·375	·370	·245	·235	·310	·220	·230	·215	·370	·280
1450	·415	·400	·255	·250	·330	·235	·240	·230	·390	·295
1500	....	·430	·270	·265	·355	·250	·255	·245	·415	·320
1550	....	....	·290	·280	....	·265	·275	·260	....	·350
1600	....	....	·315	·305	....	·285	·295	·280	....	·380
0	....	....	·055	·040	....	·040	·040	·040	....	·070
1600	....	....	·330	·310	....	·290	·300	·290		
1650	....	....	·375	·335	....	·310	·325	·310		
1700	....	....	....	·375	....	....	....	·335		
1750	....	....	....	....	....	....	....	·370		
Breaking Weight (*in pounds*).	1472·	1536·	1675·	1717·	1519·	1653·	1686·	1800·	1545·	1600·

TABLE XXIX.

TRANSVERSE STRAINS IN WHITE PINE.

Length 1·6 Feet between Bearings.

See Arts. **704** *and* **705.**

Number of Experiment.	61	62	63	64	65	66	67	68	69
Depth (*in inches*).	1·02	·99	·99	1·03	1·02	·99	·99	1·00	·99
Breadth (*in inches*).	1·00	1·02	1·02	1·00	1·01	1·01	1·01	·99	1·02
Pressure (*in pounds*).	Deflection (*in inches*).								
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
25	·035	·030	·040	·030	·035	·040	·030	·035	·030
50	·070	·060	·075	·060	·070	·080	·065	·065	·065
75	·100	·095	·115	·090	·100	·115	·095	·095	·095
100	·130	·125	·150	·120	·130	·150	·130	·125	·125
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
100	·130	·125	·150	·120	·130	·150	·130	·125	·125
125	·160	·155	·185	·150	·165	·185	·160	·160	·155
150	·190	·185	·220	·180	·195	·220	·190	·190	·185
175	·220	·215	·260	·210	·230	·250	·220	·225	·215
200	·250	·245	·295	·240	·260	·285	·250	·255	·245
0	·005	·000	·005	·000	·000	·010	·000	·005	·000
200	·250	·250	·295	·240	·260	·290	·250	·255	·245
225	·280	·280	·335	·270	·295	·325	·285	·285	·280
250	·315	·310	·380	·300	·330	·365	·320	·320	·310
275	·345	·345	·425	·335	·365	·410	·350	·355	·345
300	·380	·375	·470	·365	·405	·460	·385	·385	·375
0	·010	·005	....	·005	·010	·030	·005	·010	·010
300	·385	·380	....	·370	·410	·465	·385	·390	·380
325	·430	·415	....	·405	·460	·520	·430	·425	·415
350	·485	·450	....	·440	·535	....	....	·465	·455
375	....	·500	....	·540	....	....	....	·535	·495
Breaking Weight (*in pounds*).	376·	385·	300·	382·	356·	350·	349·	383·	399·

TABLE XXX.

TRANSVERSE STRAINS IN WHITE PINE.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705**.

Number of Experiment.	70	71	72	73	74	75	76	77	78
Depth (*in inches*).	1·53	1·50	1·52	1·53	1·51	1·49	1·53	1·52	1·49
Breadth (*in inches*).	1·02	1·03	1·03	1·02	1·03	1·01	1·02	1·02	1·01
Pressure (*in pounds*).	Deflection (*in inches*).								
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
25	·015	·010	·010	·015	·010	·015	·010	·010	·010
50	·025	·020	·020	·025	·020	·025	·020	·015	·020
75	·035	·030	·030	·035	·030	·035	·030	·025	·030
100	·045	·035	·040	·045	·040	·045	·040	·035	·040
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
100	·045	·035	·040	·045	·040	·045	·040	·035	·040
125	·055	·045	·050	·055	·050	·055	·045	·040	·050
150	·065	·050	·060	·065	·055	·065	·055	·050	·060
175	·075	·060	·070	·075	·065	·075	·065	·060	·070
200	·085	·070	·080	·090	·075	·085	·075	·070	·080
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
200	·085	·070	·080	·090	·075	·085	·075	·070	·080
225	·095	·080	·090	·100	·085	·095	·080	·075	·090
250	·105	·090	·100	·110	·095	·105	·090	·085	·100
275	·115	·100	·110	·120	·105	·115	·100	·095	·110
300	·125	·105	·120	·130	·110	·125	·105	·105	·125
0	·005	·000	·000	·000	·000	·000	·000	·000	·000
300	·125	·105	·120	·130	·110	·125	·105	·105	·125
325	·135	·115	·130	·140	·120	·135	·115	·115	·135
350	·145	·125	·140	·150	·125	·145	·125	·120	·145
375	·155	·135	·145	·160	·135	·155	·135	·130	·160
400	·165	·145	·155	·170	·145	·165	·145	·140	·170
0	·005	·005	·005	·005	·000	·000	·000	·000	·000
400	·165	·145	·155	·170	·145	·165	·145	·140	·170
425	·175	·150	·165	·180	·150	·175	·150	·145	·180
450	·185	·160	·175	·190	·160	·185	·160	·155	·190
475	·195	·170	·185	·200	·170	·195	·170	·165	·200
500	·205	·180	·195	·210	·180	·205	·175	·175	·210

TABLE XXX.—(*Continued.*)

TRANSVERSE STRAINS IN WHITE PINE.

Length 1·6 Feet between Bearings.

See Arts. **704** *and* **705**.

Number of Experiment.	70	71	72	73	74	75	76	77	78
Depth (*in inches*).	1·53	1·50	1·52	1·53	1·51	1·49	1·53	1·52	1·49
Breadth (*in inches*).	1·02	1·03	1·03	1·02	1·03	1·01	1·02	1·02	1·01
Pressure (*in pounds*).	Deflection (*in inches*).								
0	·005	·005	·005	·005	·000	·005	·000	·005	·000
500	·205	·180	·195	·215	·180	·205	·175	·175	·210
525	·215	·190	·205	·225	·190	·220	·185	·185	·225
550	·225	·200	·215	·235	·200	·230	·195	·195	·235
575	·235	·210	·225	·250	·210	·240	·205	·205	·250
600	·245	·220	·235	·260	·220	·250	·215	·215	·260
0	·005	·005	·010	·010	·005	·010	·005	·005	·005
600	·245	·220	·235	·260	·225	·255	·215	·215	·260
625	·260	·230	·245	·275	·235	·265	·225	·225	·270
650	·275	·240	·255	·290	·245	·280	·235	·235	·285
675	·290	·250	·265	·305	·255	·295	·245	·245	·300
700	·305	·260	·275	·325	·265	·310	·255	·255	·320
0	·020	·010	·010	·025	·010	·020	·005	·005	·015
700	·315	·265	·275	·335	·265	·315	·260	·255	·320
725	·345	·275	·290	....	·275	·335	·275	·265	·340
750	·445	·290	·300	....	·285	·355	·285	·280	·365
775	....	·310	·315	....	·305	·375	·300	·295	
800	....	·330	·335	....	·325	·395	·320	·315	
0	....	·030	·020	....	·020	·040	·025	·025	
800	....	·340	·340	....	·335	·405	·325	·320	
825	....	....	·365	....	·355	·430	·355	·340	
850	....	....	·390	....	·380	·455	·390	·355	
875	....	....	·465	....	....	....	....	·375	
900	....	....	....	....	....	....	....	·400	
0	....	....	....	....	....	....	....	·045	
900	....	....	....	....	....	....	....	·410	
925	....	....	....	....	....	....	....	·450	
Breaking Weight (*in pounds*).	753·	824·	877·	720·	874·	854·	869·	947·	773·

TABLE XXXI.

TRANSVERSE STRAINS IN WHITE PINE.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705**.

NUMBER OF EXPERIMENT.	79	80	81	82	83	84	85	86	87
DEPTH (*in inches*).	2·11	2·10	2·05	2·09	2·09	2·08	2·06	2·09	2·11
BREADTH (*in inches*).	1·04	1·04	1·03	1·03	1·03	1·03	1·04	1·03	1·03
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).								
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
50	·010	·010	·015	·015	·010	·015	·010	·010	·015
100	·020	·020	·025	·025	·020	·030	·020	·020	·025
150	·030	·030	·040	·035	·030	·040	·030	·030	·035
200	·035	·040	·050	·040	·035	·050	·040	·040	·040
0	·005	·000	·005	·005	·005	·005	·000	·000	·005
200	·035	·040	·050	·040	·035	·050	·040	·040	·040
250	·045	·045	·060	·050	·045	·060	·045	·050	·050
300	·055	·055	·070	·055	·055	·070	·055	·060	·055
350	·060	·060	·080	·065	·060	·080	·065	·065	·065
400	·070	·070	·090	·075	·070	·090	·070	·075	·070
0	·010	·005	·010	·010	·010	·010	·005	·005	·010
400	·070	·070	·090	·075	·070	·090	·070	·075	·070
450	·075	·080	·100	·080	·075	·100	·080	·080	·080
500	·085	·085	·110	·090	·085	·110	·090	·090	·085
550	·095	·095	·120	·095	·090	·120	·100	·100	·095
600	·105	·100	·130	·105	·100	·130	·110	·110	·100
0	·015	·010	·015	·015	·015	·020	·010	·010	·015
600	·105	·100	·130	·105	·100	·130	·110	·110	·100
650	·110	·110	·140	·115	·110	·140	·120	·120	·110
700	·120	·115	·155	·125	·120	·150	·130	·130	·115
750	·125	·125	·165	·135	·130	·160	·140	·140	·125
800	·135	·130	·175	·145	·135	·170	·150	·150	·135

TABLE XXXI.—(*Continued.*)

TRANSVERSE STRAINS IN WHITE PINE.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705**.

NUMBER OF EXPERIMENT.	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**
DEPTH (*in inches*).	2·11	2·10	2·05	2·09	2·09	2·08	2·06	2·09	2·11
BREADTH (*in inches*).	1·04	1·04	1·03	1·03	1·03	1·03	1·04	1·03	1·03
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).								
0	·020	·015	·020	·020	·020	·030	·015	·015	·020
800	·135	·130	·175	·145	·135	·170	·150	·150	·135
850	·140	·140	·190	·150	·145	·180	·160	·160	·140
900	·150	·145	·200	·160	·150	·195	·170	·170	·150
950	·155	·155	·215	·170	·160	·205	·180	·180	·155
1000	·165	·160	·230	·185	·170	·220	·190	·190	·165
0	·025	·020	·035	·025	·025	·035	·020	·025	·025
1000	·165	·160	·235	·185	·170	·220	·190	·195	·165
1050	·175	·170	·250	·195	·180	·235	·200	·205	·175
1100	·180	·180	·275	·205	·190	·250	·215	·215	·185
1150	·185	·190	....	·220	·200	·270	·235	·230	·195
1200	·195	·200	....	·240	·210	·295	·255	·245	·205
0	·030	·025	....	·045	·030	·060	·035	·040	·035
1200	·205	·200	....	·245	·210	·310	·265	·255	·210
1250	·210	·210	....	·270	·220	·340	·285	·275	·220
1300	·220	·220	....	·305	·230	....	·310	·335	·230
1350	·230	·230	....	·390	·245	....	....	....	·245
1400	·240	·240	....	....	·260	....	....	....	·260
0	·040	·030	....	....	·045	....	....	....	·055
1400	·245	·245	....	....	·270	....	....	....	·265
1450	·260	·260	....	....	·300	....	....	....	·290
1500	·285	·280	....	....	....	....	....	....	·315
1550	·315	....	....	....	....	....	....	....	·360
1600	·355								
BREAKING WEIGHT (*in pounds*).	1629·	1536·	1150·	1383·	1500·	1280·	1349·	1303·	1553·

TABLE XXXII.

TRANSVERSE STRAINS IN WHITE PINE.

Length 1 Foot between Bearings.

See Arts. **704** *and* **705**.

Number of Experiment.	278	279	280
Depth (*in inches*).	·251	·253	·258
Breadth (*in inches*).	1·000	1·000	1·000
Pressure (*in pounds*).	**Deflection (*in inches*).**		
0	·000	·000	·000
1	·019	·022	·016
2	·038	·044	·032
3	·057	·066	·048
4	·076	·088	·064
5	·095	·110	·080
6	·114	·132	·096
7	·133	·154	·112
8	·152	·176	·128
9	·171	·198	·144
10	·190	·220	·160
11	·209	·242	·176
12	·228	·264	·192
13	·247	·286	·208
14	·267	·308	·225
15	·286	·330	·241
16	·305	·352	·257
17	·324	·374	·273
18	·343	·396	·290
19	·362	·419	·306
20	·381	·442	·322
21	·400	·466	·339
22	·419	·490	·356
23	·438	·515	·373
24	·457	·541	·390
25	·477	·567	·407
26	·497	·594	·425
27	·518	....	·443
28	·539	....	·462
29	·561	....	·482
30	·583	....	·505
31	·606	....	·531
32	·629	....	·560
33	·654		

Number of Experiment.	281	282	283
Depth (*in inches*).	·498	·502	·503
Breadth (*in inches*).	1·000	1·000	1·000
Pressure (*in pounds*).	**Deflection (*in inches*).**		
0	·000	·000	·000
4	·009	·011	·009
8	·018	·021	·018
12	·026	·032	·027
16	·034	·042	·035
20	·043	·052	·044
24	·051	·062	·053
28	·059	·072	·062
32	·068	·082	·071
36	·076	·092	·079
40	·085	·103	·088
44	·094	·114	·096
48	·102	·125	·105
52	·110	·136	·114
56	·118	·147	·123
60	·127	·157	·132
64	·136	·167	·141
68	·145	·178	·150
72	·154	·189	·159
76	·163	·199	·168
80	·172	·210	·176
84	·181	·221	·185
88	·191	·232	·194
92	·200	·244	·203
96	·209	·256	·213
100	·219	·268	·225
104	·229	·280	·237
108	·239	·294	·250
112	·249	·308	·262
116	·258	·323	·277
120	·268	·338	·296
124	·278	·354	·317
128	·288	·373	
132	·299		
136	·310		
140	·321		
144	·332		
148	·344		
152	·357		
156	·371		
160	·387		

Number of Experiment.	284	285	286
Depth (*in inches*).	·748	·746	·747
Breadth (*in inches*).	1·000	1·000	1·000
Pressure (*in pounds*).	**Deflection (*in inches*).**		
0	·000	·000	·000
10	·007	·009	·007
20	·014	·018	·014
30	·021	·027	·022
40	·029	·036	·029
50	·036	·045	·037
60	·043	·054	·045
70	·050	·063	·052
80	·057	·072	·059
90	·064	·081	·066
100	·072	·090	·074
110	·079	·099	·082
120	·086	·108	·089
130	·093	·118	·097
140	·100	·127	·104
150	·107	·136	·112
160	·114	·146	·119
170	·121	·155	·127
180	·128	·165	·135
190	·135	·176	·143
200	·142	·188	·152
210	·149	·200	·161
220	·156	·213	·171
230	·164	·227	·181
240	·171	·247	·195
250	·179	....	·210
260	·186	....	·233
270	·194		
280	·201		
290	·209		
300	·218		
310	·228		
320	·239		
330	·251		
340	·265		

TABLE XXXIII.

TRANSVERSE STRAINS IN HEMLOCK.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705.**

NUMBER OF EXPERIMENT.	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**
DEPTH (*in inches*).	1·07	1·08	1·07	1·07	1·09	1·10	1·09	1·08	1·11
BREADTH (*in inches*).	1·07	1·06	1·09	1·06	1·05	1·06	1·07	1·08	1·09
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).								
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
25	·050	·050	·030	·025	·035	·025	·040	·035	·040
50	·090	·085	·060	·050	·070	·050	·080	·065	·075
75	·120	·125	·090	·075	·100	·070	·120	·095	·110
100	·150	·165	·115	·105	·135	·095	·160	·125	·145
0	·010	·005	·005	·000	·000	·005	·000	·000	·000
100	·160	·165	·120	·105	·140	·095	·160	·125	·145
125	·190	·210	·150	·130	·170	·120	·200	·155	·185
150	·225	·250	·185	·160	·200	·140	·240	·185	·220
175	·265	·295	·215	·190	·230	·165	·285	·220	·260
200	·300	·340	·245	·220	·260	·190	·330	·250	·300
0	·010	·015	·005	·000	·005	·005	·010	·005	·010
200	·305	·345	·250	·220	·265	·185	·330	·250	·300
225	·340	·400	·285	·250	·305	·210	·380	·285	·340
250	·380	·455	·315	·280	....	·235	·430	....	·385
275	·420	·510	·350	·310	....	·260	·480	....	·430
300	....	....	·395	·345	....	·285	·540	....	·475
0	....	....	·025	·005	....	·010	·045	....	·040
300	....	....	·400	·350	....	·285	·570	....	·490
325	....	....	....	·390	....	·315	....	....	·545
350	....	....	....	....	....	·350			
375	....	....	....	....	....	·385			
400	....	....	....	....	....	·445			
0	....	....	....	....	....	·045			
400	....	....	....	....	....	·465			
425	....	....	....	....	....	·530			
BREAKING WEIGHT (*in pounds*).	292·	277·	324·	350·	234·	433·	313·	238·	348·

TABLE XXXIV.

TRANSVERSE STRAINS IN HEMLOCK.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705**.

NUMBER OF EXPERIMENT.	97	98	99	100	101	102	103	104	105	106
DEPTH (*in inches*).	1·56	1·60	1·60	1·59	1·56	1·60	1·54	1·54	1·58	1·58
BREADTH (*in inches*).	1·04	1·06	1·07	1·03	1·01	1·08	1·09	1·11	1·08	1·09
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).									
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
25	·010	·010	·010	·015	·010	·015	·015	·015	·010	·015
50	·020	·025	·025	·030	·015	·030	·030	·030	·025	·030
75	·030	·035	·035	·040	·025	·045	·045	·045	·040	·045
100	·040	·050	·050	·055	·035	·060	·060	·055	·055	·055
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
100	·040	·050	·050	·055	·035	·060	·060	·055	·055	·055
125	·045	·060	·060	·065	·045	·075	·075	·070	·070	·070
150	·055	·075	·070	·080	·055	·090	·090	·085	·085	·080
175	·065	·085	·080	·090	·065	·105	·105	·100	·100	·090
200	·070	·095	·090	·105	·075	·115	·125	·115	·115	·105
0	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
200	·070	·095	·090	·105	·075	·115	·125	·115	·115	·105
225	·080	·105	·105	·115	·080	·130	·140	·130	·130	·120
250	·085	·115	·120	·130	·090	·145	·155	·145	·140	·130
275	·095	·130	·130	·145	·095	·160	·170	·160	·155	·145
300	·100	·140	·140	·160	·105	·175	·185	·175	·170	·155
0	·005	·000	·005	·000	·000	·005	·005	·000	·000	·000
300	·100	·140	·145	·160	·105	·175	·190	·175	·170	·155
325	·110	·155	·155	·175	·115	·190	·205	·190	·185	·165
350	·120	·170	·170	·190	·125	·210	·225	·205	·200	·175
375	·125	·180	·180	·205	·135	·225	·240	·220	·215	·190
400	·135	·190	·190	·220	·145	·240	·260	·235	·230	·200

TABLE XXXIV.—(*Continued.*)

TRANSVERSE STRAINS IN HEMLOCK.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705.**

NUMBER OF EXPERIMENT.	**97**	**98**	**99**	**100**	**101**	**102**	**103**	**104**	**105**	**106**
DEPTH (*in inches*).	1·56	1·60	1·60	1·59	1·56	1·60	1·54	1·54	1·58	1·58
BREADTH (*in inches*).	1·04	1·06	1·07	1·03	1·01	1·08	1·09	1·11	1·08	1·09
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).									
0	·005	·000	·010	·005	·005	·015	·015	·010	·010	·000
400	·135	·190	·190	·220	·145	·240	·265	·235	·230	·200
425	·145	·205	·205	·240	·150	·255	·285	·255	·250	·215
450	·150	·220	·220	·255	·160	·275	·305	·270	·265	·230
475	·160	·230	·230	·270	·170	·290	·325	·285	·285	·240
500	·170	·240	·245	·295	·175	·310	·345	·305	·305	·255
0	·005	·010	·015	·020	·010	·025	·030	·020	·020	·010
500	·170	·245	·245	·300	·180	·315	·355	·310	·305	·255
525	·180	·260	·260	·340	·190	·335	·380	·330	·325	·270
550	·185	·275	·275	....	·200	·355	·405	·350	·345	·285
575	·195	·300	·290	....	·210	·380	·430	·370	·365	·300
600	·205	....	·305	....	·220	·400	·460	....	·385	·315
0	·010	....	·025	....	·015	·045	·060	....	·030	·020
600	·205	....	·310	....	·225	·405	·470	....	·390	·320
625	·215	....	·325	....	·235	·435	·500	....	·415	·335
650	·225	....	·340	....	·245	·465	....	....	·435	·350
675	·240	....	·360	....	·265	....	....	....	·460	·370
700	·260	....	·380	....	·290	....	....	....	....	·385
0	·020	....	....	....	·030	....	....	....	....	·035
700	·260	....	....	....	·305	....	....	....	....	·390
725	·280	....	....	....	·345	....	....	....	....	·405
750	·295	....	....	....	....	....	....	....	....	·425
775	·360	....	....	....	....	....	....	....	....	·450
BREAKING WEIGHT (*in pounds*).	777·	575·	700·	548·	727·	651·	650·	600·	687·	800·

TABLE XXXV.

TRANSVERSE STRAINS IN HEMLOCK.

LENGTH 1·6 FEET BETWEEN BEARINGS.

See Arts. **704** *and* **705.**

NUMBER OF EXPERIMENT.	107	108	109	110	111	112	113	114	115
DEPTH (*in inches*).	2·02	2·02	2·00	2·03	2·01	2·01	1·99	2·00	2·03
BREADTH (*in inches*).	1·03	1·05	1·03	1·04	1·05	1·04	1·02	1·03	1·05
PRESSURE (*in pounds*).	DEFLECTION (*in inches*).								
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
50	·015	·010	·010	·010	·015	·015	·020	·010	·010
100	·025	·020	·020	·020	·025	·030	·030	·020	·025
150	·035	·035	·030	·035	·035	·045	·045	·035	·035
200	·045	·045	·045	·050	·045	·055	·055	·045	·045
0	·000	·000	·000	·000	·000	·000	·000	·000	·000
200	·045	·045	·045	·050	·045	·055	·055	·045	·045
250	·055	·060	·055	·065	·055	·065	·065	·055	·055
300	·065	·070	·065	·075	·070	·075	·075	·065	·065
350	·075	·085	·075	·090	·085	·085	·090	·075	·075
400	·085	·095	·085	·100	·095	·095	·100	·090	·085
0	·005	·000	·005	·005	·005	·005	·000	·000	·000
400	·085	·095	·085	·100	·095	·095	·100	·090	·085
450	·095	·105	·095	·115	·105	·105	·110	·100	·100
500	·105	·120	·105	·130	·120	·115	.120	·110	·110
550	·115	·130	·115	·145	·135	·125	·130	·120	·120
600	·125	·145	·125	·155	·145	·135	·140	·130	·130
0	·005	·005	·005	·005	·005	·005	·005	·005	·005
600	·125	·145	·125	·155	·145	·135	·140	·130	·130
650	·135	·160	·135	·170	·160	·145	·150	·140	·140
700	·145	·170	·150	·185	·170	·155	·165	·155	·150
750	·160	·185	·165	·200	·185	·165	·175	·165	·165
800	·170	·200	·175	·220	·205	·175	·190	·180	·180
0	·010	·005	·005	·010	·005	·010	·010	·005	·010
800	·170	·200	·175	·220	·205	·175	·190	·180	·180
850	·185	·215	·185	·240	·225	·185	·200	·190	·195
900	·200	·230	·195	·260	·245	·195	·215	·205	·210
950	·215	·250	·210	·285	·265	·210	·235	·215	·225
1000	·230	·275	·220	....	·305	·220	·260	....	·250
0	·025	·025	·010	....	·025	·015	·025	....	·020
1000	·240	·280	·225	....	·310	·220	·270	....	·255
1050	·255	·315	·240	....	....	·235	....	....	·280
1100	·280	·370	·260						
1150	·320	...	·290						
BREAKING WEIGHT (*in pounds*).	1154·	1111·	1181·	991·	1049·	1099·	1036·	985·	1075·

TABLE XXXVI.

TENSILE STRAINS IN GEORGIA PINE.

See Arts. **704** *and* **706**.

Number of Experiment.	116	117	118	119	120	121	122	123	124
Diameter (*in inches*).	·355	·355	·350	·355	·355	·345	·345	·355	·365
Breaking Weight (*in pounds*).	2005·	Less than 1600·	Less than 1300·	2152·	1400·	1924·	1091·	1306·	1268·

TENSILE STRAINS IN LOCUST.

Number of Experiment.	125	126	127	128	129	130	131	132	133
Diameter (*in inches*).	·355	·345	·305	·305	·300	·300	·300	·300	·300
Breaking Weight (*in pounds*).	1137·	2265·	More than 2400·	1592·	2074·	1561·	2131·	1799·	2395·

TENSILE STRAINS IN WHITE OAK.

Number of Experiment.	134	135	136	137	138	139	140	141	142
Diameter (*in inches*).	·355	·365	·345	·355	·305	·300	·305	·300	·300
Breaking Weight (*in pounds*).	1908·	1303·	1182·	2375·	1700·	1442·	1003·	1319·	2205·

TABLE XXXVII.

TENSILE STRAINS IN SPRUCE.

See Arts. **704** *and* **706.**

Number of Experiment.	143	144	145	146	147	148	149	150	151
Diameter (*in inches*).	·305	·305	·300	·305	·305	·305	·355	·365	·360
Breaking Weight (*in pounds*).	1573·	1402·	1560·	1368·	1385·	1533·	1882·	2078·	1600·

TENSILE STRAINS IN WHITE PINE.

Number of Experiment.	152	153	154	155	156	157	158	159	160
Diameter (*in inches*).	·395	·365	·365	·360	·365	·355	·350	·365	·360
Breaking Weight (*in pounds*).	1363·	1157·	1127·	1316·	1431·	1487·	1192·	1024·	1400·

TENSILE STRAINS IN HEMLOCK.

Number of Experiment.	161	162	163	164	165	166	167	168	169
Diameter (*in inches*).	·365	·355	·345	·360	·355	·355	·350	·335	·355
Breaking Weight (*in pounds*).	645·	897·	864·	999·	863·	726·	895·	809·	977·

TABLE XXXVIII.

SLIDING STRAINS IN GEORGIA PINE.

See Arts. **704** *and* **706.**

NUMBER OF EXPERIMENT.	170	171	172	173	174	175	176	177	178
DIAMETER (*in inches*).	·525	·520	·530	·530	·520	·520	·525	·520	·530
LENGTH (*in inches*).	1·065	Broke in two.	1·020	1·010	Broke in two.	1·040	1·020	1·015	1·050
BREAKING WEIGHT (*in pounds*).	1546·	1295·	1411·	1571·	1281·	1347·	1520·	1401·	1247·

SLIDING STRAINS IN LOCUST.

NUMBER OF EXPERIMENT.	179	180	181	182	183	184	185	186	187
DIAMETER (*in inches*).	·530	·530	·535	·525	·525	·530	·535	·525	·525
LENGTH (*in inches*).	·735	·730	·715	·745	Broke in two.	·760	·730	·715	·760
BREAKING WEIGHT (*in pounds*).	1490·	1236·	1533·	1192·	1492·	1758·	1403·	1331·	1483·

SLIDING STRAINS IN WHITE OAK.

NUMBER OF EXPERIMENT.	188	189	190	191	192	193	194	195	196
DIAMETER (*in inches*).	·530	·525	·535	·540	·535	·530	·530	·535	·530
LENGTH (*in inches*).	·730	·755	·740	·750	·750	·740	·725	·725	·730
BREAKING WEIGHT (*in pounds*).	1308·	1801·	1834·	1502·	1701·	1359·	1667·	1321·	1399·

TABLE XXXIX.

SLIDING STRAINS IN SPRUCE.

See Arts. **704** *and* **706.**

Number of Experiment.	197	198	199	200	201	202	203	204	205
Diameter (*in inches*).	·565	·535	·550	·525	·550	·550	·545	·550	·550
Length (*in inches*).	1·010	·990	1·010	1·010	1·030	1·020	1·005	1·010	·990
Breaking Weight (*in pounds*).	988·	770·	1130·	882·	927·	976·	1043·	838·	902·

SLIDING STRAINS IN WHITE PINE.

Number of Experiment.	206	207	208	209	210	211	212	213	214
Diameter (*in inches*).	·540	·545	·555	·545	·545	·545	·550	·545	·545
Length (*in inches*).	·995	1·000	·990	1·010	1·025	1·005	1·010	1·040	·995
Breaking Weight (*in pounds*).	730·	907·	792·	803·	842·	800·	881·	852·	885·

SLIDING STRAINS IN HEMLOCK.

Number of Experiment.	215	216	217	218	219	220	221	222	223
Diameter (*in inches*).	·540	·540	·545	·540	·530	·540	·540	·535	·530
Length (*in inches*).	·995	1·010	·995	Broke in two.	1·025	1·015	·995	1·010	·990
Breaking Weight (*in pounds*).	607·	702·	620·	796·	700·	674·	556·	627·	530·

TABLE XL.

CRUSHING STRAINS IN GEORGIA PINE.

See Arts. **704** *and* **707.**

Number of Experiment.	224	225	226	227	228	229	230	231	232
Diameter (*in inches*).	·515	·515	·520	·520	·505	·515	·510	·500	·515
Length (*in inches*).	1·035	1·025	1·040	1·035	1·035	·505	·515	·505	·510
Breaking Weight (*in pounds*).	2010·	1878·	2061·	1735·	2304·	2002·	1845·	1705·	2141·

CRUSHING STRAINS IN LOCUST.

Number of Experiment.	233	234	235	236	237	238	239	240	241
Diameter (*in inches*).	·520	·520	·520	·525	·530	·520	·525	·515	·520
Length (*in inches*).	1·055	1·020	1·035	1·045	·490	·515	·500	·490	·495
Breaking Weight (*in pounds*).	2338·	2391·	2547·	2539·	2695·	2500·	2495·	2374·	2672·

CRUSHING STRAINS IN WHITE OAK.

Number of Experiment.	242	243	244	245	246	247	248	249	250
Diameter (*in inches*).	·525	·530	·520	·530	·525	·520	·525	·520	·515
Length (*in inches*).	1·035	1·035	1·035	1·030	1·035	·505	·500	·485	·515
Breaking Weight (*in pounds*).	1546·	1978·	1992·	1455·	1989·	1650·	2116·	1387·	1376·

TABLE XLI.

CRUSHING STRAINS IN SPRUCE.

See Arts. **704** *and* **707**.

Number of Experiment.	251	252	253	254	255	256	257	258	259
Diameter (*in inches*).	·535	·535	·535	·535	·535	·530	·540	·530	·530
Length (*in inches*).	1·035	1·025	1·040	1·030	·490	·480	·485	·495	·490
Breaking Weight (*in pounds*).	1692·	1715·	1611·	1633·	1871·	1818·	1812·	1855·	1832·

CRUSHING STRAINS IN WHITE PINE.

Number of Experiment.	260	261	262	263	264	265	266	267	268
Diameter (*in inches*).	·540	·525	·535	·515	·530	·535	·525	·510	·540
Length (*in inches*).	1·035	1·035	1·040	1·040	1·030	·495	·490	·505	·495
Breaking Weight (*in pounds*).	1454·	1536·	1473·	1322·	1297·	1503·	1624·	1353·	1540·

CRUSHING STRAINS IN HEMLOCK.

Number of Experiment.	269	270	271	272	273	274	275	276	277
Diameter (*in inches*).	·520	·520	·525	·530	·530	·520	·520	·525	·520
Length (*in inches*).	1·035	1·030	1·030	1·030	·480	·525	·520	·495	·490
Breaking Weight (*in pounds*).	1137·	1178·	1130·	1156·	1150·	1334·	1290·	1317·	1320·

TABLE XLII.

TRANSVERSE STRAINS.

BREAKING WEIGHTS (*in pounds*) PER UNIT OF MATERIAL, $= B$.

See Arts. **704** *and* **708.**

GEORGIA PINE. 1" × 1".	LOCUST. 1" × 1".	WHITE OAK. 1" × 1".	SPRUCE. 1" × 1".	SPRUCE. 1" × 1½".	SPRUCE. 1" × 2".	WHITE PINE. 1" × 1".	WHITE PINE. 1" × 1½".	WHITE PINE. 1" × 2".	HEMLOCK. 1" × 1".	HEMLOCK. 1" × 1½".	HEMLOCK. 1" × 2".
950·	664·	686·	576·	604·	540·	578·	504·	563·	381·	491·	439·
1023·	555·	533·	654·	650·	569·	616·	569·	536·	358·	339·	415·
758·	1406·	660·	533·	574·	627·	480·	590·	425·	415·	409·	459·
951·	1286·	626·	694·	598·	642·	576·	482·	492·	461·	337·	370·
945·	1283·	476·	632·	538·	552·	542·	595·	533·	300·	473·	396·
1014·	1156·	567·	637·	652·	630·	566·	609·	460·	540·	377·	418·
993·	1342·	536·	702·	660·	637·	564·	582·	489·	394·	402·	410·
785·	530·	501·	593·	614·	654·	619·	643·	463·	302·	365·	383·
949·	1212·	664·	627·	592·	572·	639·	552·	542·	415·	408·	398·
	1281·	529·	722·	642·	576·					470·	
..	952·										
AVERAGE BREAKING WEIGHTS, $= B$.											
930·	1061·	578·	637·	612·	600·	576·	570·	500·	396·	407·	410·

TABLE XLIII.

DEFLECTION.

Values of Constant, *F*.

See Arts. **704** *and* **705**.

Georgia Pine. 1″×1″.	Locust. 1″×1″.	White Oak. 1″×1″.	Spruce. 1″×1″.	Spruce. 1″×1½″.	Spruce. 1″×2″.	White Pine. 1″×1″.	White Pine. 1″×1½″.	White Pine. 1″×2″.	Hemlock. 1″×1″.	Hemlock. 1″×1½″.	Hemlock. 1″×2″.
5155·	4983·	2599·	3649·	3329·	2577·	3088·	2746·	2484·	2083·	3112·	2316·
6302·	4645·	2033·	3640·	2909·	2299·	3378·	3191·	2658·	1859·	1986·	1958·
5199·	5616·	2360·	2215·	2679·	2962·	2806·	2891·	2130·	2667·	2077·	2386·
5555·	5000·	2057·	3867·	2965·	3105·	3124·	2624·	2489·	2868·	2940·	1822·
5498·	5442·	1704·	3384·	2766·	2539·	2940·	3176·	2581·	2259·	3052·	1988·
6007·	4998·	1837·	2932·	3291·	3382·	2850·	2932·	2040·	3056·	1606·	2190·
6007·	5239·	1907·	4004·	3302·	3127·	3257·	3101·	2371·	1847·	1660·	2184·
4807·	4312·	1842·	3572·	3344·	3191·	3267·	3225·	2323·	2441·	1725·	2294·
6336·	5239·	2253·	3678·	3714·	2176·	3338·	2919·	2509·	1895·	1683·	2152·
	5033·	1930·	3967·	3387·	2682·					1864·	
	4952·										
Average Values of Constant, *F*.											
5652·	5042·	2052·	3491·	3169·	2804·	3116·	2978·	2398·	2331·	2170·	2143·

TABLE XLIV.

TENSILE STRAINS.

BREAKING WEIGHTS (*in pounds*) PER SQUARE INCH OF SECTIONAL AREA, = *T*.

See Arts. 704 *and* 706.

GEORGIA PINE.	LOCUST.	WHITE OAK.	SPRUCE.	WHITE PINE.	HEMLOCK.
20257·	11487·	19277·	21530·	11123·	6164·
.....	24229·	12453·	19189·	11057·	9062·
.....	.....	12644·	22069·	10771·	9242·
21742·	21790·	23995·	18724·	12929·	9815·
14144·	29341·	23268·	18957·	13676·	8719·
20582·	22084·	20400·	20982·	15023·	7335·
11671·	30147·	13728·	19014·	12389·	9302·
13195·	25451·	18660·	19860·	9786·	9178·
12118·	33882·	31194·	15719·	13754·	9871·
AVERAGE WEIGHTS PRODUCING RUPTURE, = *T*.					
16244·	24801·	19513·	19560·	12279·	8743·

TABLE XLV.

SLIDING STRAINS.

BREAKING WEIGHTS (*in pounds*) PER SQUARE INCH OF SLIDING SURFACE, = G.

See Arts. 704 *and* 706.

GEORGIA PINE.	LOCUST.	WHITE OAK.	SPRUCE.	WHITE PINE.	HEMLOCK.
6706.	9189.	8122.	3902.	3204.	2664.
.....	7675.	11019.	3460.	3870.	3035.
6270.	9538.	11025.	4709.	3307.	2671.
7050.	7391.	8744.	4034.	3408.	.....
.....	.....	10089.	3788.	3521.	3095.
6099.	10485.	8324.	4028.	3412.	2899.
6884.	8549.	10422.	4449.	3672.	2440.
6499.	8599.	8105.	3492.	3512.	2762.
5383.	9014.	8687.	3835.	3813.	2427.
AVERAGE RESISTANCE TO RUPTURE PER SQUARE INCH, = G.					
6413.	8805.	9393.	3966.	3524.	2749.

TABLE XLVI.

CRUSHING STRAINS.

CRUSHING WEIGHTS (*in pounds*) PER SQUARE INCH OF SECTIONAL AREA, = *C*.

See Arts. **704** *and* **707**.

GEORGIA PINE.	LOCUST.	WHITE OAK.	SPRUCE.	WHITE PINE.	HEMLOCK.
9649·	11009·	7142·	7527·	6349·	5354·
9015·	11259·	8966·	7629·	7095·	5547·
9705·	11993·	9380·	7166·	6552·	5220·
8170·	11729·	6595·	7264·	6346·	5240·
11503·	12216·	9188·	8323·	5879·	5213·
9611·	11772·	7769·	8240·	6686·	6281·
9032·	11525·	9775·	7912·	7502·	6074·
8683·	11396·	6531·	8408·	6623·	6084·
10278·	12582·	6606·	8304·	6724·	6216·
AVERAGE RESISTANCE TO CRUSHING PER SQUARE INCH, = *C*.					
9516·	11720·	7995·	7864·	6640·	5692·

DIRECTORY,

OR

DIGEST OF THE PRINCIPAL RULES.

BELOW may be found the numbers of such formulas, articles, figures and tables as are particularly applicable in any given problem.

By reference to these, the rules needed in any certain case, occurring in practice, may be more readily found than by either the index or table of contents.

LEVERS—WOOD.

Load at end.	Rupture.	Strain at wall,	(*6.*)
		" " any point, *Figs.* 27, 28, 33,	(*42.*), (*43.*)
		Size when at the point of rupture,	(*15.*)
		" to resist rupture safely,	(*19.*), (*36.*)
	Flexure.	Weight,	(*123.*)
		Length,	(*127.*)
		Breadth,	(*128.*)
		Depth,	(*129.*)
		Deflection,	(*121.*)

LEVERS—ROLLED-IRON.

SINGLE BEAMS—WOOD.

2 loads.	Rupture.	Strain at any point,	*Figs.* 36, 37, (*53.*), (*54.*), (*55.*)
		" " locations of weights,	*Art.* **153**, (*51.*), (*52.*)
		Size to resist rupture safely,	(*30.*), (*31.*), (*56.*), (*57.*)
3 loads.		Strain at any point,	*Fig.* 38, (*61.*), (*62.*), (*63.*), (*64.*)
		" " locations of weights,	(*58.*), (*59.*), (*60.*)
		Size to resist rupture safely,	(*65.*), (*66.*), (*67.*)

Loaded promiscuously.	Rupture.	Strain at any point,	*Figs.* 52, 53, (*81.*), (*95.*), (*98.*)
	Rupture.	Size to resist safely,	(*82.*), (*91.*), (*96.*), (*99.*)
	Flexure.	" " " "	(*189.*), (*193.*)

Loaded symmetrically.	Rupture.	Strains,	*Figs.* 39, 40, 41, *Arts.* **196** to **203**, **210**, **211**

SINGLE BEAMS—ROLLED-IRON.

Load at middle.	Flexure.	General rule,	(*216.*)
		Weight,	(*217.*)
		Length,	(*218.*)
		Deflection,	(*219.*)
		Moment of inertia,	(*220.*)

Load at any point.	Flexure.	Weight,	(*223.*)
" " " "	"	Size,	(*221.*), (*222.*)

Load uniformly distributed.	Flexure.	Weight,	(*228.*)
" " "	"	Size,	(*227.*)

FLOOR TIMBERS—WOOD.

FLOOR BEAMS—ROLLED-IRON.

FRAMED GIRDERS.

TUBULAR IRON GIRDERS.

CAST-IRON GIRDERS.

ROOF TRUSSES.

FLOOR-ARCHES—TIE-RODS.

SHEARING.

PROMISCUOUS.

INDEX.

ANSWERS TO QUESTIONS.

37.—Transverse.

38.—In proportion directly as the breadth.

39.—In proportion directly as the square of the depth.

40.—The elements are the strength of the unit of material, the area of cross-section and the depth.

The expression is $R = Bbd^2$.

41.—The amount is equal to the total load.

42.—One half.

43.—The sum is equal to the total load.

44.—The portion of the weight borne at either point is equal to the product of the weight into its distance from the other support, divided by the length between the two supports.

45.—$R = W\frac{n}{l}$

46.—$P = W\frac{m}{l}$

47.—10000 pounds.

5000 pounds.

48.—The moment, or the product of half the weight into half the length of the beam.

49.— $Wl = Bbd^2$.

50.—$22666\frac{2}{3}$ pounds.

51.—As many times as the breadth is contained in the depth.

62.—$22781\frac{1}{4}$ pounds.

63.—40500 pounds.

64.—20250 pounds.

65.—$5062\frac{1}{2}$ pounds.

84.—Depth, 6·6 inches; breadth, 3·3 inches.

85.—5·24 inches square.

86.—6·93 inches.

87.—4 inches.

126.—2 feet $10\frac{3}{4}$ inches.

127.—2 feet $4\frac{1}{2}$ inches.

128.—2 feet $7\frac{5}{8}$ inches.

129.—2 feet $1\frac{3}{8}$ inches.

130.—1 foot $8\frac{3}{4}$ inches.

131.—1 foot $11\frac{1}{8}$ inches.

132.—2 feet 0 inches.

133.—1 foot $7\frac{5}{8}$ inches.

134.—1 foot $9\frac{3}{4}$ inches.

135.—3 feet 1 inch.

160.—Breadth, 6·78 inches; depth, 12·34 inches.

161.—2·94 inches.

162.—2·86 inches.

163.—0·291$\frac{2}{3}$ inches.

164.—1·763 inches.

165.—1·244 inches.

166.—3·111 inches.

179.—36720 foot-pounds.

180.—36·72 inches.

181.—

	Ordinates.	Strains.
For 5 ft.,	18·36	18360 foot-pounds.
" 6 "	22·032	22032 " "
" 7 "	25·704	25704 " "
" 8 "	29·376	29376 " "
" 9 "	33·048	33048 " "

182.—10·733, 10·182 and 9·6 inches respectively.

183.—6·245, 8·062, 9·539 and 10·198 inches respectively.

184.—Depth, 8·1565 inches.
Weight, 652·218 pounds.
Shearing strain at wall, 733·783 pounds.
" " " 5 ft. from wall, 699·798 pounds.

185.—302·222 pounds.

186.—Shearing strain, 4973$\frac{4}{9}$ pounds.
Height, 0·93$\frac{1}{4}$ inches.

187.—1·46 inches.

204.—10666$\frac{2}{3}$ pounds.

205.—2666$\frac{2}{3}$, 5333$\frac{1}{3}$ and 8000 pounds.

206.—Strain at *A*, 17142$\frac{6}{7}$; at *B*, 22285$\frac{5}{7}$.

207.—$8571\frac{3}{7}$, $7428\frac{4}{7}$ and 21000 pounds respectively.

208.—12750, 22750, 19000, 8500, 9500, 20500 and 21500 pounds respectively.

209.—3920 pounds.

217.—A parabolic curve.

218.—800, 1050 and 1200 pounds.

219.—5·1087 inches.

220.—Elliptical.

228.—0, 300, 600, 900 and 1700 pounds.
At the wall 2500 pounds.

229.—A parabolic curve.

230.—1250 pounds.

231.—Triangular.

232.—3450, 6250 and 9450 pounds.

233.—200, 1600, 6150 and 11050 pounds.

269.—19200 pounds; located at the concentrated weight.

270.—7·01 inches.

287.—Resistance to flexure.

288.—To any amount within the limits of elasticity.

289.—The extensions are directly as the forces.

290.—The deflections are directly as the extensions.

291.—The deflections are as the weights into the cube of the lengths.

307.—By the power of reaction.

308.—To the number of fibres, to the distance they are extended, and to the leverage with which they act.

309.— $Wl^2 = Fbd^2\delta$.

310.—0·266 of an inch.

315.—The rules for strength are the more simple.

316.— $\delta = \frac{72el^2}{d}$

317.— $e = \frac{d\delta}{72l^2}$

318.— $a = \frac{B}{72Fe}$

319.— $r = \frac{72cl}{d}$

354.—Formulas (*122.*), (*124.*), (*125.*), (*126.*) and (*120.*).

355.—Formulas (*123.*), (*127.*), (*128.*), (*129.*) and (*121.*).

356.—Formulas (*131.*), (*132.*), (*133.*), (*134.*) and (*135.*).

357.—Formulas (*136.*), (*137.*), (*138.*), (*139.*) and (*140.*).

437.—12·345 inches.

438.—4·176 inches.

439.—6·16 inches.

440.—8·185 inches.

441.—10·453 inches.

442.—9·197 inches.

537.— $\frac{1}{12}bd^3$ (*form. 205.*).

538.— $\frac{1}{12}(bd^3 - b_1d_1^3)$ (*form. 213.*).

539.—The Buffalo 12¼ inch 180 pound beam.

540.—9475·58 pounds.

541.—2004·52 pounds.

542.—The Trenton 9 inch 85 pound beam.

543.—Two Trenton 12¼ inch 125 pound beams.

544.—It should be a 15 inch 200 pound beam.

545.—It should be a 10½ inch 135 pound beam.

574.—41⅝ inches.

575.—35 inches.

576.—At 5 feet from the end of girder, 15 inches each;
" 10 " " " " " " 26⅝ " "
" 15 " " " " " " 35 " "
" 20 " " " " " " 40 " "
" 25 " or at middle, 41⅝ " "

577.—At end of girder, 0·38 inch;
" 5 feet from end of girder, 0·30 "
" 10 " " " " " 0·23 "
" 15 " " " " " 0·15 "
" 20 " " " " " 0·08 "
" 25 " or at middle, 0·0 "

578.—At 5 feet from end of girder, 8·95 inches;
" 10 " " " " " 15·34 "
" 15 " " " " " 19·18 "
" 20 " or at middle, 20·46 "

579.—4·2155 feet.

597.—Bottom flange, 16 × 2·195 inches;
Top " 5⅛ × 1·646 "
Web, 1·372 " thick.

598.—Bottom flange, 16 $\times$ 1·646 inches;
Top " $5\frac{1}{8} \times$ 1·234 "
Web, 1·029 " thick.

599.—Bottom flange, 16 $\times$ 2·49 inches;
Top " $5\frac{1}{8} \times$ 1·867 "
Web, 1·556 " thick.

600.—32·99 inches.

601.—At the location of the 25000 pounds;
The bottom flange, 16 $\times$ 1·663 inches;
" top " $5\frac{1}{8} \times$ 1·247 "
" web, 1·039 " thick.
At the location of the 30000 pounds;
The bottom flange, 16 $\times$ 1·588 inches;
" top " $5\frac{1}{8} \times$ 1·191 "
" web, 0·992 " thick.

602.—3·68 inches.

651.—The strain in *AB* is 3550 pounds;
" " " *BC* " 10280 "
" " " *EA* " 10740 "
" " " *AF* " 15240 "
" " " *BG* " 10130 "

652.—7·8125 feet.

653.—Six.

654.—The strain in *DE* is 3600 pounds;
" " " *CD* " 5425 "
" " " *BC* " 12700 "
" " " *AB* " 14500 "
" " " *KA* " 21725 "
" " " *AU* " 15700 "
" " " *CT* " 35275 "

The strain in *ES* is 41800 pounds;
“ “ “ *BL* “ 26125 “
“ “ “ *DM* “ 39200 “

655.—The strain in *DE* is 3614 pounds;
“ “ “ *CD* “ 5420 “
“ “ “ *BC* “ 12647 “
“ “ “ *AB* “ 14454 “
“ “ “ *KA* “ 21681 “
“ “ “ *AU* “ 15655 “
“ “ “ *CT* “ 35223 “
“ “ “ *ES* “ 41746 “
“ “ “ *BL* “ 26091 “
“ “ “ *DM* “ 39137 “

656.—The area at *AU* should be 16·103 inches;
“ “ “ *CT* “ “ 36·180 “
“ “ “ *ES* “ “ 42·872 “
The size of *BL* “ “ 6·626 × 7·951 inches;
“ “ “ *DM* “ “ 7·715 × 9.258 “
“ “ “ *DE* “ “ 3·004 × 3·605 “
“ “ “ *BC* “ “ 4·612 × 5·534 “
“ “ “ *KA* “ “ 5·651 × 6·781 “
The area of *CD* “ “ 0·603 inches;
“ “ “ *AB* “ “ 1·611 “

688.—The computed strain in *AG* is 22535 pounds;
“ “ “ “ *BH* “ 18028 “
“ “ “ “ *AB* “ 4507 “
“ “ “ “ *AF* “ 18750 “
“ “ “ “ *BC* “ 15000 “
“ measured “ “ *AG* “ 22500 ‘
“ “ “ “ *BH* “ 18000 ‘
“ “ “ “ *AB* “ 4500 “
“ “ “ “ *AF* “ 18750 “
“ “ “ “ *BC* “ 15000 “

689.—The strain in			*AN*	is	44200	pounds;
"	"	"	*BO*	"	38720	"
"	"	"	*DP*	"	29160	"
"	"	"	*AB*	"	5400	"
"	"	"	*CD*	"	13300	"
"	"	"	*AM*	"	36760	"
"	"	"	*CL*	"	32240	"
"	"	"	*BC*	"	10000	"
"	"	"	*DE*	"	26320	"

690.—The strain in			*AJ*	is	41000	pounds;
"	"	"	*BK*	"	36150	"
"	"	"	*AB*	"	4950	"
"	"	"	*AH*	"	32400	"
"	"	"	*BC*	"	24700	"
"	"	"	*CG*	"	14500	"

691.—The strain is 16000 pounds.

692.—Six.

That shown in *Fig.* 115.

The strain in			*AO*	is	96200	pounds;
"	"	"	*BP*	"	88200	"
"	"	"	*CH*	"	53000	"
"	"	"	*CQ*	"	25700	"
"	"	"	*DR*	"	17600	"
"	"	"	*AB*	"	8000	"
"	"	"	*CD*	"	8000	"
"	"	"	*AN*	"	81600	"
"	"	"	*HM*	"	74800	"
"	"	"	*DE*	"	10200	"
"	"	"	*BH*	"	24700	"

AO	should be	$9 \times 15 \cdot 80$	} 9×16 tapered to 9×14	
BP	" "	$9 \times 14 \cdot 49$		
CH	" "	$9 \times 13 \cdot 41$,	or	9×14
CQ	" "	$6 \times 8 \cdot 61$	} 6×9 tapered to 6×6	
DR	" "	$6 \times 5 \cdot 89$		
AB	" "	$4 \times 6 \cdot 41$,	or	4×7
CD	" "	$4 \times 6 \cdot 41$	"	4×7
AN	" "	$8 \cdot 83 \times 11 \cdot 77$	"	9×12
HM	" "	$14 \cdot 85 \times 19 \cdot 80$	"	15×20
DE	" "	$1 \cdot 133$ area,	or	$1\frac{1}{4}$ diameter.
BH	" "	$2 \cdot 744$ "	"	2 "